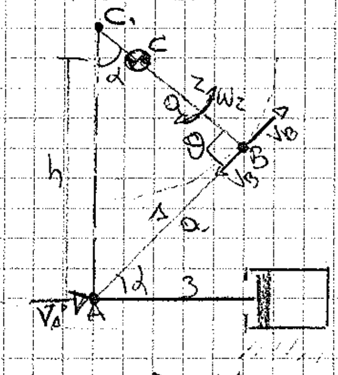
03-03-2010

ESERCIZIO 1.10



COPPIA PRISMATICA

$a = 125\,mm$
$h = 175\,mm$
$\theta = 80°$

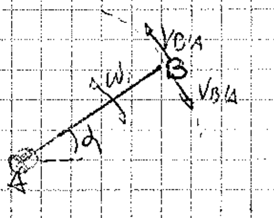
$V_A = 0.5\,m/s$
$\vec{\omega}_1 = ?$
$\vec{\omega}_2 = ?$

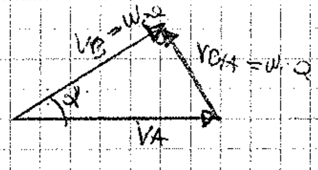
3 MOTO TRASLATORIO
2 MOTO ROTATORIO
1 MOTO ROTO TRASLATORIO
punto B MOTO CIRCOLARE

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

modulo: $? \; (\omega_2 (B)) \quad V_A \quad (\omega_1 \cdot AB)\,?$

direzione





$V_B = \omega_2 a = V_A \cos\alpha \qquad \omega_2 = \frac{V_A}{a}\cos\alpha \qquad \omega_2 = 3.2\,rad/s$

$V_{B/A} = \omega_1 a = V_A \sin\alpha \qquad \omega_1 = \frac{V_A}{a}\sin\alpha \qquad \omega_1 = 2.4\,rad/s$

$h = CB \cdot \cos\alpha + AB \cdot \sin\alpha = a(\sin\alpha + \cos\alpha)$

$h^2 = a^2(\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha) = a^2(1 + \sin 2\alpha) \qquad \frac{h^2}{a^2} - 1 = \sin 2\alpha$

$\alpha = \frac{1}{2}\arcsin\left(\frac{h^2}{a^2} - 1\right)$

Si può anche risolvere con il metodo di centro di istantaneo rotazione

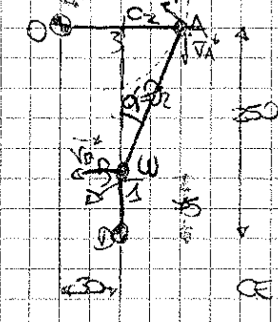
C_1 centro di istantaneo rotazione del corpo 1

$AC_1 = \frac{a}{\sin\alpha} \qquad V_A = \omega_1 \cdot AC_1 \qquad \omega_1 = \frac{V_A}{a}\sin\alpha$

$BC_1 = \frac{a}{\cos\alpha} \qquad V_B = \omega_1 \cdot BC_1 = \frac{V_A}{\cancel{a}}\sin\alpha \cdot \frac{\cancel{a}}{\cos\alpha} = V_A \sin\alpha \frac{\cos\alpha}{\sin\alpha} = V_A \cos\alpha$

$V_B = V_A \cos\alpha = \omega_2 \cdot a \qquad \omega_2 = \frac{V_A}{a}\cos\alpha$

ESERCIZIO 1.11



$\tan\theta = \frac{AC_2}{BC_2} = \frac{50}{175}$
$\theta = 2$

1 MOTO ROTATORIO
2 MOTO ROTO-TRASLATORIO
3 MOTO ROTATORIO

$\omega_1 = 2\,rad/s \rightarrow \dot{\omega}_1 = 0$

$\vec{\omega}_2 ? \quad \vec{\omega}_3 ? \qquad \dot{\vec{\omega}}_2 ? \quad \dot{\vec{\omega}}_3 ? \qquad \vec{V}_A ? \quad \vec{a}_A$

CE

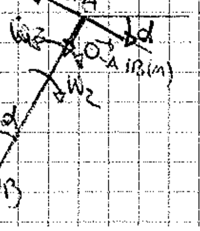
$V_B = \omega_1 \cdot DB = \omega_2 \cdot BC_2 \qquad \omega_2 = \omega_1 \frac{DB}{BC_2} = 2\frac{75}{175}\,0.857\,\frac{rad}{s}$

$V_A = \omega_2 \cdot AC_2 = \omega_3 \cdot AO \qquad \omega_3 = \omega_2 \frac{AC_2}{AO} = 0.857\frac{50}{100} = 0.428\,\frac{rad}{s}$

$V_A = 0.857 \cdot 0.05 = 0.043\,m/s$

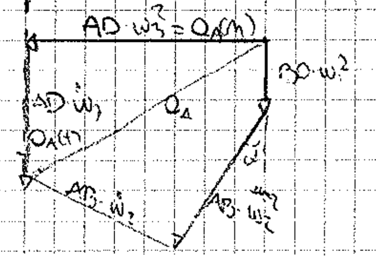
TEO DI RIVALS $\quad \vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

$\vec{a}_{A(n)} + \vec{a}_{A(t)} = \vec{a}_{B(n)} + \vec{a}_{B(t)} + \vec{a}_{A/B(n)} + \vec{a}_{A/B(t)}$



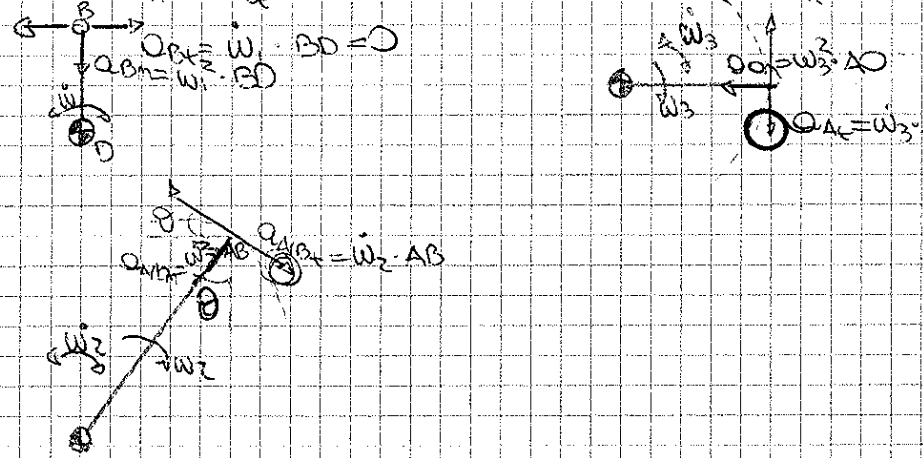
$$\vec{a}_{A(n)} + \vec{a}_{A(t)} = \vec{a}_{B(n)} + \cancel{\vec{a}_{B(t)}} + \vec{a}_{A/B(n)} + \vec{a}_{A/B(t)}$$

$AO \cdot \omega_3^2$ $\quad (AO \cdot \dot\omega_3)?$ $\quad BD \cdot \omega_1^2$ $\quad AB \cdot \omega_2^2$ $\quad (AB \cdot \dot\omega_2)?$



04-03-2010

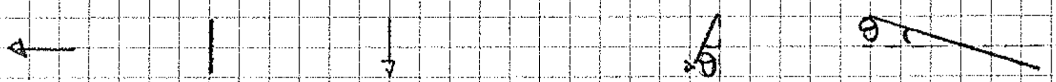
$\vec{a} = \vec{a}_{Bn} + \vec{a}_{Bt}$ $\qquad \vec{a}_A = \vec{a}_{An} + \vec{a}_{At}$

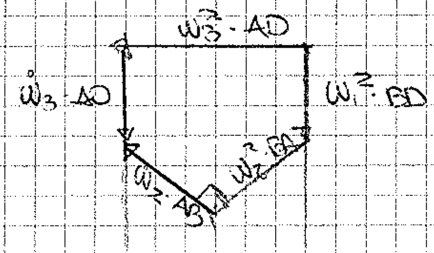


$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

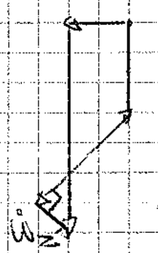
$$\vec{a}_{An} + \vec{a}_{At} = \vec{a}_{Bn} + \cancel{\vec{a}_{Bt}} + \vec{a}_{A/Bn} + \vec{a}_{A/Bt}$$

$\omega_3^2 \cdot AO$ $\quad (\dot\omega_3 \cdot AO)$ $\quad \omega_1^2 \cdot BD$ $\quad \omega_2^2 \cdot AB$ $\quad (\dot\omega_2 \cdot AB)$





I due vettori a biro sono ipotetici, infatti possono variare a seconda della lunghezza degli altri 3.



Proietto i 5 vettori sugli assi:

- orizzontale: trovo un'equazione che mi risolve le incognite (si ricava $\dot\omega_2$)

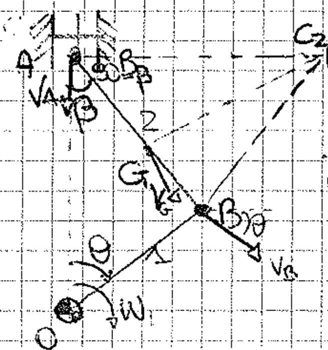
$$\omega_3^2 \cdot AO = \omega_2^2 \cdot AB \cdot \sin\theta + \dot\omega_2 \cdot AB \cdot \cos\theta \rightarrow \dot\omega_2 = -0.105 \ \frac{rad}{s^2}$$

dato che $\dot\omega_2$ è ~~positivo~~ negativo non è orientato come nel disegno, ma così

- verticale: $\dot\omega_3 \cdot AO + \dot\omega_2 \cdot AB \sin\theta = \omega_1^2 \cdot BD + \omega_2^2 \cdot \cos\theta$

$$\dot\omega_3 = 4.34 \ \frac{rad}{s^2}$$

ESERCIZIO 1.9



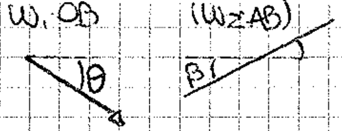
$\omega_1 = 1500 \frac{giri}{minuti} \rightarrow \dot{\omega}_1 = 0$

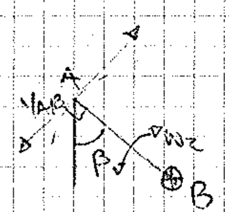
$\overline{OB} = 42.5\ mm$
$\overline{AB} = 107.5\ mm$
$\overline{AG} = 75\ mm$
$\theta = 60°$

$\vec{\dot{\omega}}_2 = ?$
$\vec{V}_A = ?$
$\vec{\omega}_2 = ?$
$\vec{a}_A = ?$

$$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$$

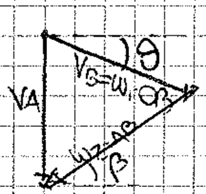
? $\quad \omega_1 \cdot OB \quad (\omega_2 \cdot AB)$





$$\frac{OB}{\sin\beta} = \frac{AB}{\sin\theta} \qquad \beta = \sin^{-1}\left(\frac{OB}{AB}\sin\theta\right) = 20.02°$$

$$\omega_1 \cdot OB = 12\ m/s$$



$-\omega_1 \cdot OB \cdot \cos\theta = \omega_2 \cdot AB \cdot \cos\beta$

$-\omega_2 = 33.05 \frac{rad}{s}$

$V_A = \omega_1 \cdot OB \cdot \sin\theta + \omega_2 \cdot AB \cdot \sin\beta = 6.33\ m/s$

C_2 è il centro di istantanea rotazione

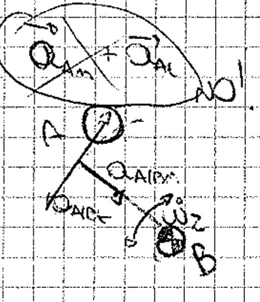
$V_G = \omega_2 \cdot \overline{GC_2} = 6.38\ m/s$

$AO = OB \cdot \cos\theta + AB \cdot \cos\beta$
$AO \cdot \tan\theta = C_2A$

Teorema di Carnot: $GC_2 = \sqrt{AG^2 + AC_2^2 - 2 AG \cdot AC_2 \cdot \cos(90-\beta)}$
$= 193\ mm$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

$\vec{a}_{A_n} + \vec{a}_{A_t}$ NO!



$$\vec{a}_A = \vec{a}_{B_n} + \cancel{\vec{a}_{B_t}} + \vec{a}_{A/B_n} + a_{A/B_t}$$

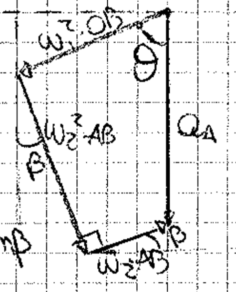
? $\quad \omega_1^2 \cdot OB \quad \dot{\omega}_1 \cdot OB = 0 \quad \omega_2^2 \cdot AB \quad (\dot{\omega}_2 \cdot AB)$

- verticale: $a_A = \omega_1^2 \cdot OB \cos\theta + \omega_2^2 \cdot AB \cdot \cos\theta - \dot{\omega}_2 \cdot AB \cdot \sin\beta$

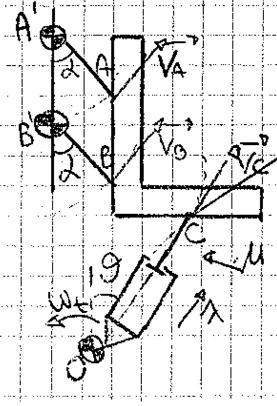
$a_A = 318.5\ m/s^2$

- orizzontale: $\dot{\omega}_2 \cdot AB \cdot \cos\beta = \omega_1^2 \cdot OB \cdot \sin\theta - \omega_2^2 \cdot AB \cdot \sin\beta$

$\dot{\omega}_2 = +8584 \frac{rad}{s^2} \rightarrow a_{A/B_t}$



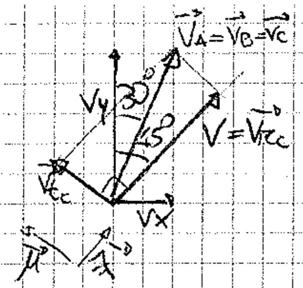
• ESERCIZIO 1.22



$\alpha = 60°$
$\theta = 45°$
$V = 0.1\ m/s$ ← velocità relativa di C
V_y ?

← il cilindro ruota

$\vec{V_C} = \vec{V_A} = \vec{V_B}$ velocità assoluta
$\vec{V_{Ct}} = \omega_t \cdot OC\ \vec{\mu}$



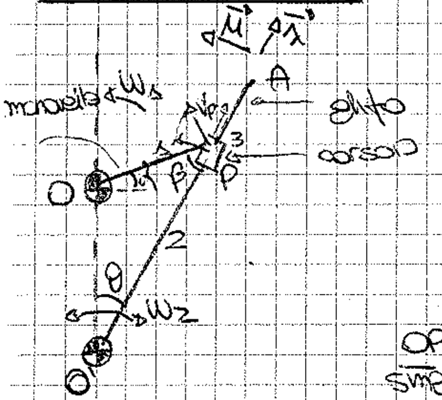
$\vec{V_C} = \vec{V_{rC}} + \vec{V_{tC}}$

$V_C = \frac{V}{\cos 15°} = \frac{V_y}{\cos 30°}$

$V_y = V \frac{\cos 30°}{\cos 15°} = 0.08\ m/s$

• ESERCIZIO 1.24

Catena cinematica
Guida prismatica



$OP = 0.3 m$
$O'A = 0.8 m$
$OO' = 0.4 m$

$\theta = 25°$
$\omega_1 = 100\ rad/s$

$\omega_2 = ?$
$V_A = ?$

$\frac{OP}{\sin\theta} = \frac{OO'}{\sin\beta} \rightarrow \beta = ...$

$\alpha = 90° - \beta - \theta$

$O'P = OP \cdot \cos\beta + OO' \cos\theta$

$\vec{V_P} = \vec{V_{rP}} + \vec{V_{tP}}$

$\omega_1 \cdot OP$? $(\omega_2 \cdot O'P)$

$V_P = \omega_1 \cdot OP$

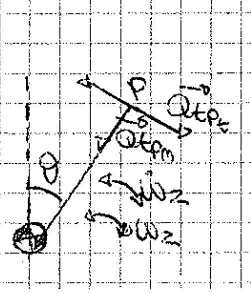
$V_{tP} = V_P \cdot \sin(\alpha+\theta) = \omega_1\ OP \cdot \sin(\alpha+\theta) = \omega_2 \cdot O'P$

$\omega_2 = \omega_1 \frac{OP}{O'P} \sin(\alpha+\theta) = \omega_1 \frac{OP \sin(\alpha+\theta)}{OP\cos\beta + OO' \cos\theta}$

$V_A = \omega_2 \cdot O'A\ \vec{\mu}$

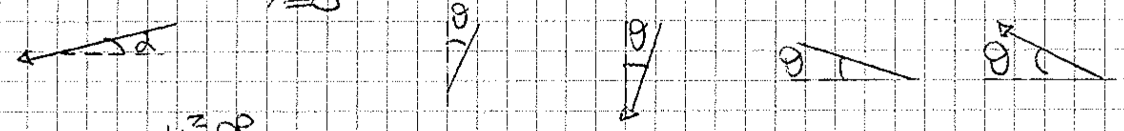
$\vec{a_P} = \vec{a_{rP}} + \vec{a_{tP}} + \vec{a_{cP}}$
$\vec{a_P} = \vec{a_{P_n}} + \vec{a_{P_t}}$

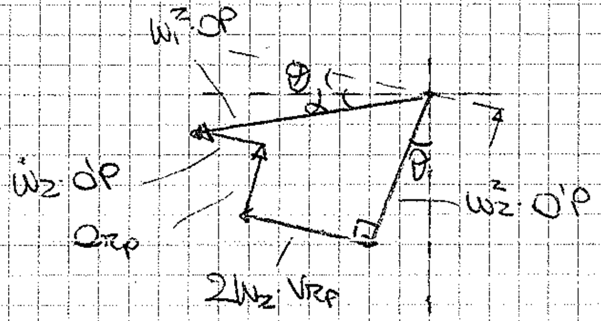
$\vec{a_{rP}} \parallel \vec{\lambda}$ $\vec{a_{cP}} = 2\vec{\omega_2} \times \vec{V_{rP}} = 2\omega_2 V_{rP}\ \vec{\mu}$



$$\vec{a}_{P_n} + \vec{a}_{P_t} = \vec{a}_{r_P} + \vec{a}_{tp(n)} + \vec{a}_{tp(t)} + \vec{a}_{cP}$$

$$\omega_1^2 \cdot OP \qquad \dot{\omega}_1 \cdot \cancel{OP} = 0 \qquad ? \qquad \omega_2^2 \cdot O'P \qquad (\dot{\omega}_2 \cdot O'P) \qquad 2\omega_2 \cdot V_{rP}$$





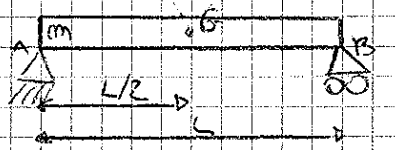
Proiettiamo su una retta ruotata di θ

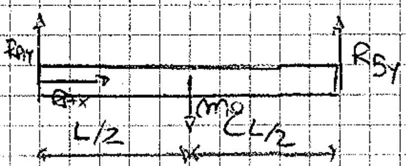
$$\dot{\omega}_2 \cdot O'P = \omega_1^2 \cdot OP \cdot \cos(\alpha+\theta) - 2\omega_2 \cdot V_{rP}$$

$$\dot{\omega}_2 = 518 \text{ rad/s}^2$$

$$\vec{a} = O'A\sqrt{\dot{\omega}_2^2 + \omega_2^4} = 1384 \text{ m/s}^2$$

ESERCIZIO 2.21



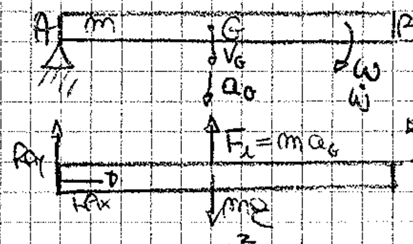


$\rightarrow$ $R_{Ax} = 0$

$\uparrow Y$ $\quad R_{AY} + R_{BY} - mg = 0 \qquad R_{AY} = \frac{1}{2}mg$

$A)$ $\quad R_{BY} \cdot L - mg \cdot \frac{L}{2} = 0 \qquad R_{BY} = \frac{1}{2}mg$

Immaginiamo di togliere il carrello in B



$I_G \dot{\omega} = I_G a_G \frac{2}{L}$
$= m\frac{L^2}{12} a_G \frac{2}{L} = m \frac{L}{6} a_G$

CONDIZIONE DI MOTO INCIPIENTE

all'inizio $\quad V_G = 0 \quad \omega = 0$
$\qquad a_G \neq 0 \quad \dot{\omega} \neq 0$

$$I_G = \frac{mL^2}{12}$$

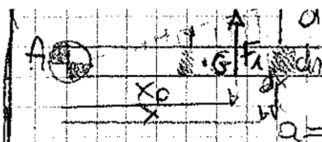
$$\vec{a}_G = \overset{=0}{\vec{a}_{G_n}} + \vec{a}_{G_t}$$
$$\qquad \omega^2 \frac{L}{2} \qquad \dot{\omega}\frac{L}{2}$$

$\rightarrow$ $R_{Ax} = 0$

$\uparrow$ $\quad R_{AY} + m a_G - mg = 0 \qquad R_{AY} = \frac{1}{4}mg$

$A)$ $\quad m a_G \frac{L}{2} - mg\frac{L}{2} + m\frac{L}{6}\cdot\frac{a_G}{3} = 0 \quad a_G + \frac{1}{3}a_G - g = 0 \quad a_G = \frac{3}{4}g$

DIAGRAMMA DI FORZE DISTRIBUITE



$dF_i = dm \cdot x \cdot \dot{\omega}$

$a = x \cdot \dot{\omega}$

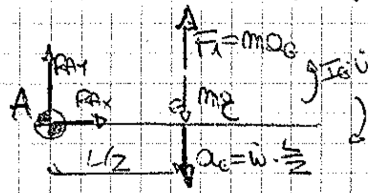
$$m = \rho L \qquad dm = \rho\, dx$$

$$F_i = \int dF_i = \int dm\, x \cdot \dot{\omega} = \dot{\omega}\rho \int_0^L x\, dx = \dot{\omega} \cdot \rho \cdot \frac{L^2}{2} = \frac{1}{2}\dot{\omega}\, mL \qquad a_G = \dot{\omega}\frac{L}{2}$$

$$F_i = m \cdot a_G$$

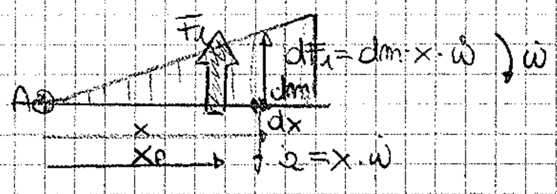
~~Applico la risultante delle forze d'inerzia F_i nel punto x_P~~

$$\cancel{F_i \cdot x_P = \int_m dF_i \cdot x = \int_0^L x\, \dot{\omega}\, \rho dx = \dot{\omega}\rho \int_0^L x dx = \dot{\omega}\rho\frac{L}{2} = x_P \cdot m\dot{\omega}\frac{L}{2}}$$



$$m = L \cdot \rho$$

$$I_G = \frac{mL^2}{12}$$



$dm = dx \cdot \rho$

Calcolo la risultante delle F_i:

$$F_i = \int_m dF_i = \int x \cdot \dot{\omega} \cdot \rho \cdot dx = \dot{\omega}\, \rho \int_0^L x dx = \dot{\omega} \cdot \rho \cdot \frac{L^2}{2} = (\rho L) \cdot \left(\dot{\omega}\frac{L}{2}\right) = m \cdot a_G$$ risultante delle forze d'inerzia

Questa risultante F_i deve essere applicata in un punto tale che abbia lo stesso effetto di tutte le forze inerziali

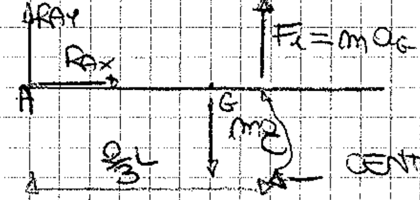
$$\underbrace{F_i \cdot x_P}_{\text{momento della risultante delle forze d'inerzia}} = \underbrace{\int_m dF_i\, x}_{\text{momento risultante delle forze d'inerzia}}$$

$$\int x \cdot dm\, \dot{\omega}\, x = \int \dot{\omega} x^2 \rho\, dx = \rho\dot{\omega}\int_0^L x^2 dx = \rho \cdot \dot{\omega}\frac{L^3}{3} = (\rho \cdot L)\left(\dot{\omega}\frac{L^2}{3}\right) =$$

$$= \dot{\omega} \cdot \left(m\frac{L^2}{3}\right) = \dot{\omega} \cdot I_A$$

$$F_i \cdot x_P = m \cdot a_G \cdot x_P = \dot{\omega} \cdot I_A$$

$$\cancel{m} \cdot x_P \cdot \cancel{\dot{\omega}}\frac{\cancel{L}}{2} = \cancel{\dot{\omega}}\, \cancel{m}\frac{L^{\cancel{2}}}{3} \qquad \boxed{x_P = \frac{2}{3}L}$$

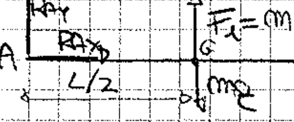


CENTRO DI PERCOSSA

DIAGRAMMA DI CORPO LIBERO DEL SISTEMA

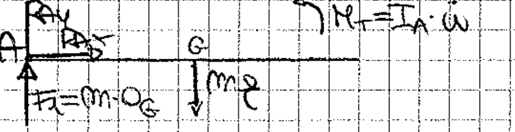
RIDUZIONE DELLE FORZE D'INERZIA

Spostando la forza d'inerzia nel baricentro



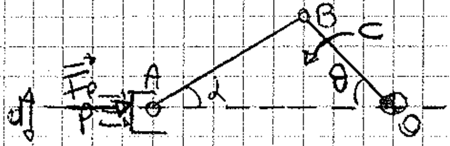
$$M = I_G \dot{\omega} \qquad M = F_i \cdot \frac{L}{6} = m \cdot a_G \cdot \frac{L}{6} = m\frac{L}{6}\dot{\omega}\frac{L}{2} = \dot{\omega} \cdot m\frac{L^2}{12}$$

Posso anche ridurre le forze d'inerzia ad un punto fisso del corpo (il punto A in questo caso)



$$M_T = F_i \cdot \frac{2}{3}L = \frac{2}{3}L\, m\dot{\omega}\frac{L}{2} = \dot{\omega} \cdot m\frac{L^2}{3} = \dot{\omega} \cdot I_A$$

ESERCIZIO 2.16



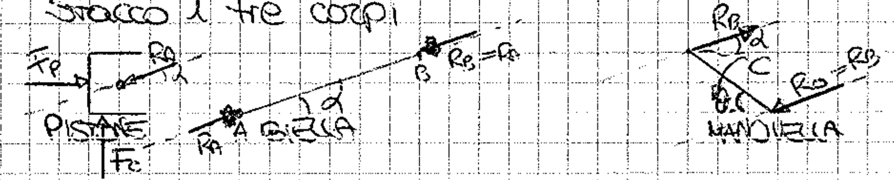
$p = 100$ kPa $= 10^5\ \frac{N}{m^2} = 1$ bar

$OB = 42.5$ mm $d = 40$ mm
$AB = 107.5$ mm
$\theta = 60°$

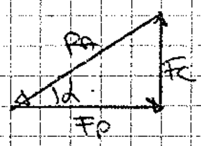
C ?
R_A ?
R_B ?
R_O ?

$F_p = p \cdot \pi \frac{d^2}{4} = 40\pi$ N $\cong 120$ N

Stacco i tre corpi



$AB \sin\alpha = OB \sin\theta$ $\alpha = \sin^{-1}\left(\frac{OB}{AB}\sin\theta\right) \cong 20.02°$



$\vec{F_p} + \vec{R_A} - \vec{F_c} = 0$

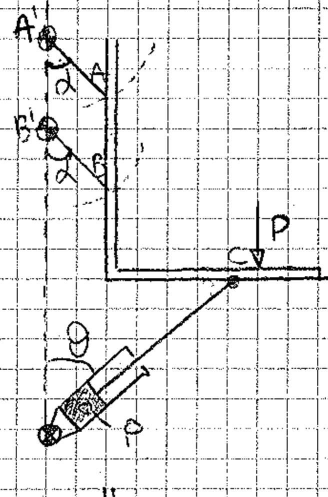
$R_A = \frac{F_p}{\cos\alpha} = 133.75$ N

$R_B = R_A = 133.75$ N

$R_O = R_B = 133.75$ N

$C = R_O \cdot b = R_O \cdot OB \cdot \cos(90 - \theta - \alpha) = 5.6\ N/m$

ESERCIZIO 2.18

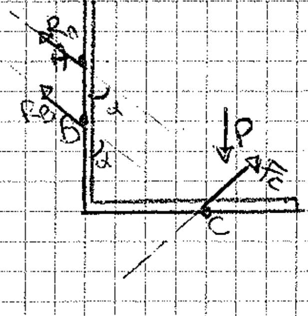


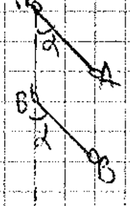
$\theta = 45°$
$\alpha = 60°$
$P = 500$ kg

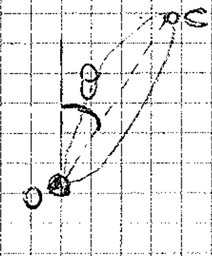
$\phi = 5$ cm alesaggio = diametro interno del cilindro

Il cilindro viene visto come un unico corpo rigido

Non abbiamo le dimensioni di questo corpo
↳ non posso scrivere le equazioni dei momenti







$F_c = p \cdot \frac{\pi \phi^2}{4}$

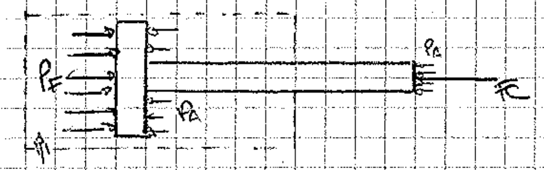
x, y

$$\begin{cases} F_c \sin\theta - R_A \sin\alpha - R_B \sin\alpha = 0 \\ F_c \cos\theta - P + R_A \cos\alpha + R_B \cos\alpha = 0 \end{cases} \qquad \begin{cases} F_c \sin\theta = (R_A + R_B)\sin\alpha \\ F_c \cos\theta - P = -(R_A + R_B)\cos\alpha \end{cases}$$

$$F_c \cdot \cos\theta - P = -F_c \frac{\sin\theta}{\sin\alpha}\cos\alpha$$

$$F_c \left(\cos\theta + \sin\theta \frac{\cos\alpha}{\sin\alpha}\right) = P \qquad F_c = \frac{P}{\cos\theta + \frac{\sin\theta}{tg\,\alpha}} = 4388\text{N}$$

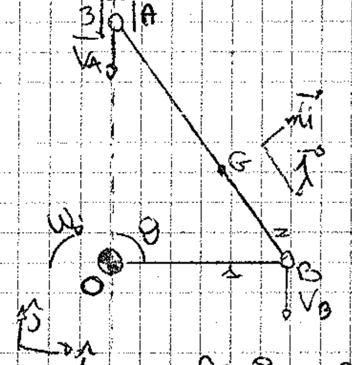
$$p\,\frac{\pi \phi^2}{4} = 4388\text{N} \rightarrow p = 2.24 \cdot 10^6\ N/m^2\ [= Pa] = 22.6\ \text{bar}$$



$$F_C = p \cdot A = (p_F - p_A) \cdot A = p \cdot \frac{\pi \phi^2}{4}$$

pressione RELATIVA del fluido

ESERCIZIO 2.22



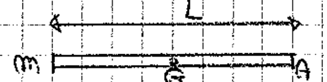
$OB = 42.5\,mm$
$AB = 107.5\,mm$
$AG = 75\,mm$

$\theta = 90°$
$m_2 = 0.6\,kg$
$m_3 = 0.82\,kg$

$\rho_2 = 28\,mm$ → RAGGIO D'INERZIA BARICENTRICO
$\omega_1 = 3000\,rpm = \frac{giri}{min}$

$\alpha = \sin^{-1}\left(\frac{OB}{AB}\right) = 23.3°$

$R_A = ?$



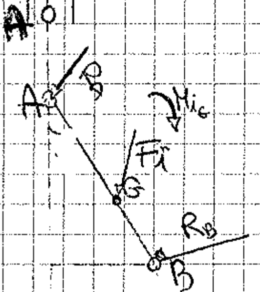
$I_G = \frac{mL^2}{12}$ $I_A = \frac{mL^2}{3}$

$I_A = I_G + m\,AG^2$ Huygens-Steiner

$I_G = m\rho_G^2$ $\rho_G = \sqrt{\frac{L^2}{12}}$

$I_A = m\rho_A^2$ $\rho_A = \sqrt{\frac{L^2}{3}}$

Analisi cinematica



$$\vec{F}_i = -m\vec{a}_G$$
$$\vec{M}_{i_G} = -\vec{I}_G \cdot \dot{\vec{\omega}}_2 = -m_2 \cdot \rho_2^2 \cdot \dot{\vec{\omega}}_2$$

- $V_B = \omega_1 \cdot OB(-\vec{j})$

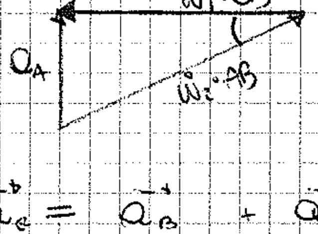
$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$
$?$ $\omega_1 \cdot OB$ $(\omega_2 \cdot AB)$

è unica possibilità per rappresentare questa relazione è

$\vec{V}_{A/B} = 0 \Rightarrow \omega_2 = 0$
$\Downarrow$
$\vec{V}_A = \vec{V}_B$

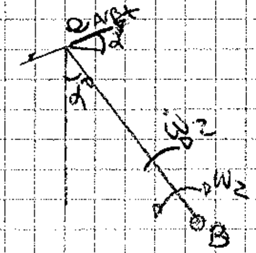
$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

$\vec{a}_A = \vec{a}_{Bn} + \vec{a}_{Bt} + \vec{a}_{A/Bn} + \vec{a}_{A/Bt}$
$?$ $\omega_1^2 \cdot OB$ $\dot{\omega}_1 \cdot OB$ $\omega_2^2 \cdot AB$ $(\dot{\omega}_2 \cdot AB)$



$\vec{a}_A = \omega_1^2 \cdot OB \cdot \tan\alpha = 1805\,m/s^2$

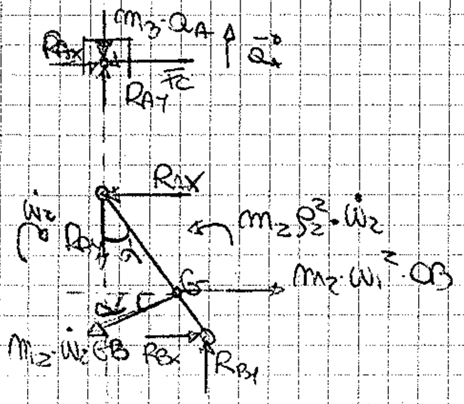
$\dot{\omega}_2 = \frac{\omega_1^2 \cdot OB}{\cos\alpha \cdot AB} = 42.481\,rad/s^2$



$\vec{a}_G = \vec{a}_B + \vec{a}_{G/B_n} + \vec{a}_{GB_t} = \vec{a}_A + \vec{a}_{G/A}$
$?$ $\omega_1^2 \cdot OB$ $\omega_2^2 \cdot GB$ $\dot{\omega}_2 \cdot GB$

$\vec{a}_G = \omega_1^2 \cdot OB(-\vec{i}) + \dot{\omega}_2 \cdot GB(\vec{\mu})$

Possiamo ora all'analisi della Dinamica



$R_A = \sqrt{R_{AX}^2 + R_{AY}^2}$

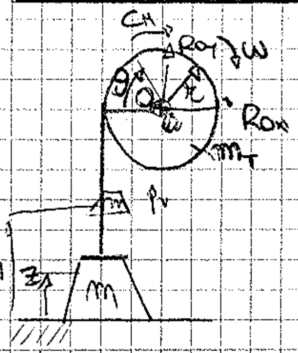
$m_3 \cdot a_A = R_{AY}$ $\qquad R_{AY} = 1680\,N$

(B) $\quad R_{AX} \cdot AB \cdot \cos\alpha + R_{AY} \cdot AB \cdot \sin\alpha + m_2 \rho_2^2 \cdot \dot{\omega}_2 + \dot{\omega}_2 \cdot m_2 \cdot GB^2 - m_2 \omega_1^2 \cdot OB \cdot GB\cos\alpha = 0$

$R_{AX} = \ldots\ldots = 351\,N$

$R_A = 1521\,N$

ESERCIZIO 2.4 : ARGANO



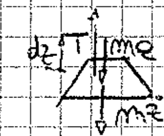
$C_M = ?$

$z = r \cdot \theta$

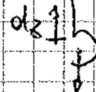
$m_T = 100\,kg$
$r = 0.15\,m$
$I_G = \dfrac{m_T r^2}{2}$
$m = 200\,kg$

$t = 0$	$t = T$
$z = 0$	$z = h = 2\,m$
$\dot{z} = 0$	$\dot{z} = V = 1\,m/s$

$\cancel{L_i} + L_e = \Delta E_C + \Delta E_E + \Delta E_G$



$T \cdot dz = dL$



$dL = -T \cdot dz \quad \Rightarrow L_i = 0$

R_{OX} e R_{OY} non producono lavoro → sono applicate ad un punto fisso

$dL = \vec{C} \cdot d\vec{\theta}$

$L_e = C_M \int_1^2 d\theta = C_M(\theta_2 - \theta_1) = C_M \cdot \dfrac{h}{r}$

$\Delta E_C = E_{C_2} - \cancel{E_{C_1}} = \frac{1}{2} m V^2 + \frac{1}{2} I_G \cdot \omega^2 = \frac{1}{2} m V^2 + \frac{1}{2} \frac{m_T r^2}{2} \cdot \frac{V^2}{r^2}$

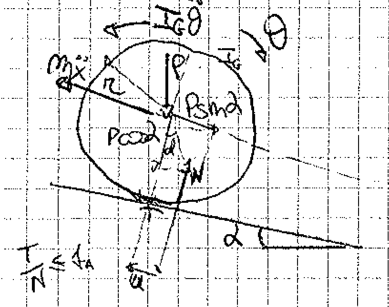
non ci sono corpi che si deformano → $\Delta E_E = 0$

$\Delta E_G = E_{G_2} - E_{G_1} = mgh$

$$C_M \cdot \frac{h}{r} = \frac{1}{2} m V^2 + \frac{1}{4} m_T V^2 + mgh$$

$$C_M = 304\,Nm$$

ESERCIZIO 3.18



$\alpha = \begin{cases} 10^\circ \\ 45^\circ \end{cases}$

$d = 2r = 1m$
$M = 10\,000\ kg$
$f_A = 0.2$
$f = 0.15$
$u = 2cm$
$L = 200m$

$t = ?$ tempo

$n = ?$ n° di giri

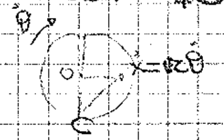
$I_G = \frac{Mr^2}{2}$

x: $P\sin\alpha - T - m\ddot{x} = 0$

y: $N - P\cos\alpha = 0$

(G): $I_G\ddot{\theta} + N\mu - Tr = 0$

	V_R	T/N
Aderenza	$=0$	$< f_A$
Ad. limite	$=0$	$= f_A$
Strisc.	$\neq 0$	$= f$



- aderenza $\ddot{x} = r\ddot{\theta}$

ipotesi $\rightarrow$ poi devo verificare se $\frac{T}{N} \le f_A$

- se non è verificato $\rightarrow$ strisciamento

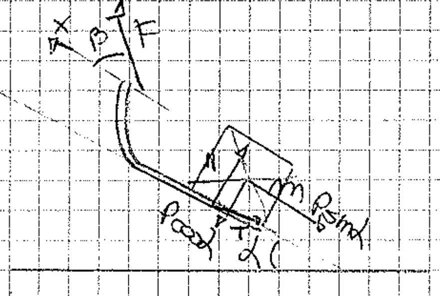
$T = fN$

α	T/N (f)	$\ddot{x}$ (m/s^2)	$\ddot{\theta}$ (rad/s^2)	t (s)	n
10°	0.09	0.88	1.76	21.3	63.6 giri
45°	~~0.86~~ 0.15	5.9	3.05	3.05	16.5 giri

$L = \frac{1}{2}\ddot{x}t^2 \rightarrow t = \sqrt{\frac{2L}{\ddot{x}}}$ $\quad n = \frac{L}{2\pi R} \times 10^\circ$

$\theta = \frac{1}{2}\ddot{\theta}t^2 \quad \times 45^\circ$

ESERCIZIO 3.17



$m = 500\ kg$

$f = 0.2$
$i = 30\%$ $\quad \alpha = tg^{-1}\frac{3}{10}$

x: $F\cos\beta - T - P\sin\alpha = 0$

y: $F\sin\beta + N - P\cos\alpha = 0$

$T = f \cdot N$

Devo fare in modo che ~~F~~ F sia minimo in funzione di β

$\begin{cases} F\cos\beta - P\sin\alpha = fN \\ -F\sin\beta + P\cos\alpha = +N \end{cases}$ $\quad f = \frac{F\cos\beta - P\sin\alpha}{-F\sin\beta + P\cos\alpha}$

$F\cos\beta - P\sin\alpha = fP\cos\alpha - fF\sin\beta$

$F = \frac{fP\cos\alpha + P\sin\alpha}{\cos\beta + f\sin\beta} = P\frac{f\cos\alpha + \sin\alpha}{\cos\beta + f\sin\beta}$

devo

devo cercare il valore min di $F \rightarrow$ valore max di $\kappa(\beta) = \cos\beta + f\sin\beta$

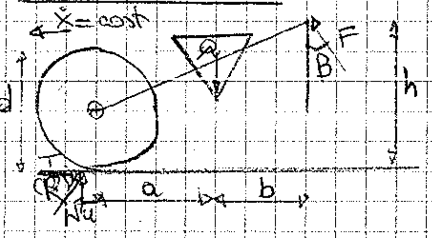
$$\frac{dk}{d\beta} = 0 \qquad -\sin\beta + f\cos\beta = 0 \qquad f = \frac{\sin\beta}{\cos\beta} = \tan\beta \qquad \beta = \tan^{-1} f$$

CONDIZIONE DI MINIMO SFORZO

$$\frac{d^2 k(\beta)}{d\beta^2} = -\cos\beta - f\sin\beta < 0 \rightarrow \text{è un massimo}$$

• ESERCIZIO 3.16

13-04-2010



$M = 80\,kg$ $\qquad Q = M \cdot g$

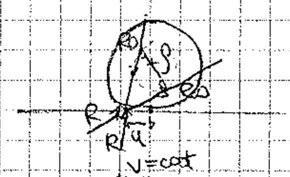
$a = 0.7\,m$ $\qquad u = 10\,mm$

$b = 0.5\,m$

$h = 0.9\,m$ $\qquad d_p = 30\,mm$ } perno

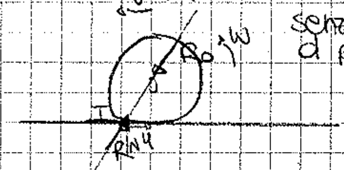
$d = 0.4\,m$ $\qquad f = 0.2$

$F = ?$ $\quad \beta = ?$ $\quad$ a velocità costante



$\rho = \frac{d_p}{2} \cdot \sin\varphi$ $\quad$ raggio di attrito del perno

$\vec{R} = \vec{N} + \vec{T}$



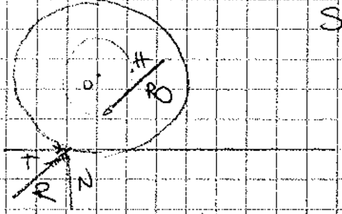
senza attrito del perno $\qquad R = R_0$

$N \cdot u = T \cdot r$

$\rightarrow \quad T - F\sin\beta = 0 \qquad (1)$

$\uparrow \quad N - Q + F\cos\beta \qquad (2)$

$\circlearrowleft_K \quad T \cdot h - N(a + b + u) + Qb = 0 \qquad (3)$



Suppongo che la R_0 passi per il punto H (trascuro il momento)

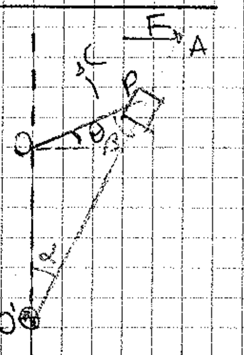
$$\circlearrowleft_H \quad T \cdot r - N(\mu + \rho) + \underbrace{R_0 \cdot \varepsilon}_{\text{trascurabile}}$$

$$T \cdot r = N(\mu + \rho) \qquad (4)$$

$$N = \frac{Q \cdot b}{(a+b+u) - \frac{(u+\rho)}{d/2} \cdot h} = 340.7\,N \qquad T = 22\,N$$

$$\tan\beta = \frac{T}{Q - N} \rightarrow \beta = 2.84° \rightarrow F = 444\,N$$

ESERCIZIO 3.20 $\quad$ (2.20)



$OP = 0.3\,m$

$O'A = 0.8\,m$

$O'O = 0.4\,m$

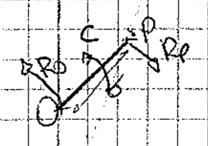
$\theta = 25°$

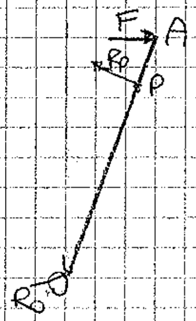
$F = 100\,N$

$f = 0.5$

$$O'P = \sqrt{\overline{OP}^2 + \overline{OO'}^2 - 2\,\overline{OP}\,\overline{O'O}\cos(90 + \theta)} = 0.592\,m \qquad \text{teo carnot (cos)}$$

teorema seni: $\alpha = 27.3°$ $\quad \beta = 37.7°$





$\vec{F} + \vec{R}_P + \vec{R}_{O'} = 0 \qquad \Sigma \vec{F}_e = 0$

$\widehat{O'}$) $\quad -F \cdot \overline{O'A} \cos\alpha + R_P \cdot \overline{O'P} = 0$

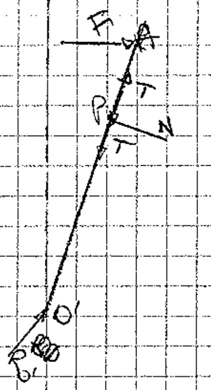
$R_P = 120N$

$|R_O| = |R_P|$

$\widehat{O}$) $\quad -R_P \cdot b + C = 0 \qquad b = \overline{OP} \cdot \cos\beta$

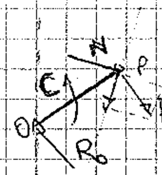
$C = R_P \cdot b = 28{,}5 \ Nm$

CON ATTRITO AL PERNO



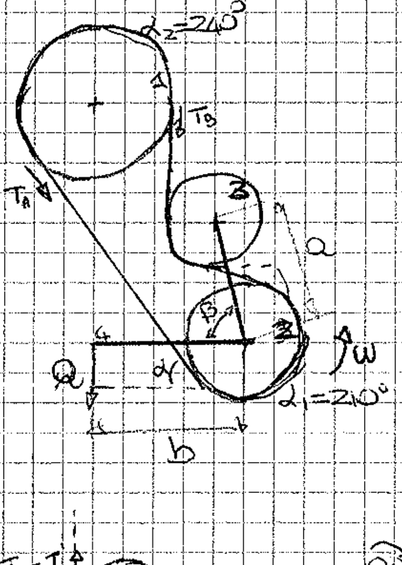
$\widehat{O'}$) $\Rightarrow N = 120N$

$T = f N = 60N$



$|R_O| = |R|$

- ESERCIZIO 5.22



3: galoppino spinto attraverso la leva

$n_2 = 360$ rpm

$Q = 180$ N

$D_2 = 300$mm

$f_a = 0.3$

$a = 300$mm

$b = 405$mm

$\alpha = 30°$

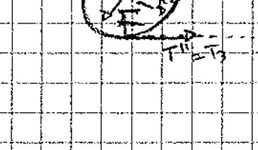
$\beta = 60°$

T_A?

T_B?

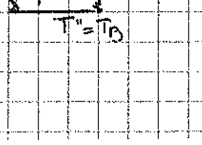
p_{MAX}?

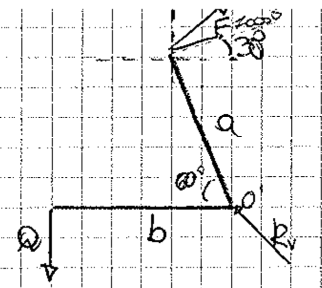
2 PULEGGIA MOTRICE



$\widehat{O_3}$ $\quad T' \cdot r_3 = T'' \cdot \cancel{r_3}$

$\vec{F} + \vec{T'} + \vec{T''} = 0$





$\overset{\curvearrowright}{O'}$ $\quad F\cos 15°\cdot a - Q\cdot b = 0$

$$F = \frac{Q\cdot b}{a\cos 15°} = 252\,N$$

$$T_B = \frac{F}{\sqrt{2}} = \frac{Q\cdot b}{a\cos 15°\sqrt{2}} = 178\,N$$

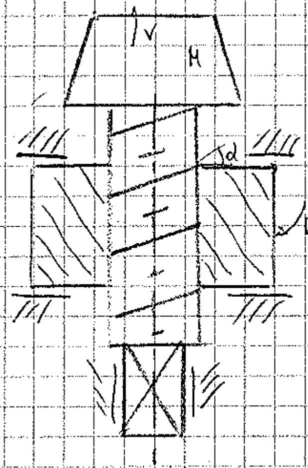
$\frac{T_A}{T_B} \le e^{f_a\alpha_1} \Rightarrow \frac{T_A}{T_B} = e^{f_0\alpha_1}$ CASO DI ADERENZA LIMITE

$$T_A = T_B\, e^{f_a\alpha_{(1-2)}}$$

$$T_A = T_B\cdot e^{f_0\cdot\alpha_1} = 178\cdot e^{0.3\cdot\frac{7}{6}\pi} = 534\,N$$

$$P_M = C_M\cdot\omega_2 = (T_A - T_B)\frac{D_2}{2}\cdot\omega_2 = \underset{N}{(534-178)}\cdot\underset{m}{0.15}\cdot\underset{\frac{giri}{min}}{360}\cdot\underset{\frac{min}{s}}{\frac{2\pi}{60}} = 2013\,W$$

ESERCIZIO 5.24



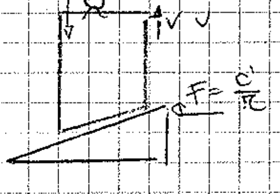
- $M = 100\,kg$
- $d = 30\,mm\ (= 2r)$
- $\alpha = 3°$
- $f = 0.1$
- $C = 5\,N\cdot m$

$C = ?$

$v = ?$

- $\varphi = tg^{-1}(0.1)$

$C = 100\cdot 9.81\cdot 0.015\cdot tg(3° + 5.71°) = 2.25\,N\,m$

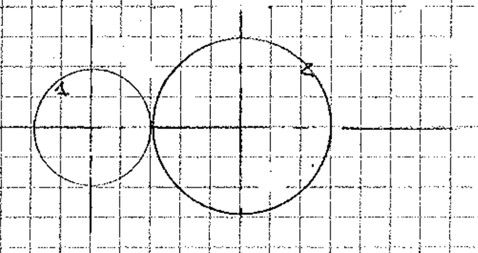


$$C' = rQ'\cdot tg(\alpha+\varphi) = r\, tg(\alpha+\varphi)\, M(g+\dot{v})$$

$$\dot{v} = 11.85\,m/s^2$$

RUOTE DENTATE

ESERCIZIO 5.4



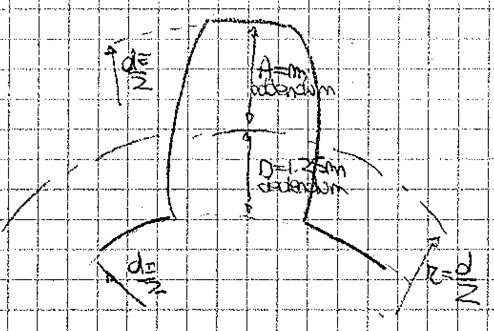
$i = 3$
$z_1 = 30$
$d_{E1} = 128\,mm$ diametro di troncatura esterno
$p = 12.57\,mm$ passo

$m \quad d_{I1} \quad z_2$
$d_1 \quad d_{I2} \quad a$?
$d_2 \quad d_{E2}$

$m = \frac{p}{\pi} = \frac{12.57}{\pi} = 4\,mm \qquad m = \frac{p}{\pi} = \frac{2\pi r}{z\pi} = \frac{d}{z}$

$d_1 = d_{E1} - 2m = z_1 \cdot m = 120\,mm$

$i = 3 = \frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} \rightarrow d_2 = i \cdot d_1 = 3 \cdot 120 = 360\,mm$



$d_{I1} = d_1 - 2D = 120 - 2 \cdot 5 \cdot 4 = 110\,mm$

$d_{I2} = d_2 - 2D = 360 - 10 = 350\,mm$

$d_{E2} = d_2 + 2m = 368\,mm$

$a = \frac{d_1}{2} + \frac{d_2}{2} = 240\,mm$

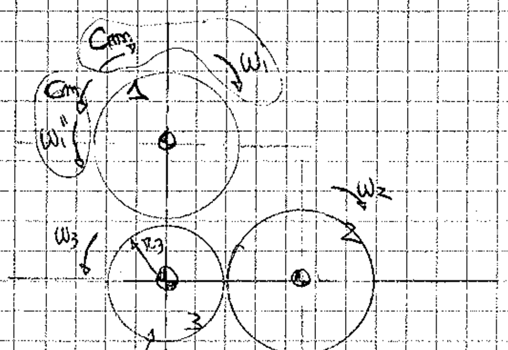
$i = \frac{z_2}{z_1} \rightarrow z_2 = 3 \cdot 30 = 90$

ESERCIZIO 5.5

$C_M = 150\,Nm \quad \alpha = 20°$ $\qquad F = ?$ $\qquad$ Dati dell'esercizio precedente

$C_m = F_t \cdot \frac{d_1}{2} = F \cdot \cos\alpha \cdot \frac{d_1}{2} \quad \rightarrow F = \frac{2 C_M}{d_1 \cos\alpha} = \frac{300}{0.12 \cdot \cos 20°} = 2660\,N$

ESERCIZIO 5.6



RUOTA OZIOSA

$i = \frac{\omega_1}{\omega_2} = \frac{\omega_1}{\omega_3} \cdot \frac{\omega_3}{\omega_2} = \frac{z_3}{z_1} \cdot \frac{z_2}{z_3} = \frac{z_2}{z_1}$

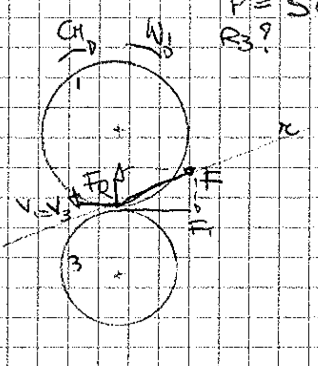
Avrei potuto fare lo stesso ingranaggio togliendo la ruota 3 e collegando insieme 1 e 2, ma in questo modo il verso di ω_2 cambierebbe

$z_3 = 40$
$n_3 = 360\,rpm$
$m = 6$
$\alpha = 20°$
$P = 5\,CV$
R_3 ?

$1\,CV = 735\,W$

$r_3 = \frac{m z_3}{2} = 120\,mm$

$P = C_m \cdot \omega_1$



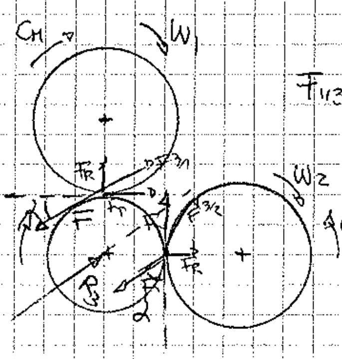
$C_m \cdot \omega_1 = F_T \cdot V = P \quad \Rightarrow F_T = \frac{P}{\omega_3 r_3}$

$V_3 = \omega_3 \cdot r_3$

$F = \frac{F_T}{\cos\alpha} = \frac{P}{\omega_3 r_3 \cos\alpha} = \frac{5 \cdot 735}{\cos 20° \cdot 0.12} \cdot \frac{30}{360\pi} = 864.5\,N$

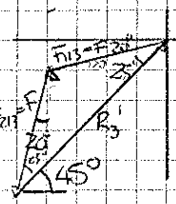
valido per entrambi i versi di rotazione

• m_1



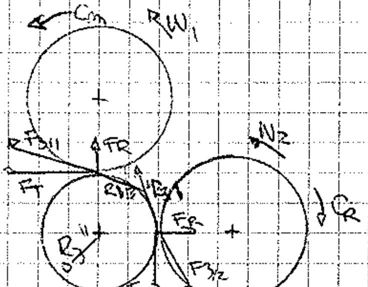
$$\vec{R}_3' + \vec{F}_{1/3} + \vec{F}_{2/3} = 0$$

$$F_{1/3} = F_{2/3} = F$$

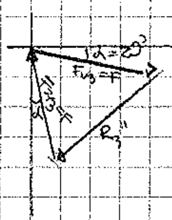


$$R_3' = 2F\cos 25° = 1567\,N$$

• m_2

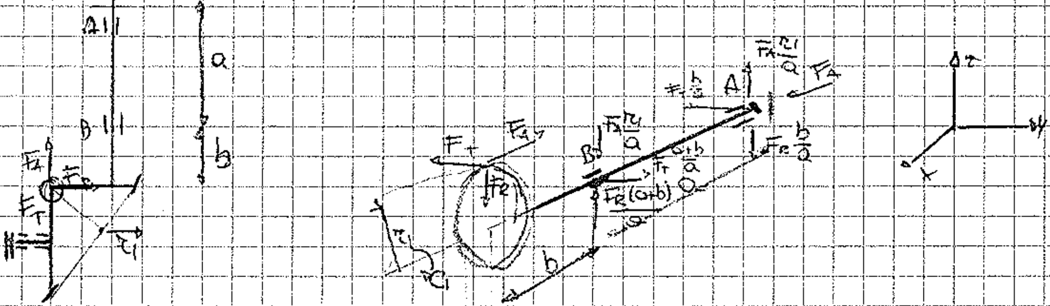


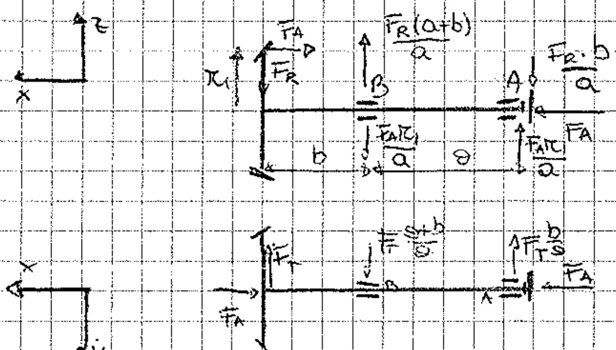
$$\vec{F}_{1/3} + \vec{F}_{2/3} + \vec{R}_3'' = 0$$



$$R_3'' = 2F\sin 25° = 731\,N$$

ESERCIZIO 5.10





$$R_A = \sqrt{\left(F_T\frac{b}{a}\right)^2 + \left(F_R\frac{b}{a} - F_A\frac{r_1}{a}\right)^2}$$

$$R_B = \sqrt{\left(F_T\frac{a+b}{a}\right)^2 + \left(F_R\frac{a+b}{a} - F_A\frac{r_1}{a}\right)^2}$$

ESERCIZIO 5.11

$W_u = 30\,kW$

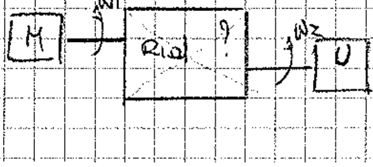
$\omega_1 = 1500\,rpm$

$i = 50$

$\eta = 0.9$

$C_1,\ C_2 = ?$

$W_p = ?$



$i = \frac{\omega_1}{\omega_2} = 50 = \frac{1500}{\omega_2}$ $\qquad \omega_2 = 30\ \text{rpm}$ $\qquad \omega_1 = 50\pi\ \text{rad/s}$

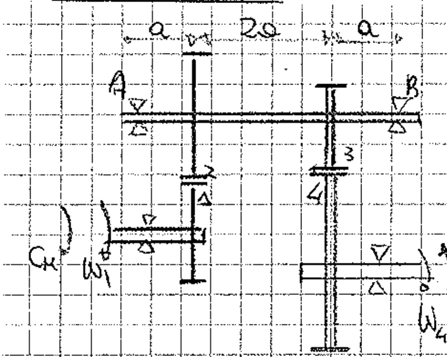
$\omega_2 = \pi\ \text{rad/s}$

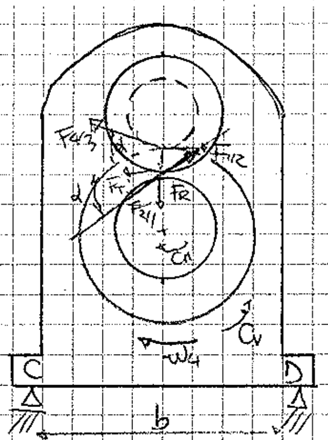
$W_u = C_2 \cdot \omega_2 \qquad C_2 = \frac{30 \cdot 10^3}{\pi} = 9549\ \text{N}\cdot\text{m}$

$\eta = \frac{W_u}{W_E} = \frac{C_2 \cdot \omega_2}{C_1 \cdot \omega_1} = \frac{C_2}{C_1\, i} \qquad C_1 = \frac{C_2}{\eta \cdot i} = 212.2\ \text{N/m}$

$\eta = \frac{W_u}{W_E} = \frac{W_E - W_P}{W_E} = 1 - \frac{W_P}{W_E} \rightarrow W_P = W_E\,(1 - \eta) = 3333.\bar{3}\ \text{W}$

ESERCIZIO 5.12





$z_1 = z_3 = 17$

$z_2 = z_4 = 52$

$m = 2.5\ \text{mm}$

$\alpha = 20°$

$C_M = 10\ \text{Nm}$

$b = 180\ \text{mm}$

$i_{14} = \frac{\omega_1}{\omega_2} \cdot \frac{\omega_3}{\omega_4} = \left(-\frac{z_2}{z_1}\right)\left(-\frac{z_4}{z_3}\right) > 0 \qquad R_A, R_B = ? \qquad C_S = ?$

coppia di reazione telaio

$= \left(\frac{52}{17}\right)^2 = 9.356$

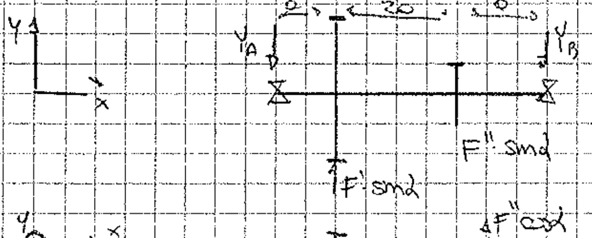
$r = \frac{m z}{2} \rightarrow r_1 = r_3 = 21.25\ \text{mm}$

$r_2 = r_4 = 65\ \text{mm}$

$F_T = \frac{C_M}{r_1} = F_{1/2} \cos\alpha = \frac{C_M}{r_1} = F' \cos\alpha \rightarrow F' = \frac{C_M}{r_1 \cdot \cos\alpha} \qquad F'' = \frac{C_U}{r_4 \cdot \cos\alpha}$

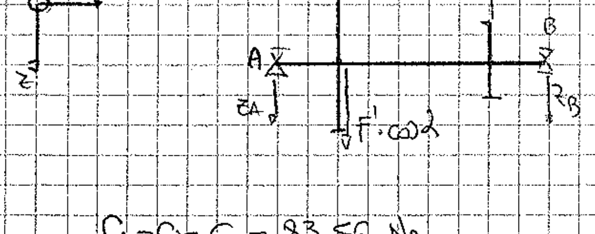
$\frac{C_U}{r_4} = F'' \cdot \cos\alpha$

$\eta = 1 = \frac{C_U \cdot \omega_4}{C_M \cdot \omega_1} = \frac{C_U}{C_M \cdot i_{14}} \qquad C_U = \eta \cdot i_{14} \cdot C_M$



$Y_A \cdot 4a = F' \sin\alpha \cdot 3a + F'' \sin\alpha \cdot a \rightarrow Y_A = \ldots$

$Y_B = (F' + F'') \sin\alpha - Y_A = \ldots$



$Z_A \cdot 4a = F'' \cos\alpha \cdot a - F' \cos\alpha \cdot 3a \rightarrow Z_A$

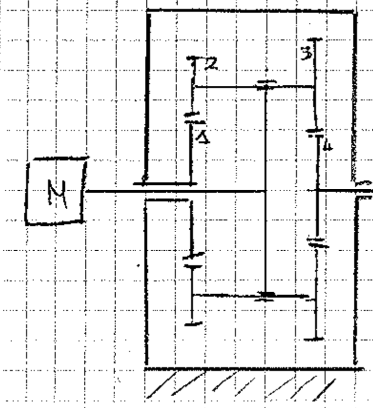
$Z_B = F'' \cos\alpha - F' \cos\alpha - Z_A$

$R_A = \sqrt{Z_A^2 + Y_A^2} = 259.5\ \text{N} \qquad R_B = \sqrt{Z_B^2 + Y_B^2} = 1056\ \text{N}$

$C_S = C_U - C_M = 83.56\ \text{Nm}$

$C_S = R \cdot b \qquad R = \frac{C_S}{b} = 464\ \text{N}$

ESERCIZIO 5.15



$\omega_1 = 0$ $\qquad z_2 = 17$

$P = 1.2\ kW$ $\qquad z_3 = 18$

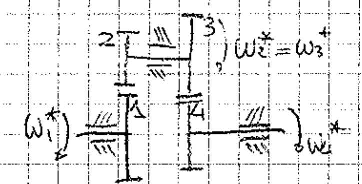
$n_M = 300\ rpm$ $\qquad m = 5\ mm$

$z_1 = 97$ $\qquad \alpha = 20°$

$i = \dfrac{\omega_M}{\omega_U} = ?$ $\qquad C_R = ?$

$F_{1,2} = ?$ $\qquad F_{3,4} = ?$

Sistema relativo al portatreno



Ω del portatreno $= \omega_M$ del motore

$$i^* = \frac{\omega_1^*}{\omega_4^*} = \frac{\omega_1^*}{\omega_2^*} \cdot \frac{\omega_3^*}{\omega_4^*} = \frac{-z_2}{z_1} \cdot \frac{-z_4}{z_3}$$

$$\underbrace{\frac{\omega_1 - \omega_M}{\omega_U - \omega_M}}_{\Delta} = \frac{z_2 z_4}{z_1 z_3} \qquad \omega_1 = 0$$

ω_U dell'utilizzatore $= \omega_4$

$$\frac{-\omega_M}{\omega_U - \omega_M} = i^* \qquad -\omega_M = i^*\omega_U - i^*\omega_M$$

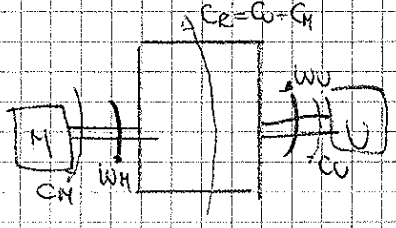
$$\omega_M(i^* - 1) = i^*\omega_U \quad \Rightarrow \quad \boxed{\frac{\omega_M}{\omega_U} = \frac{i^*}{i^* - 1} = -14.32} = i$$

Si nota che $r_1 + r_2 = r_3 + r_4$ $\qquad m = \dfrac{2r}{z}$

$\Downarrow$

$z_1 + z_2 = z_3 + z_4$ perché tutte e 4 le ruote hanno lo stesso modulo

$z_4 = 96$

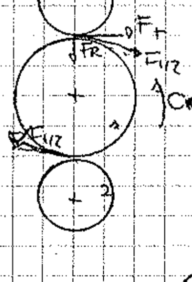


$$C_M = \frac{P_M}{\omega_M} = \frac{1200 \cdot 30}{300\,\pi} = \frac{120}{\pi}\ Nm$$

$$\eta = 1 = \frac{C_U \cdot \omega_U}{C_M \cdot \omega_M} = \frac{C_U}{C_M \cdot i}$$

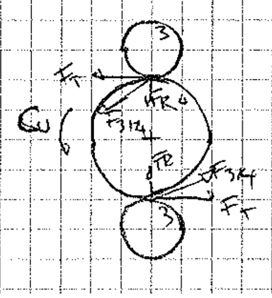
$$|C_U| = \eta \cdot C_M \cdot i = |C_M||i|$$

$$|C_R| = C_U + C_M = 15.32$$



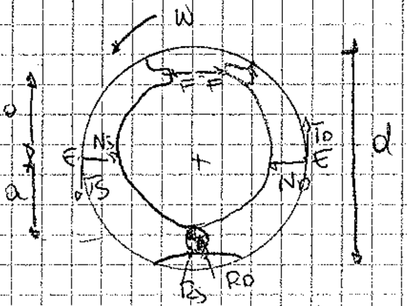
$$C_R = 2 r_1 F_{1/2} \cdot \cos\alpha \quad \Rightarrow \quad F_{1/2} = \frac{C_R \cdot 2}{2\cos\alpha \cdot m z_1} = 1284\,N$$

$$r = \frac{m z}{2}$$



$$C_U = 2 r_4 \cdot F_{3/4} \cos\alpha \quad \Rightarrow \quad F_{3/4} = \frac{C_U \cdot 2}{2\cos\alpha \cdot m z_4} = 1213\,N$$

ESERCIZIO 4.12



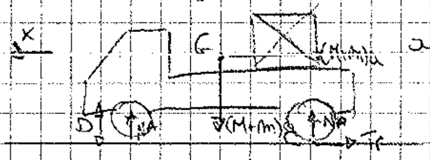
decelerazione $= 3\ m/s^2$ $\quad D = 0.8\ m$

$V_0 = 50\ km/h = \frac{50}{3.6}\ m/s$ $\quad d = 60\ cm$

$M = 3600\ kg$ $\quad a = 20\ cm$

$m = 400\ kg$

$f = 0.25$ $\quad t_f,\ S,\ f_{A(MIN)},\ F$?

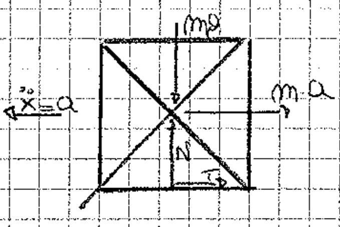


$a = \frac{d^2x}{dt^2}$ $\quad dec = -\frac{d^2x}{dt^2}$ $\Rightarrow$ $a = -3\ m/s^2$

$$\begin{cases} S = V_0 t + a\frac{t^2}{2} \\ V = V_0 + at \end{cases}$$

$0 = V_0 + a t_F$ $\quad t_F = -\frac{V_0}{a} = -\frac{50}{3.6(-3)} = 4.63\ s$

$S = 32.16\ m$



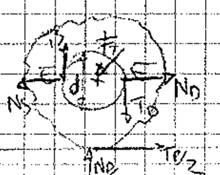
$T = f_{A(MIN)} \cdot N$

$N = mg$ $\quad T = -ma$

$-\cancel{m}\, a = f_{A(MIN)} \cdot \cancel{m} g$ $\quad f_{A(MIN)} = -\frac{a}{g} = \frac{3}{9.81} = 0.31$

Lo forza tangenziale è solo sulle ruote posteriori perchè sono quelle frenanti

Ci sono 2 ruote



$T_P = -(M+m)a = 12000\ N$ $\quad T_P/2 = 6000\ N$

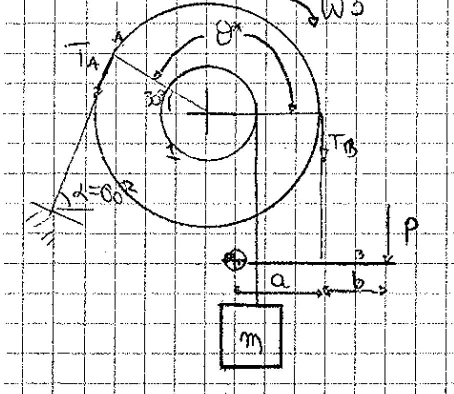
$\circlearrowleft O \quad \frac{T_P}{2}\frac{D}{2} = (T_S + T_D)\frac{d}{2}$

$\circlearrowleft A \quad F \cdot 2a + T_S\frac{d}{2} - N_S \cdot a = 0$

$F \cdot 2a - T_D\frac{d}{2} - N_D \cdot a = 0$ $\quad$ 5 equazioni in 5 incognite

$T_D = f N_D$ $\quad T_S = f N_S$ $\quad \Rightarrow F = 6875\ N$

ESERCIZIO 4.15



$W_0 = 32\ rpm$ $\quad C_f = ?$

$T = ?$ tempo di arresto

$D_1 = 350\ mm$

$D_2 = 800\ mm$

$e_1 = 2\ cm$ $\quad$ rigidezza elastica fune

$e_2 = 2\ cm$ $\quad$ " anelastico "

$P = 800\ N$

$m = 420\ kg$

$I = 52\ kg \cdot m^2$

$a = 300\ mm$

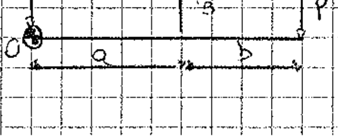
$b = 600\ mm$

$\alpha = 60°$

$f = 0.22$

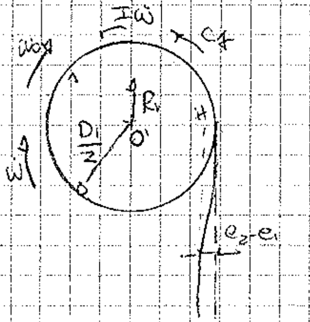
$\frac{T_A}{T_B} = e^{f\theta^*}$ $\quad (>1)$ $\quad \theta^* = 150° = \frac{150}{180}\pi = \frac{5}{6}\pi$ rad

$T_A > T_B$ $\quad C_f = (T_A - T_B)\frac{D_2}{2}$

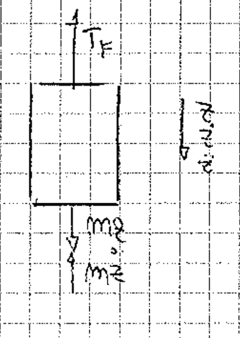


$T_B = P\frac{a+b}{a}$ $\quad T_A = T_B \cdot e^{f\theta^*}$

$$C_f = \frac{D_2}{2} P\left(e^{f\theta^*} - 1\right)\frac{a+b}{a} = \frac{0.8}{4}\, 800\left(e^{0.22\cdot\frac{2\pi}{6}} - 1\right)\frac{300}{300} = 789\ N\cdot m$$



$$\dot{z} = \omega\frac{D}{2}$$



$\curvearrowleft O'$ $\quad C_f + I\dot{\omega} - T_F\left(\frac{D_1}{2} - e_2 + e_1\right) = 0$

$$T_F = m(g - \ddot{z})$$

$$\dot{\omega} = \frac{2\ddot{z}}{D_1}$$

$$C_f + I\dot{\omega} - m\left(g - \frac{D_1}{2}\dot{\omega}\right)\left(\frac{D_1}{2} - e_2 + e_1\right) = 0$$

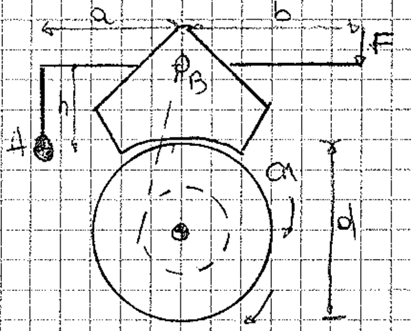
$$\dot{\omega}\left[I + m\frac{D_1}{2}\left(\frac{D_1}{2} - e_2 + e_1\right)\right] = -C_f + mg\left(\frac{D_1}{2} - e_2 + e_1\right)$$

$$\dot{\omega} = -3.06\ ^{rad}/_{s^2}$$

$$\omega = \omega_0 + \dot{\omega}t$$

$$0 = \omega_0 + \dot{\omega}T \qquad T = -\frac{\omega_0}{\dot{\omega}} = \frac{32\pi}{30\,(-3.06)} = 1.1\ s$$

ESERCIZIO 4.13

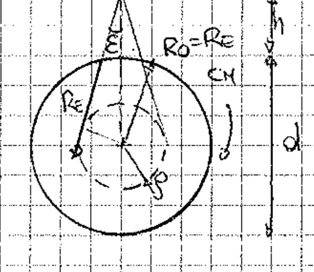


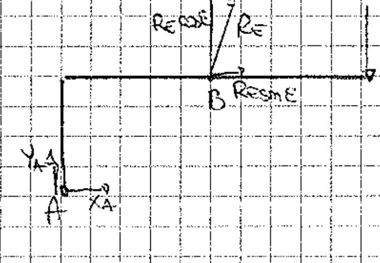
$a = 15\,cm$
$b = 30\,cm$
$h = 5\,cm$
$d = 22\,cm$
$F = 100\,N$
$f = 0.4$

C_M ?
R_A = ?

$$\sin\varepsilon = \frac{\rho}{h + d/2} \qquad \varepsilon = 14.78°$$

$$\rho = \frac{d}{2}\sin\varphi \qquad \varphi = tg^{-1} f$$





$\curvearrowleft A$ $\quad P\cdot(a+b) = R_E\cdot\cos\varepsilon\cdot a - R_E\sin\varepsilon\cdot h$

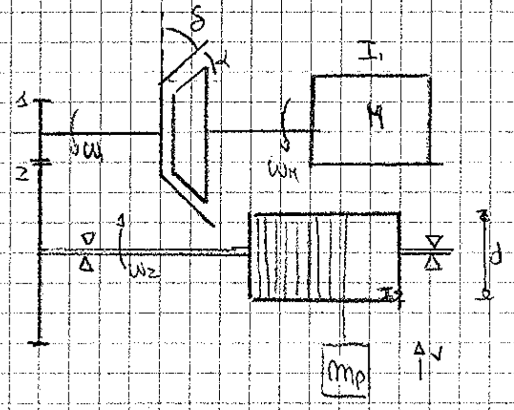
$$R_E = \frac{P(a+b)}{a\cos\varepsilon - h\sin\varepsilon}$$

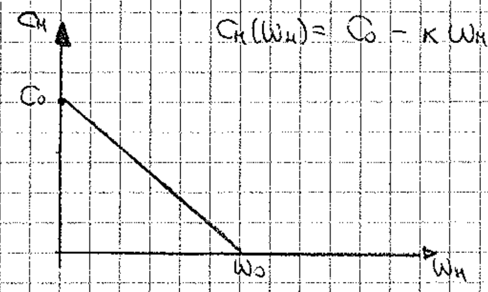
$$C_M = R_E\cdot\rho = 13.3\ N\cdot m$$

$$X_A = -R_E\sin\varepsilon = -86.85\,N \qquad Y_A = P - R_E\cos\varepsilon = -228.35\,N$$

$$R_A = \sqrt{X_A^2 + Y_A^2} = 245\,N$$

ESERCIZIO 6.17





$C_0 = 500\ Nm$ $\qquad \omega_0 = 300\ rpm$

$\eta = 1$

$m_M = 40\ kg$ massa delle parti rotanti del motore

$\rho_M = 15\ cm \rightarrow I_1 = m_M \cdot \rho_M^2$

$\delta = 65° \rightarrow \alpha = 25°$ $\qquad r_i = 20\ cm \qquad r_e = 24\ cm \qquad f = 0.1 \qquad F_0 = 400\ daN$

$z_1 = 17 \qquad z_2 = 90$

$m_P = 500\ kg \qquad d = 40\ cm$

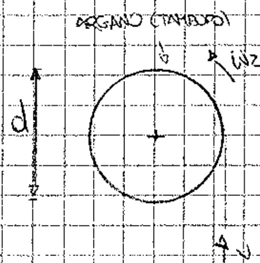
$I_2 = 40\ kg\,m^2$

1) $V_R = ?$ velocità di regime
$P = ?$

2) frizione innestata, $v = 0 \Rightarrow t_{90} = ?$
$(V_{90} = 0.9 V_R)$

3) $v = 0 \qquad \omega_M = 300\ rpm$
$\dot\omega_M = ? \qquad \dot v = ?$

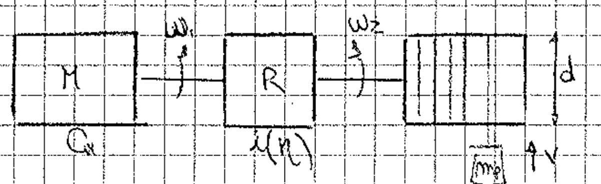
ORGANO (TAMBURO)

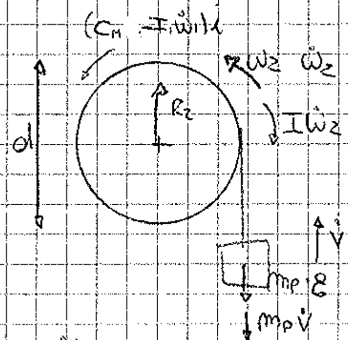


$v = \omega_2 \frac{d}{2} \qquad \dot v = \dot\omega_2 \frac{d}{2}$

$i = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{90}{17} \qquad \omega_1 = i\,\omega_2 = \frac{2v}{d} i$

$C_M = C_0 \left(1 - \frac{\omega_M}{\omega_0}\right)$





$C_M \cdot i = I_2 \dot\omega_2 + m_P g \frac{d}{2} + m_P \dot v \frac{d}{2}$

$\left(C_0 - \frac{C_0}{\omega_0}\omega_1 - I_1\dot\omega_1\right) i = I_2 \dot\omega_2 + m_P g \frac{d}{2} + m_P \frac{d}{2} \dot v \qquad \dot\omega_2 = \frac{2\dot v}{d} \qquad \omega_1 = \frac{2i}{d} v$

~~$C_0 i - \frac{C_0}{\omega_0} i^2 \frac{2}{d} v = I_2 \frac{2}{d} \dot v - m_P g \frac{d}{2} - m_P \frac{d}{2} \dot v$~~

$\left(C_0 - \frac{C_0}{\omega_0}\omega_1 - I_1 \dot\omega_1\right) i = I_2 \dot\omega_2 + m_P \frac{d}{2} g + m_P \frac{d}{2} \dot v$

$C_0 \cdot i - C_0/\omega_0\, i^2 \frac{2}{d} v - I_1 i^2 \frac{2}{d} \dot v = I_2 \frac{2}{d} \dot v + m_P g \frac{d}{2} + m_P \frac{d}{2} \dot v$

$\dot v \underbrace{\left(I_2 \frac{2}{d} + m_P \frac{d}{2} + I_1 \frac{2}{d} i^2\right)}_{K_1} = \underbrace{\left(C_0 i - m_P g \frac{d}{2}\right)}_{K_1} - \underbrace{\left(\frac{C_0}{\omega_0} \frac{2}{d} i^2\right)}_{K_2} v$

$K_1 \cdot \dot v = K_1 - K_2 \cdot v \qquad K_1 = 426\ N\,s^2 = kg\,m$

$K_1 - K_2 v = K_1 \frac{dv}{dt} \qquad K_1 = 1666\ N\,m$

$K_2 = 2230\ N\,s$

$$\int_0^{t_{90}} dt = K_1\int_0^{0.9V_R}\frac{dv}{K_1-K_2v} \qquad \dot v=0 \qquad V=V_R$$

$$K_1 = K_2\cdot V_R \qquad V_R=\frac{K_1}{K_2}=\frac{1669}{2230}\frac{N}{\frac{N s}{m}}=0.747\ \frac{m}{s}$$

$$P_{REG} = V_R\cdot m_T\cdot g = 3664\ W$$

$$\int_0^{t_{90}} dt = K_1\int_0^{0.9V_R}\frac{dV}{K_1-K_2V}=t_{90} \qquad \frac{dA}{dV}=-K_2 \qquad dA=-K_2\,dV \qquad dV=-\frac{dA}{K_2}$$

$$A=K_1-K_2V$$

$$t_{90}=-\frac{K_1}{K_2}\int_{K_1}^{K_1-K_2\cdot 0.9V_R=K_1-0.9K_1=0.1K_1}\frac{dA}{A}=-\frac{K_1}{K_2}\ln\left(\frac{0.1\cancel{K_1}}{\cancel{K_1}}\right)=\frac{K_1}{K_2}\ln 10 = 0.44\ s$$

$$V_R=\frac{K_1}{K_2}$$

$$\cdot\ C_1=\frac{1}{\sin\alpha}F_0\frac{r_i+r_e}{2}=\frac{0.1}{\sin 25°}\cdot 4000\,\frac{0.2+0.24}{2}=208.2\ N\,m$$

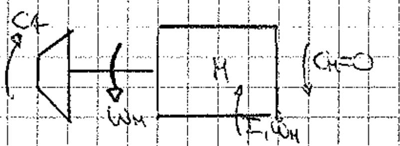
$\alpha=25°$

$$C_1\cdot i = I_2\dot\omega_2 + m_p g\frac{d}{2} + m_p\frac{d}{2}\dot v$$

$$C_1\cdot i = I_2\dot v\frac{2}{d} + m_p g\frac{d}{2} + m_p\dot v\frac{d}{2}$$

$$\dot v\left(\frac{2I_2}{d}+m_p\frac{d}{2}\right)=C_1 i - m_p g\frac{d}{2}$$

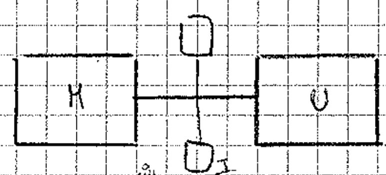
$$\dot v = 0.45\ m/s^2$$



$$C_1+I_1\dot\omega_M=0$$

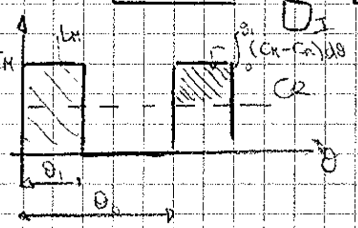
$$\dot\omega_M=-\frac{C_1}{I_1}=-231.3\ \frac{rad}{s^2}$$

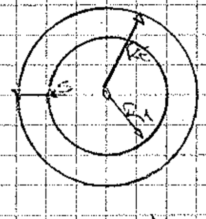
ESERCIZIO 6.19



$\theta_0=2\pi$ densità $\gamma=7860\ kg/m^3$

$R_i=80$ mm
$R_e=120$ mm
$s=40$ mm





$P_M=24$ CV

$\omega_M=4000$ rpm

$\theta_0=\frac{1}{4}2\pi$ rad

$\theta_1=\frac{1}{3}\theta_0$

$C_M=?$
$I=?$ del volano
$\omega_{MIN}=?$
$\omega_{MAX}=?$
$\lambda=?$

$$L_T=0=L_M+L_U=\int_0^{\theta_0}C_M\,d\theta+\int_0^{\theta_0}-C_R\,d\theta$$

$$C_M\theta_1+0(\theta_0-\theta_1)=C_R\theta_0$$

$$I=I_E-I_I=m_E\frac{R_e^2}{2}-m_I\frac{R_i^2}{2}$$

$$m=\gamma\cdot V=\gamma\cdot R^2\pi\cdot s$$

$$I=\frac{\gamma\pi s}{2}(R_e^4-R_i^4)=4\cdot 10^{-3}\ kg\,m^2$$

$$P_M=C_R\ \omega_{MED} \qquad C_R=24\cdot\frac{735\cdot 30}{4000\cdot\pi}=42.11\ N\cdot m$$

$$C_M=C_R\frac{\theta_0}{\theta_1}=3\cdot C_R=126.33\ Nm$$

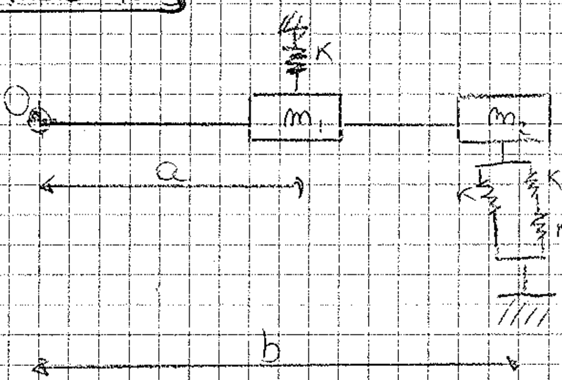
$$I\,i\,\omega^2_{med}=(C_M-C_R)\theta_1 \qquad i=0.01436$$

$$\begin{cases} 2\omega_{MED} = \omega_{MAX} + \omega_{MIN} & (1) \\ \omega_{MED}\, i = \omega_{MAX} - \omega_{MIN} & (2) \end{cases} \Rightarrow \overset{(1)+(2)}{2\omega_{MAX} = \omega_{MED}(2+i)} \qquad \overset{(1)-(2)}{2\omega_{MIN} = \omega_{MED}(2-i)}$$

$\omega_{MIN} = 3971.3\ rpm = 415.9\ rad/s$

$\omega_{MAX} = 4028.7\ rpm = 421.9\ rad/s$

ESERCIZIO 7.13

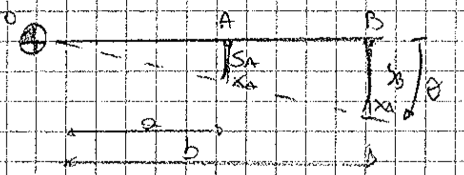


$m_1 = 10\ kg$
$m_2 = 20\ kg$

$K = 1000\ N/m$

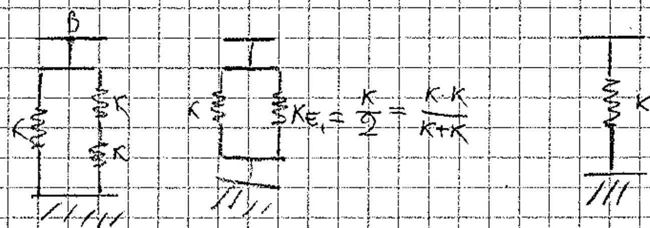
$a = 1m$
$b = 2m$

$T = f(K, m_1, m_2, a, b)$?



$S_A = a \cdot \theta \cong x_A$

$S_B = b \cdot \theta \cong x_B$



$K_{E_1} = \frac{K}{2} = \frac{K \cdot K}{K+K}$

$K_E = K + \frac{K}{2} = \frac{3}{2}K$

DIAGRAMMA DI CORPO LIBERO DINAMICO

$F_{E_1} = K \cdot a \cdot \theta$

$F_{E_2} = \frac{3}{2}K \cdot b \cdot \theta$

$F_{i_1} = m_1 \cdot \ddot{x}_A = m_1 \cdot a \cdot \ddot{\theta} \qquad F_{i_2} = m_2\, b\ddot{\theta}$

$\circlearrowleft O)$

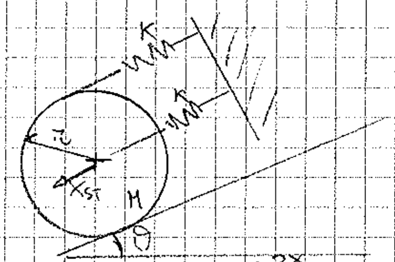
$$m_1 a^2 \ddot{\theta} + m_2 b^2 \ddot{\theta} + K a^2 \theta + \frac{3}{2} K b^2 \theta = 0$$

$$(m_1 a^2 + m_2 b^2)\ddot{\theta} + \left(K a^2 + \frac{3}{2} K b^2\right)\theta = 0$$

$$\ddot{\theta} + \underbrace{\frac{K a^2 + \frac{3}{2} K b^2}{m_1 a^2 + m_2 b^2}}_{\omega_n^2} \cdot \theta = 0$$

$$\omega_n = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{m_1 a^2 + m_2 b^2}{K a^2 + \frac{3}{2} K b^2}} = 0.71\, s$$

ESERCIZIO 7.14



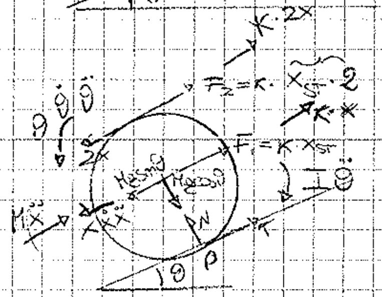
$M = 500\ kg$

$k = 10\ \frac{kN}{m}$

$\theta = 30°$

$x_{st} = ?$

$f_m = ? = \frac{\omega_m}{2\pi}$



$\curvearrowright P \quad k \cdot 2x_{ST} \cdot 2r + k \cdot x_{ST} \cdot r = Mg \sin\theta\ r$

$$x_{ST} = \frac{Mg \sin\theta}{5k} = 49.1\,mm$$

$x = r\theta$

$\ddot{x} = r\ddot{\theta}$

$I = \frac{Mr^2}{2}$

$\curvearrowright P$

$$I\ddot{\theta} + Mr\ddot{x} + krx + k2r \cdot 2x = 0$$

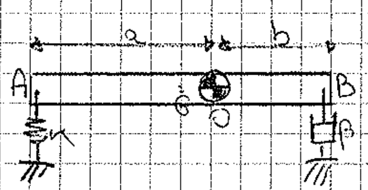
$$\frac{Mr^2}{2}\ddot{\theta} + Mr^2\ddot{\theta} + 5kr^2\theta = 0$$

$$\frac{3}{2}M\ddot{\theta} + 5k\theta = 0$$

$$\ddot{\theta} + \underbrace{\frac{10k}{3M}}_{\omega_m^2}\theta = 0 \qquad \omega_m = \sqrt{\frac{10}{3}\frac{k}{M}}$$

$$f_m = \frac{1}{2\pi}\sqrt{\frac{10k}{3M}} = 1.3\,Hz$$

ESERCIZIO 7.22



$a = 1.2\,m$

$b = 0.8\,m$

$m = 80\,kg$

$k = 50\,kN/m$

$\zeta = 0.5$

$I_G = \frac{m\ell^2}{12}$

$\beta = ?$

$j\,O\,\theta\,\ddot{\theta}$

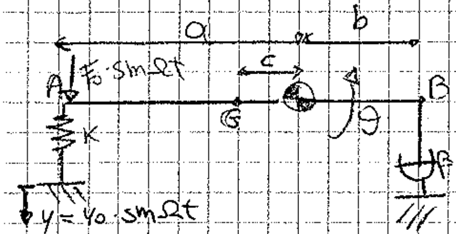
$\curvearrowright O \quad I_G\ddot{\theta} + m\,OG\,\ddot{\theta} + \beta b^2\dot{\theta} + ka^2\theta = 0$

$$\underbrace{\left(\frac{m(a+b)^2}{12} + m\,OG^2\right)}_{I_O}\ddot{\theta} + \beta b^2\dot{\theta} + ka^2\theta = 0$$

$$\ddot{\theta} + \underbrace{\left(\frac{\beta b^2}{I_O}\right)}_{2\zeta\omega_m}\dot{\theta} + \underbrace{\left(\frac{ka^2}{I_O}\right)}_{\omega_m^2}\theta = 0 \qquad \omega_m = \sqrt{\frac{ka^2}{I_O}} \qquad \frac{\beta b^2}{I_O} = 2\zeta\omega_m$$

$$\beta = \frac{2\zeta\omega_m I_O}{b^2} = 2231\ \frac{Ns}{m}$$

ESERCIZIO 7.23



$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$$

$$\theta = \Theta \sin(\omega_s t - \varphi) \cdot e^{-\zeta\omega_n t}$$

$$\omega_n\sqrt{1-\zeta^2} = \omega_s$$

$\beta = ?$

$\zeta = 0{,}5$

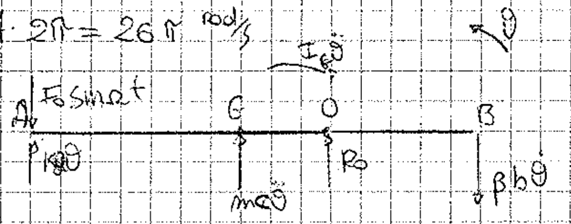
$\omega_n = 48.1 \ \text{rad/s}$

$F_0 = 200\,N$

$f = 13\,Hz$

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = A \cdot \sin \Omega t$$

$$\Omega = 4 \cdot 2\pi = 26\pi \ \text{rad/s}$$



$$I_G = \frac{m\ell^2}{12}$$

$$I_G + mc^2 = I_O$$

$$I_G\ddot{\theta} + mc^2\ddot{\theta} + \beta b^2\dot{\theta} + ka^2\theta - F_0\, a \sin \Omega t = 0$$

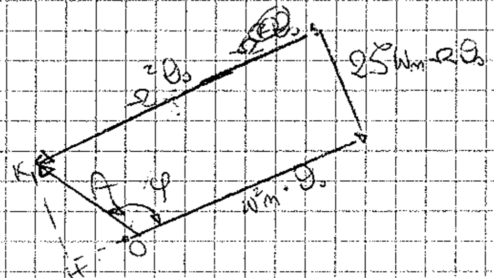
$$\ddot{\theta} + \frac{\beta b^2}{(I_G + mc^2)}\dot{\theta} + \frac{ka^2}{(I_G + mc^2)}\theta = \frac{F_0 \cdot a}{(I_G + mc^2)}\sin \Omega t$$

$2\zeta\omega_n \to \frac{\beta b^2}{I_O}$

$\omega_n^2 \to \frac{ka^2}{I_O}$

$A \to \frac{F_0 \cdot a}{I_O}$

$A = 8.04 \ \text{rad/s}^2$ $\qquad \theta(t) = \theta_0 \sin(\Omega t - \varphi)$



$$A^2 = (OH)^2 + (HK)^2 = \theta_0^2(\Omega^2 - \omega_n^2)^2 + (2\zeta\omega_n\Omega\theta_0)^2$$

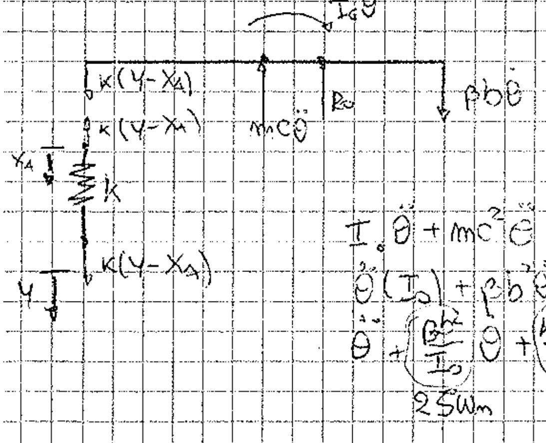
$$\theta_0 = \frac{A}{\sqrt{(-\Omega^2 - \omega_n^2)^2 + (2\zeta\omega_n\Omega)^2}} = 1{,}37 \cdot 10^{-3} \ \text{rad}$$

$$x_A = \theta \cdot a = X_{A0} \cdot \sin(\Omega t - \varphi)$$

$$X_{A0} = \theta_0 \cdot a = 1.65 \ \text{mm}$$

* Variante: non applico la forza, ma sposto la molla di $y = Y_0 \sin \Omega t$

$$\theta = \theta_0 \cdot \sin(\Omega t - \varphi)$$



$$I_G\ddot{\theta} + mc^2\ddot{\theta} + \beta b^2\dot{\theta} - ka(y - a\theta) = 0$$

$$\ddot{\theta}(I_O) + \beta b^2\dot{\theta} + ka^2\theta = kY_0 \sin \Omega t$$

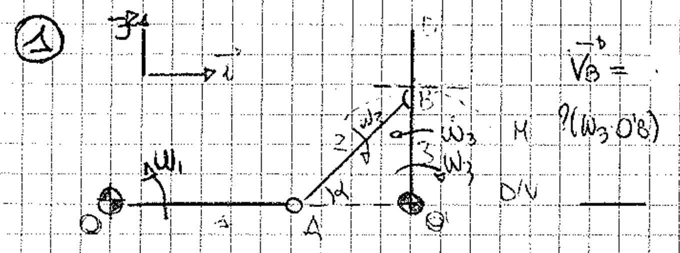
$$\ddot{\theta} + \underbrace{\left(\frac{\beta b^2}{I_O}\right)}_{2\zeta\omega_n}\dot{\theta} + \underbrace{\left(\frac{ka^2}{I_O}\right)}_{\omega_n^2}\theta = \underbrace{\left(\frac{kY_0 a}{I_O}\right)}_{B}\sin \Omega t$$

$Y_0 = 10\,mm$ $\qquad f = 7\,Hz$ $\qquad \Omega = 7 \cdot 2\pi \ \text{rad/s}$

$\theta_0 = 0.081 \ \text{rad}$ $\qquad X_{A0} = 10.8 \ \text{mm}$

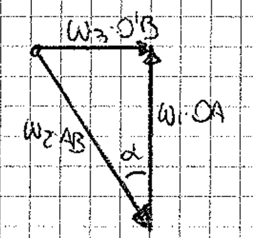
TEMA D'ESAME

(1)



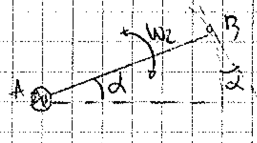
$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}}$

$?(\omega_3 \cdot O'B) \quad \omega_1 \cdot OA \quad ?(\omega_2 \cdot AB)$



$\alpha = tg^{-1} \dfrac{O'B}{OO'-OA} = 26.56°$

$OO'-OA = O'S$



$\cos\alpha = \dfrac{AO'}{AB}$

$\omega_3 \cdot O'B = \omega_1 \cdot OA \cdot tg\alpha$

$\omega_3 = \omega_1 \cdot \dfrac{OA}{O'B} \cdot \dfrac{O'B}{OO'-OA} = 2.5 \; rad/s$

$\omega_1 \cdot OA = \omega_2 \cdot AB \cdot \cos\alpha$

$\omega_2 = \omega_1 \cdot \dfrac{OA}{AB \cos\alpha} = \omega_1 \dfrac{OA}{AB} \cdot \dfrac{AB}{AO'} = 2.5 \; rad/s$

$AO' = OO'-OA$

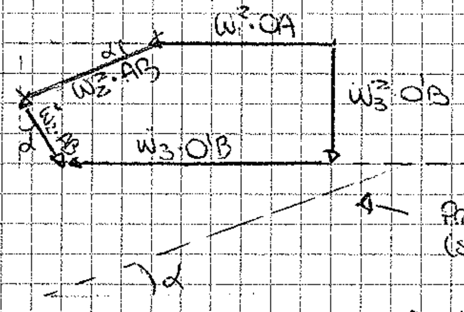
Il centro di istantanea rotazione del corpo 2 è O' in questo momento (centro fisso di rotazione di 3) ⇓ 2 e 3 hanno la stessa veloc.

$V_D = O'D \cdot \omega_3 = 0.3 \; m/s \; \vec{i}$

$\vec{a_B} = \vec{a_A} + \vec{a_{B/A}}$

$\vec{a_{Bn}} + \vec{a_{Bt}} = \vec{a_{An}} + \vec{a_{At}} + \vec{a_{B/An}} + \vec{a_{B/At}}$

$\omega_3^2 O'B \quad (\dot{\omega}_3 O'B) \quad \omega_1^2 OA \quad =0 \quad \omega_2^2 \cdot AB \quad (\dot{\omega}_2 \cdot AB)$

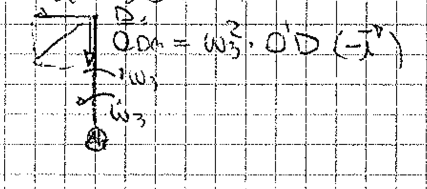


Proietto su questa direzione (sparisce $\dot{\omega}_2$ che non è richiesto)

$\omega_2^2 \cdot AB + \omega_1^2 \cdot AO \cos\alpha = \omega_3^2 \cdot O'B \sin\alpha + \dot{\omega}_3 \cdot O'B \cos\alpha$

$\dot{\omega}_3 = \dots = 62.5 \; rad/s^2$

$\vec{a}_D = \vec{a}_{Dn} + \vec{a}_{Dt}$

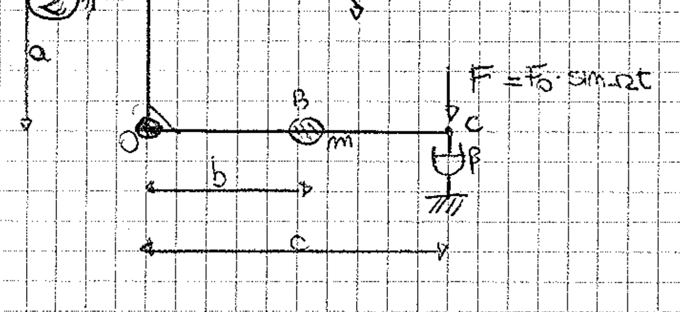


$\vec{a}_{Dt} = \dot{\omega}_3 \cdot O'D (-\vec{j})$

$\vec{a}_{Dn} = \omega_3^2 \cdot O'D (-\vec{i})$

$|a_D| = \sqrt{\omega_3^4 + \dot{\omega}_3^2} \cdot O'D = 7.54 \; m/s^2$

(2)



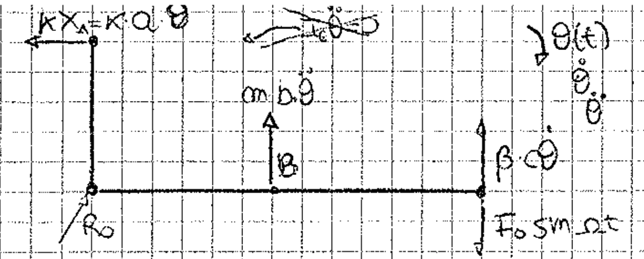
$F = F_0 \cdot \sin\Omega t$

$m = 40 \; kg$ $\quad$ $a = 20 cm$

$\beta = 100 \; \frac{N \cdot s}{m}$ $\quad$ $b = 30 cm$

$K = 8000 \; \frac{N}{m}$ $\quad$ $c = 60 cm$

$F_0 = 8 N$ $\quad$ $\Omega = 8 \; rad/s$



$$\sum M_O = 0$$

$$mb^2\ddot\theta + \beta c^2 \dot\theta + ka^2\theta = F_0 \cdot c \cdot \sin \Omega t$$

$$\ddot\theta + \underbrace{\frac{\beta c^2}{mb^2}}_{2\zeta\omega_n}\dot\theta + \underbrace{\frac{ka^2}{mb^2}}_{\omega_n^2}\theta = \underbrace{\frac{F_0 \cdot c}{mb^2}}_{A}\sin \Omega t$$

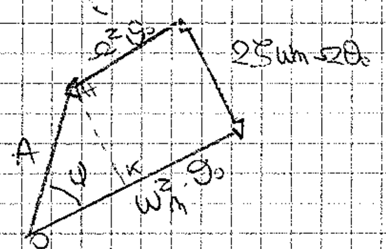
$\omega_n = 10\ rad/s$

$\zeta = 0.5$

$A = 1.5\ rad/s^2$

$x_C = \theta \cdot c$

$x_C = x_{C0} \cdot \sin(\Omega t - \varphi)$

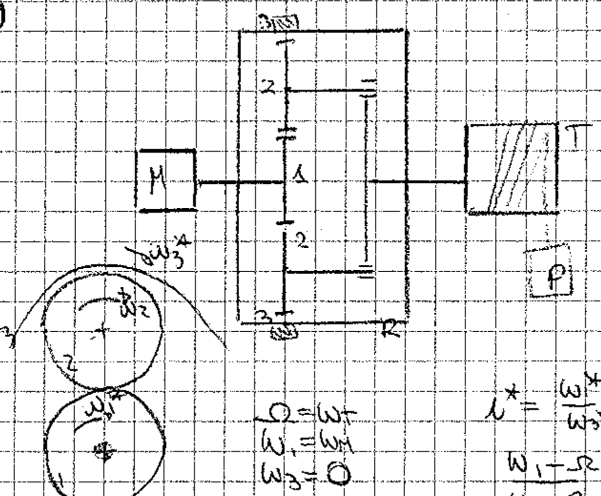


$$A^2 = \overline{OK}^2 + \overline{HK}^2 = \theta_0^2(\omega_n^2 - \Omega^2)^2 + (2\zeta\omega_n\Omega)^2 \cdot \theta_0^2$$

$\theta_0 = 0.0171\ rad/s$

$x_{C0} = \theta_0 \cdot c = 10.3\ mm$

3



$z_1 = 20$

$z_2 = 30$

$P = 200 N$

$r = 20 cm$

$C_M = 20 Nm$

$I_M = 0.1\ kg\, m^2$

$I_T = 0.4\ kg\, m^2$

$\eta = 0.9$

$r_3 = r_1 + 2r_2 \qquad m = \frac{p}{\pi} = \frac{2\pi r}{\pi z} = \frac{2r}{z}$

$z_3 = z_1 + 2z_2 = 80$

$\Omega = \omega_T$

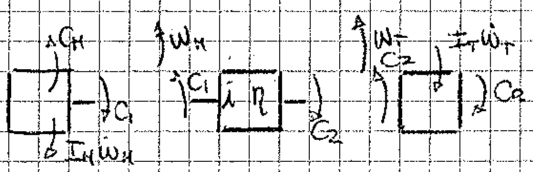
$\omega_1 = \omega_M$

$\omega_3 = 0$

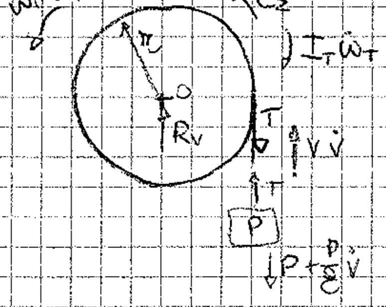
$$i^* = \frac{\omega_1^*}{\omega_3^*} = \frac{\omega_1 - \Omega}{\omega_3 - \Omega} = \frac{\omega_1^*}{\omega_2^*} \cdot \frac{\omega_2^*}{\omega_3^*} = -\frac{z_2}{z_1} \cdot \frac{z_3}{z_2} = -\frac{z_3}{z_1} \qquad \omega_1 \parallel \Omega$$

$$\frac{\omega_1 - \Omega}{\omega_3 - \Omega} = -\frac{z_3}{z_1} = \frac{\omega_M - \omega_T}{-\omega_T} \qquad \frac{z_3}{z_1} = \frac{\omega_M}{\omega_T} - 1 \qquad i = \frac{\omega_M}{\omega_T} = 1 + \frac{z_3}{z_1} = 5$$

(ω_M concorde con ω_T)



$$\eta = \frac{P_2}{P_1} = \frac{C_2\omega_T}{C_1\omega_M} = \frac{C_2}{i \cdot C_1} \qquad C_2 = C_1 \cdot i\eta$$



$T = P + \frac{P}{g}\dot v$

$v = \omega_T \cdot r$

$\dot v = \dot\omega_T \cdot r$

$C_2 = C_1 \cdot i \cdot \eta = i\eta(C_M - I_M\dot\omega_M)$

$\dot\omega_M = \dot\omega_T \cdot i$

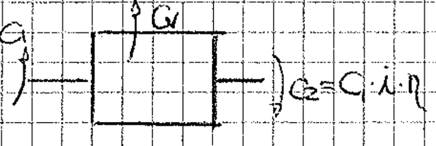
$C_2 = C_M \cdot i \cdot \eta - I_M \cdot i^2 \eta \cdot \dot\omega_T$

$\circlearrowright O)\quad C_2 - I_T\dot\omega_T - P r - \frac{P}{g} r \dot v = 0$

$$C_M i \eta - I_M \cdot i^2 \eta\, \dot\omega_T - P r - \frac{P}{g} r^2 \dot\omega_T - I_T \cdot \dot\omega_T = 0$$

$\dot\omega_T = \ldots = 14.4\ rad/s^2$

$\dot\omega_M = \dot\omega_T \cdot i \cong 72\ rad/s^2 \qquad \dot v = \dot\omega_T \cdot r = 2.88\ m/s^2$



$C_V = C_2 - C_1 = C_1(i\eta - 1) = 4.75\ Nm$

$C_1 = C_M - I_M \cdot \dot\omega_M$