

CAMPI MAGNETOSTATICI

$$\begin{cases} \operatorname{div} \vec{B} = 0 \\ \operatorname{rot} \vec{H} = \vec{J} \end{cases} \quad B = \mu H$$

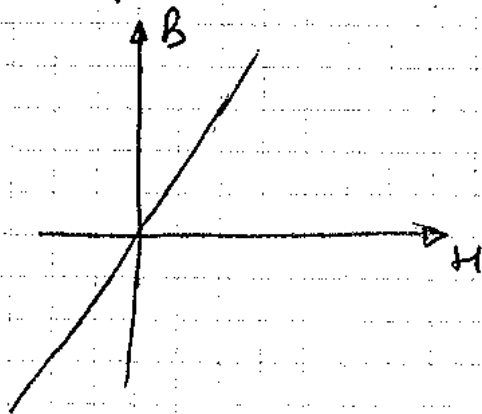
μ permeabilità magnetica $[\frac{H}{m}]$

B [T], H $[\frac{A}{m}]$

- Materiali:
- feromagnetici $\mu/\mu_0 \gg 1$ $(10^3 \div 10^5)$
 - paramagnetici $\mu/\mu_0 > 1$
 - diamagnetici $\mu/\mu_0 < 1$
 - vuoto (o vuoto) $\mu = \mu_0 = 4\pi \cdot 10^{-7} \frac{H}{m}$

$$\mu = \mu_r \mu_0$$

Comportamento di feromagnetici.



Se μ è scalare i materiali sono isotropi.

$$\begin{cases} B_x = \mu H_x \\ B_y = \mu H_y \\ B_z = \mu H_z \end{cases}$$

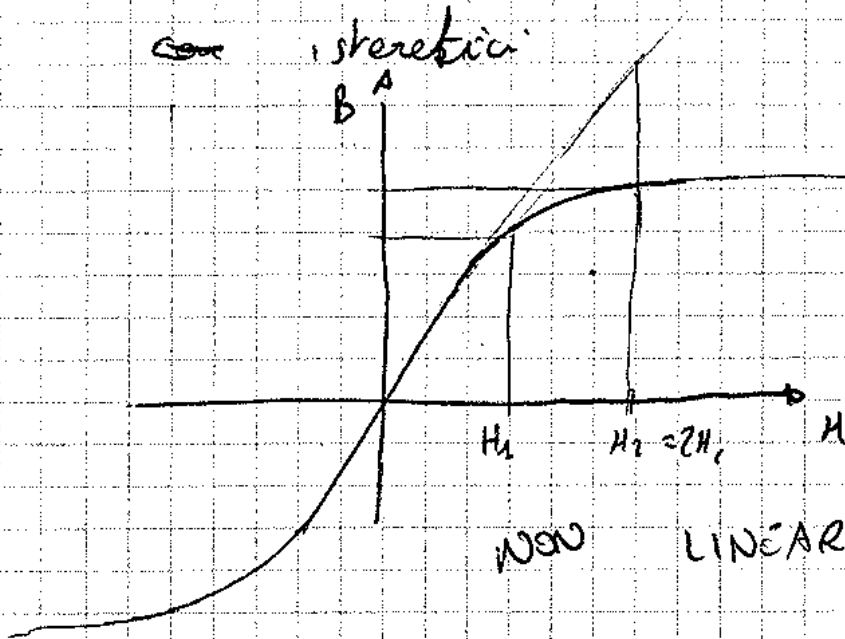
Se μ è matrice i materiali sono anisotropi o cristalli orientati.

Caso ferromagneto

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}$$

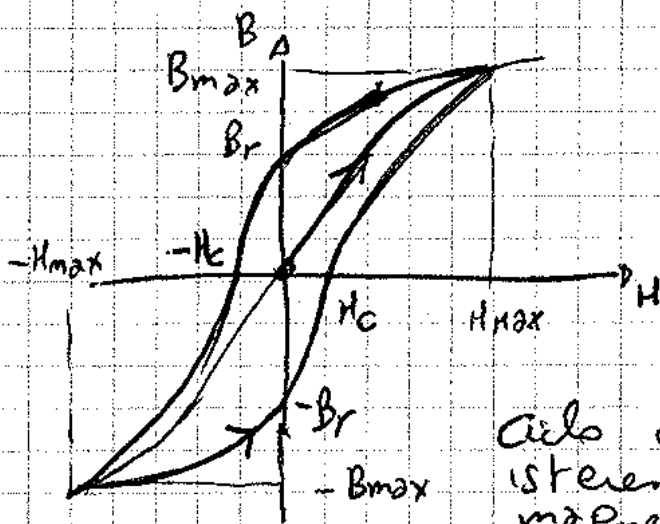
I materiali hanno comportamento non lineare e

isteretici



$$B = \mu H = \mu(H) H$$

$$B = f(H)$$



H_c campo coercitivo
 B_r induzione residua

anello di isteresi magnetica

Il campo magnetostatico non è più irrotazionale

$$\operatorname{div} \vec{B} = 0 \Rightarrow \vec{B} = \operatorname{rot} \vec{A}$$

$$\operatorname{rot} \vec{H} = \operatorname{rot} \frac{\vec{B}}{\mu} = \vec{J} \Rightarrow \operatorname{rot} \left(\frac{1}{\mu} \operatorname{rot} \vec{A} \right) = \vec{J}$$

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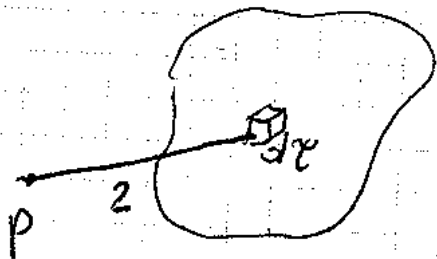
Se $\mu = \text{cost}$

$$\operatorname{rot}(\operatorname{rot} \vec{A}) = \mu \vec{J} = \operatorname{grad} \operatorname{div} \vec{A} - \nabla^2 \vec{A}$$

Però $\operatorname{div} \vec{A} = 0$ $-\nabla^2 \vec{A} = \mu \vec{J}$

$$\vec{A} = -\frac{1}{4\pi} \int_{\tau} \frac{\nabla^2 \vec{A}}{r} d\tau \quad \text{tes Green}$$

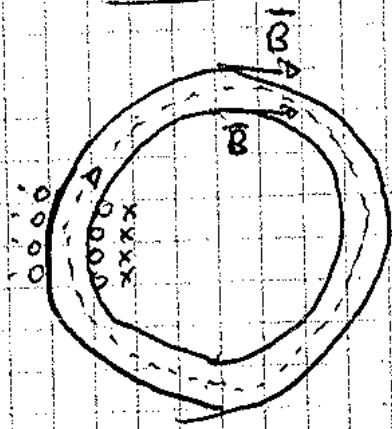
$$= -\frac{1}{4\pi} \int_{\tau} \frac{\mu \vec{J}}{r} d\tau$$



Dividiamo lo studio di campi magnetostatici in 2 ambiti:

- correnti magnetiche
- domini a μ costante (regioni aperte con conduttori)

CIRCUITI MAGNETICI



fuori del materiale
ferromagnetico

$$\vec{B} = 0$$

$$\text{div } \vec{B} = 0$$

$B_{n1} = B_{n2}$ nel
passeggio tra
due pezzi!

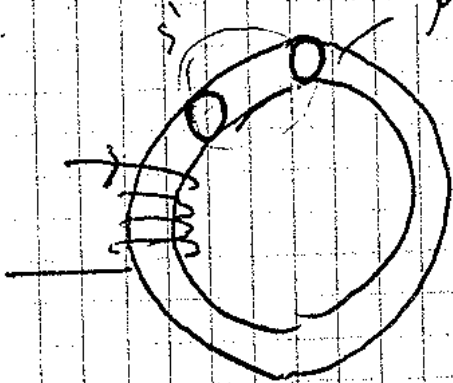
$\text{rot } \vec{H} = \vec{J} \Rightarrow H_{t1} = H_{t2}$ nel passaggio tra due pezzi!

$$\int_{\tau} \text{div } \vec{B} d\tau = \oint_{\partial\tau} \vec{B} d\vec{s} = 0$$

$$\int_S \text{rot } \vec{H} d\vec{s} = \oint_S \vec{J} d\vec{s} - \oint_{\partial S} \vec{H} d\vec{l}$$

Chiamiamo $\Phi_i = \int_{S_i} \vec{B} d\vec{s}$ $\sum_{i=1}^n \Phi_i = 0$

$\mu \gg \mu_0$



$$\int_{S_1} \vec{B} d\vec{s} = \int_{S_{\text{est}}} \vec{B} d\vec{s} + \int_{S_2} \vec{B} d\vec{s} = 0 \Rightarrow$$

$$\Rightarrow \int_{S_1} \vec{B} d\vec{s} = - \int_{S_2} \vec{B} d\vec{s}$$

$$\Phi_{\text{entrato}}_{S_1} = \Phi_{\text{uscito}}_{S_2}$$

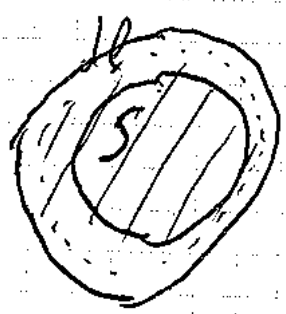
$$\int_{\text{linea di comp}} \vec{n} d\vec{l} = \int_{\text{linea di comp}} \mu dl = \int \frac{B}{\mu} dl *$$

$$\Phi = \int_S \vec{B} \cdot d\vec{s} = \int_S B ds = BmS \text{ in alcuni casi}$$

$$\approx BS \Rightarrow B = \frac{\Phi}{S}$$

$$= \int \frac{\Phi}{\mu S} dl = \Phi \int \frac{dl}{\mu S} \text{ resistenza (R)}$$

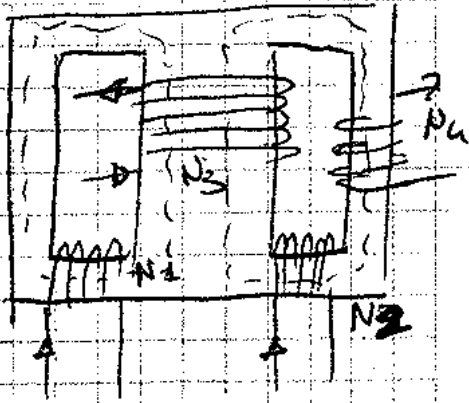
Se $\mu = \text{cost}$ e $S = \text{cost}$ allora
 $\propto \int \frac{dl}{\mu S}$



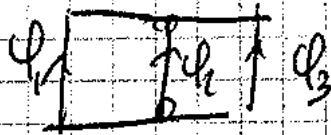
$$\int_S \vec{B} \cdot d\vec{s} = \int_{\text{cond}} \vec{B} \cdot d\vec{s} + \int_{\text{sta. cond.}} \vec{B} \cdot d\vec{s} = NI$$

C no spire

$$\Phi_R = NI \cdot f \text{ forte magnetismo}$$



lato o ass dell'arresto magnetico è una serie del circuito ottocorde delle stime Cues di fluso



le equazioni si applicano in serie.

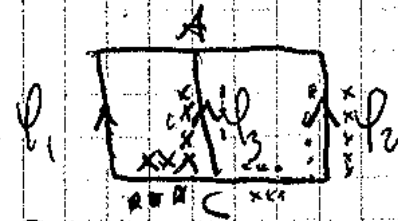
Equazioni di maglie

$$\oint H dl = \oint \frac{\Phi}{\mu S} dl = \sum_{x=1}^M \int \frac{\Phi_x dl_x}{\mu S_x} =$$

$$\sum_{x=1}^M \Phi_x R_x$$

$$\int_S \vec{J} \cdot d\vec{l} = \sum_{i=1}^{M'} N_i I_i$$

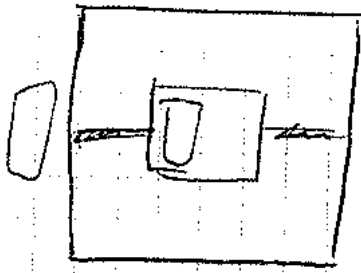
In questo caso



Eg. nodo: $\Phi_1 + \Phi_3 + \Phi_2 = 0$

Cp maglie: $R_1 \Phi_1 - R_3 \Phi_3 = N_1 I_1 + N_2 I_2 + N_3 I_3$

$R_1 \Phi_1 - R_2 \Phi_2 = N_1 I_1 - N_2 I_2 - N_3 I_3$

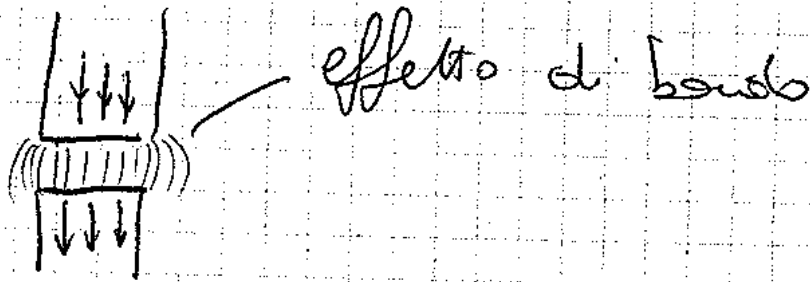


Se rimane una strofinella
di aria si crea

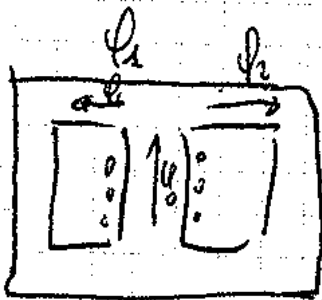
$$R = \left(\frac{l_{\text{c}} \mu_{\text{Fe}}}{\mu_{\text{Fe}} S_{\text{Fe}}} \right) + \left(\frac{2l_{\text{aio}}}{\mu_0 S_{\text{aio}}} \right)$$

In questo caso si può mischiare

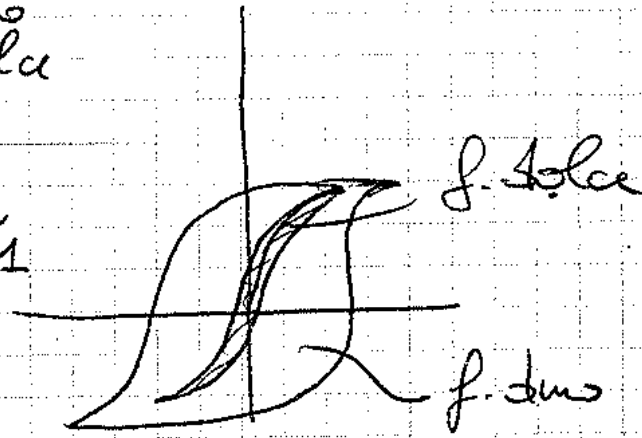
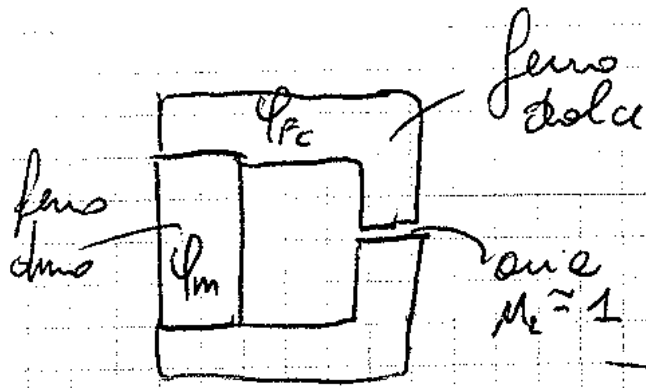
$R_{\text{Fe}} < R_{\text{aio}}$ in alcuni casi anche
+ trascurabile R_{Fe}



No: trascurare l'effetto di bobina.



I_1 e I_2 sono in parallelo



$$\Phi_{Fe} = \Phi_m = \Phi_e$$

⇓

$$B_c S_c = B_m S_m = B_e S_e$$

$$\oint H dl = 0 \quad H_m l_m + H_e l_e + H_c l_c = 0$$

$$H_m \mu_m + H_e \mu_e + \mu_p \frac{H_e S_e}{\mu_0 S_c} = 0$$

$$H_c = \mu_p \frac{H_e S_e}{\mu_0 S_c}$$

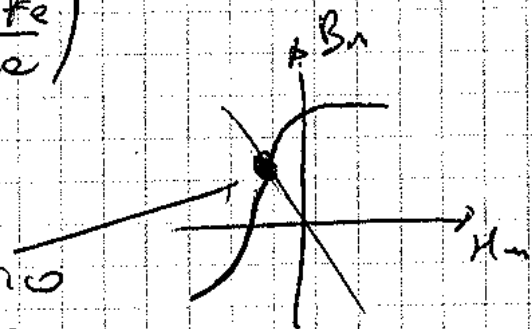
$$H_m \mu_m + B_e \left(\frac{l_e}{\mu_e} + \frac{S_e l_c}{S_c \mu_0} \right) = 0$$

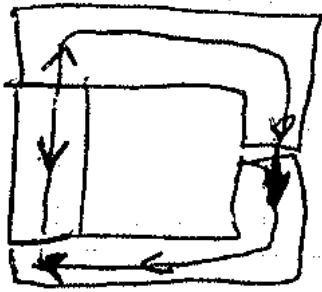
$$H_m \mu_m + B_m \frac{S_m}{S_e} \left(\frac{l_e}{\mu_e} + \frac{l_c}{\mu_0} \frac{S_e}{S_c} \right) = 0$$

$$B_m = -H_m \frac{l_m}{\frac{S_m}{S_e} \left(\frac{l_e}{\mu_e} + \frac{l_c}{\mu_0} \frac{S_e}{S_c} \right)}$$

risultato
ogni volta

induzione e campo magnetico
nel magnete permanente





le pendenze delle rette indicano il punto di lavoro.

DOMINI a μ COSTANTE

$$\begin{cases} \text{div } \vec{B} = 0 \\ \text{rot } \vec{H} = \vec{J} \\ \vec{B} = \mu \vec{H} \end{cases} \rightarrow \begin{cases} \vec{B} = \text{rot } \vec{A} \\ \vec{H} = \frac{1}{\mu} \text{rot } \vec{A} \end{cases}$$

$$\text{rot } \frac{1}{\mu} \text{rot } \vec{A} = \vec{J} \Rightarrow \frac{1}{\mu} \text{rot}(\text{rot } \vec{A}) = \vec{J}$$

$$\text{grad}(\text{div } \vec{A}) - \nabla^2 \vec{A} = \mu \vec{J}$$

$\nabla^2 = 0$
posto

Teo GREEN

$$\vec{A}(\vec{r}) = - \frac{1}{4\pi} \int \frac{\nabla'^2 \vec{A}}{r} d\tau = \frac{\mu}{4\pi} \int \frac{\vec{J}}{r} d\tau$$

$$\frac{1}{r} d\tau = S d\theta$$

$$A = \frac{\mu_0}{4\pi} \int \frac{I d\vec{\ell}}{r^2} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{\ell}}{r}$$

$$\mu_0 H = B = \text{rot } A = \text{rot} \left[\frac{\mu_0 I}{4\pi} \oint \frac{d\vec{\ell}}{r} \right]$$

$$= \mu_0 \text{rot} \left[\frac{I}{4\pi} \oint \frac{d\vec{\ell}}{r} \right] = \frac{\mu_0 I}{4\pi} \oint \text{rot} \frac{d\vec{\ell}}{r}$$

H

$$d\vec{\ell} = dx \vec{i} + dy \vec{j} + dz \vec{k}$$

$$r = \sqrt{(x_0 - x)^2 + (y_0 - y)^2 + (z_0 - z)^2}$$

$$\frac{d\vec{\ell}}{r} = \frac{dx}{r} \vec{i} + \frac{dy}{r} \vec{j} + \frac{dz}{r} \vec{k}$$

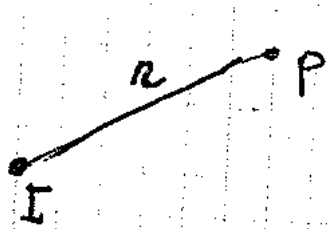
$$\text{rot} \frac{d\vec{\ell}}{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{dx}{r} & \frac{dy}{r} & \frac{dz}{r} \end{vmatrix} =$$

$$= \vec{i} \left(\frac{\partial}{\partial y} \frac{dz}{r} - \frac{\partial}{\partial z} \frac{dy}{r} \right) + \dots$$

$$\text{rot} \left(\frac{d\vec{\ell}}{r} \right) = \frac{d\vec{\ell} \times \vec{r}}{|\vec{r}|^3}$$

$$H = \frac{I}{4\pi} \oint \frac{d\vec{\ell} \times \vec{r}}{|\vec{r}|^3}$$

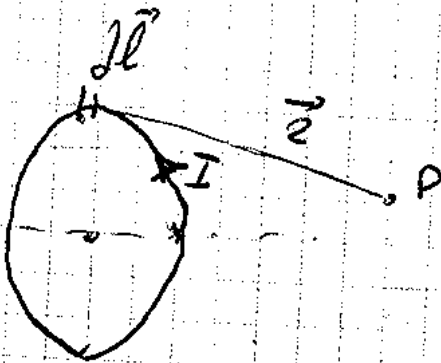
$$H = \sum_{k=1}^n \frac{I_k}{4\pi} \oint \frac{d\vec{\ell}_k \times \vec{r}_k}{|\vec{r}_k|^3}$$



filo corrente infinitesimo

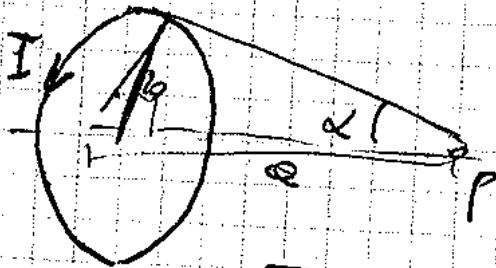
$$\vec{H} = \frac{\vec{I} \times \vec{r}}{2\pi r^2}$$

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calcol non troppo facile

Caso di punto su asse dello spiro



$$r = \sqrt{a^2 + z^2}$$

$$H_{\text{as}} = \frac{I}{2} \int \frac{a^2 \sin \alpha}{r^3} d\alpha$$

$$\frac{I a^2}{2(a^2 + z^2)^{3/2}} d\alpha = \frac{I a^2}{(a^2 + z^2)^{3/2}}$$