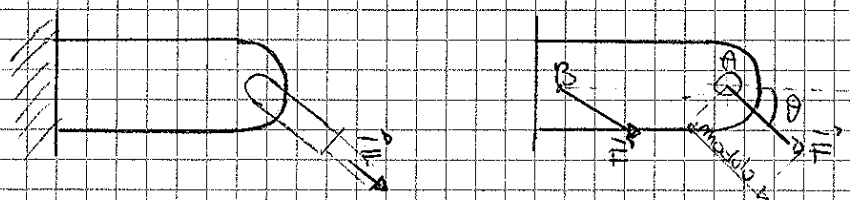
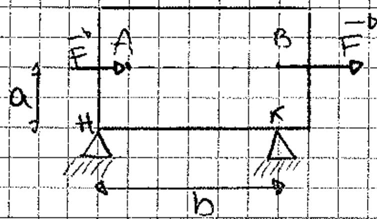
DINAMICA

Studio le caratteristiche delle forze che agiscono nei sistemi meccanici

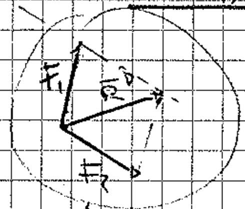


le forze sono vettori applicati, ovvero hanno una caratteristica che è il punto di applicazione



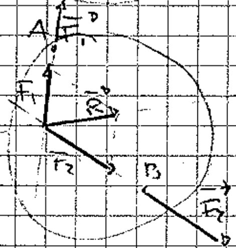
SLIDING VECTOR: il vettore può essere applicato in un punto qualsiasi lungo la sua retta d'azione x un corpo rigido e l'effetto non cambia
PRINCIPIO DI TRASMISSIBILITÀ

SOMMA/COMPOSIZIONE DELLE FORZE

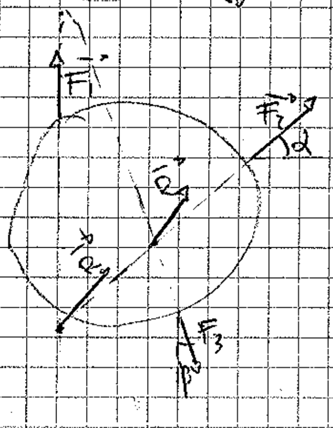


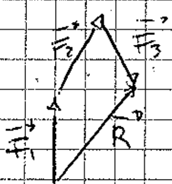
SOMMA (COMPOSIZIONE)

$\vec{R} = \vec{F_1} + \vec{F_2}$

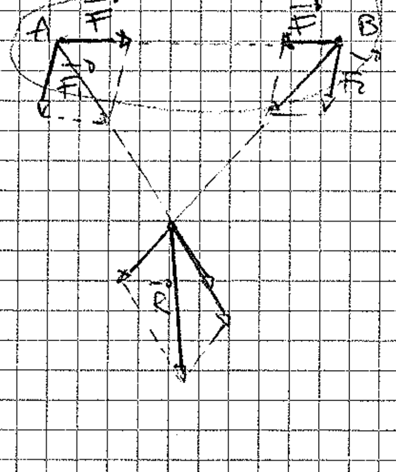


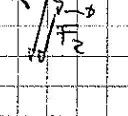
$\vec{R} = \vec{F_1} + \vec{F_2}$





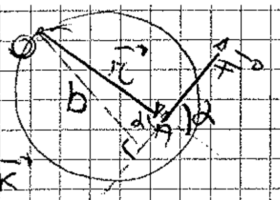
$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$





$\vec{R} = \vec{F_1} + \vec{F_2}$

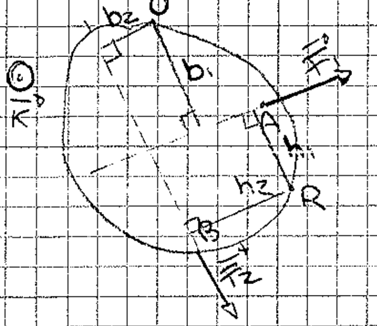
MOMENTO DI UNA FORZA



$$\vec{M}_O = \vec{r} \times \vec{F} = rF \cdot \sin\alpha\, \vec{k} = bF\vec{k}$$

$b = r\sin\alpha$ braccio della forza

~~Teorema~~

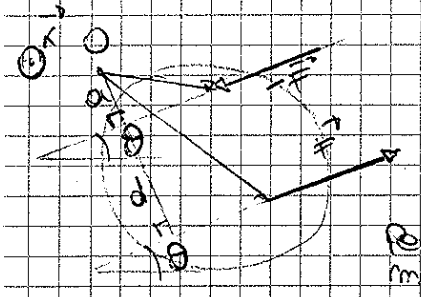


momento rispetto ad O

$$\vec{M}_O = b_1 \cdot F_1 \cdot \vec{k} + b_2 \cdot F_2 \cdot \vec{k}$$

$$\vec{M}_R = h_1 \cdot F_1 \cdot (-\vec{k}) + h_2 \cdot F_2 \cdot \vec{k}$$

momento rispetto a R

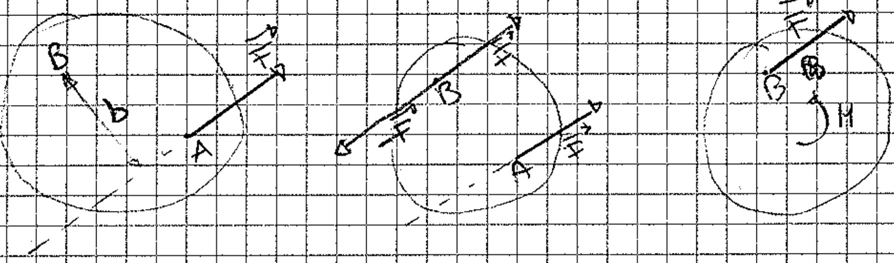


COPPIA DI FORZE

$$\vec{M}_O = \vec{r}_1 \times \vec{F} + \vec{r}_2 \times (-\vec{F}) = (a+d)F\vec{k} + aF(-\vec{k}) =$$

$$= d \cdot F \cdot \vec{k}$$

Per una coppia di forze parallele (uguali e opposte) il momento non dipende dal punto O, ma solo dalla distanza tra le 2 forze (tra le 2 rette d'azione)



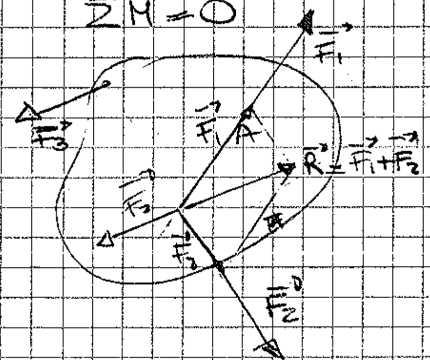
$M = bF$ MOMENTO DI TRASPORTO (o di TRASPOSIZIONE)

EQUILIBRIO DI UN SISTEMA DI FORZE

11-03-10

$$\sum \vec{F} = 0$$

$$\sum \vec{M} = 0$$

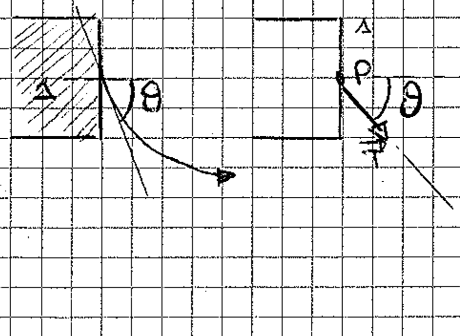


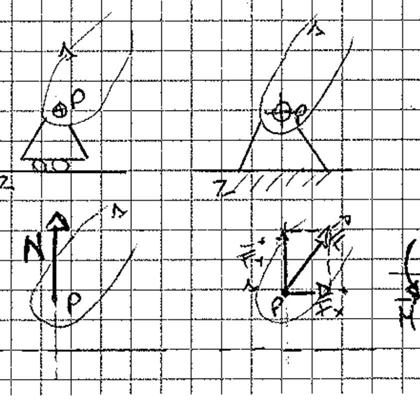
$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

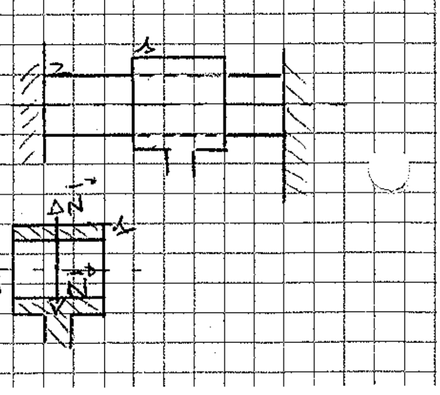
$$\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$$

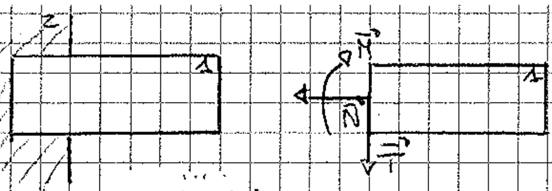
1

Incastro Prismatico

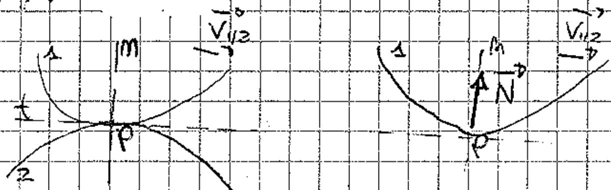




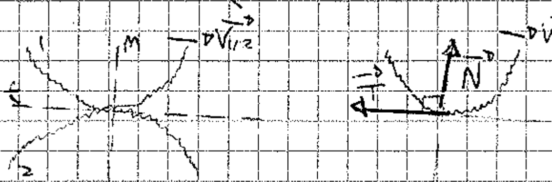




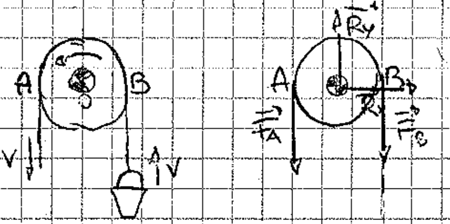
INCASTRO



SUPERFICI LISCE (senza attrito)



SUPERFICI SCABRE o RUVIDE



LEGGI DI NEWTON

1) Se la risultante delle forze su un corpo è nulla, il corpo o è in quiete o continua a muoversi di moto rettilineo uniforme

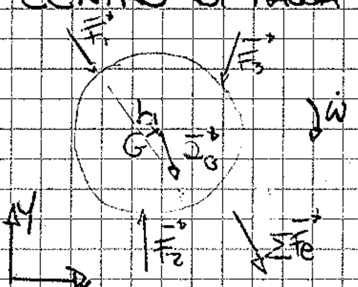
2) $\sum \vec{F_e} \neq 0 \qquad \sum \vec{F_e} = \vec{R}$

$$\underline{\sum \vec{F_e} = m\,\vec{a}}$$

3)

$$\vec{F}_{2,1} = -\vec{F}_{1,2}$$

CENTRO DI MASSA - BARICENTRO - MOMENTO D'INERZIA



$$\underline{\sum \vec{F_e} = m\,\vec{a}_G}$$

$$\sum M_{eG} = b_1 F_1 \vec{k} + b_2 F_2(-\vec{k}) + b_3 F_3(-\vec{k})$$

$$\underline{\sum \vec{M}_{eG} = I_G\,\dot{\vec{\omega}}}$$

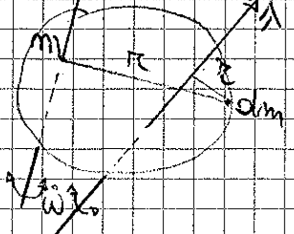
I_G – MOMENTO D'INERZIA BARICENTRICO

$\sum \vec{M}_{eG}$ – momento risultante delle forze esterne rispetto al baricentro

$[M] = N \cdot m = \frac{Kg \cdot m^2}{s^2}$

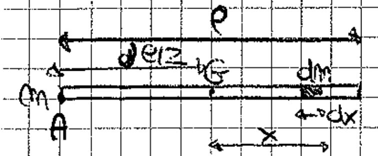
$[I] = Kg \cdot m^2$

sono due cose diverse anche se si chiamano con lo stesso nome



$$I_k = \int_m r^2\,dm = \int dI_k$$

$$I_\lambda = \int dI_\lambda = \int h^2\,dm$$

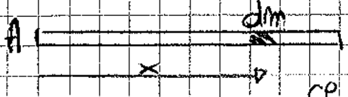


$m = \ell \cdot \rho$

$dm = \rho \cdot dx$

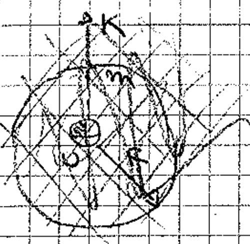
$dI_G = x^2 dm = x^2 dx \cdot \rho$

$$I_G = \rho \int_{-\ell/2}^{\ell/2} x^2 dx = \rho \left[\frac{x^3}{3} \right]_{-\ell/2}^{\ell/2} = \rho \frac{\ell^3}{12} = \frac{m\ell^2}{12}$$



$0 \le x \le \ell$

$$I_A = \rho \int_0^{\ell} x^2 dx = \rho \frac{\ell^3}{3} = \frac{m\ell^2}{3}$$



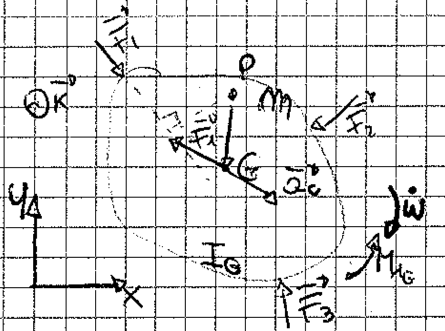
$$\boxed{\begin{cases} \sum \vec{F_e} = m \vec{a}_G \\ \sum \vec{M}_{e_G} = I_G \dot{\vec{\omega}} \end{cases}}$$

D'ALEMBERT: $-m \vec{a}_G = \vec{F_i}$ RISULTANTE DELLE FORZE D'INERZIA

$-I_G \dot{\vec{\omega}} = \vec{M}_{iG}$ MOMENTO RISULTANTE DELLE FORZE D'INERZIA (rispetto al baricentro)

$$\begin{cases} \sum \vec{F_e} + \vec{F_i} = 0 \\ \sum \vec{M}_{e_G} + \vec{M}_{iG} = 0 \end{cases}$$ PRINCIPIO DI D'ALEMBER

16-03-2010



L'applicazione delle forze al corpo rigido determina il suo moto

$\sum \vec{F_e} = m \vec{a}_G \qquad -m\vec{a}_G = \vec{F_i}$

$\underline{\sum \vec{F_e} + \vec{F_i} = 0}$ D'Alembert

Aggiungendo la forza d'inerzia ho costruito un sistema di forze equilibrato

$\sum \vec{M}_{e_G} = I_G \dot{\vec{\omega}} \qquad -I_G \cdot \dot{\vec{\omega}} = \vec{M}_{iG}$

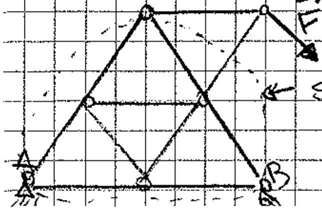
$\underline{\sum \vec{M}_{e_G} + \vec{M}_{iG} = 0}$

Le forze esterne + le forze d'inerzia introducono un sistema in equilibrio dinamico

$\sum \vec{M}_{e_P} = \ell_1 F_1 \vec{k} + \ell_2 F_2 (-\vec{k}) + \ell_3 F_3 \vec{k}$

$\sum \vec{M}_{e_P} + \underbrace{\vec{M}_{iG} + \vec{h} \wedge \vec{F_i}}_{\vec{M}_{iP}} = 0 \qquad \underline{\sum \vec{M_e} + \vec{M_i} = 0}$

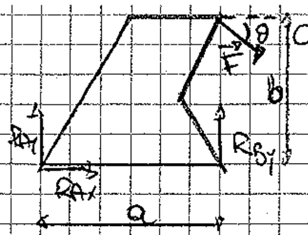
$$\begin{cases} \sum \vec{F_e} + \vec{F_i} = 0 \\ \sum \vec{M_e} + \vec{M_i} = 0 \end{cases}$$ EQUAZIONI CARDINALI DELLA DINAMICA



STRUTTURA RETICOLARE (tiranti e puntoni) → rigido e indeformabile (a parte def. elastiche)

← superficie di distacco

Stacchiamo la struttura dai vincoli (nei punti A e B)



CORPO LIBERO

L'insieme delle forze si chiama DIAGRAMMA DI CORPO LIBERO

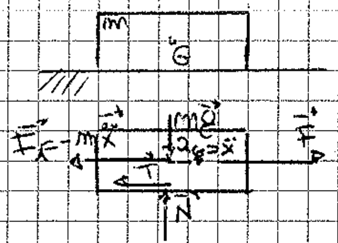
$$\sum \vec{F_e} = 0 \quad \begin{cases} \sum F_{ex} = 0 \\ \sum F_{ey} = 0 \\ \cancel{\sum F_{ez} = 0} \end{cases} \qquad \begin{aligned} F\cos\theta + R_{Ax} &= 0 \\ F\sin\theta - R_{Ay} - R_{By} &= 0 \end{aligned}$$

$$\sum \vec{M_e} = 0 \quad \begin{cases} \cancel{\sum M_{ex} = 0} \\ \cancel{\sum M_{ey} = 0} \\ \sum M_{ez} = 0 \end{cases} \qquad \curvearrowleft A \quad R_{By} \cdot a - F\sin\theta \, a - F\cos\theta \, b = 0$$

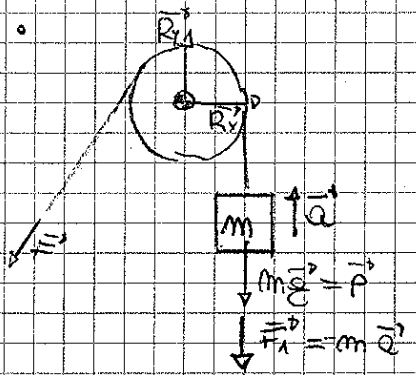
$$\sum \vec{F_e} + \vec{F_i} = 0 \quad \begin{cases} \sum F_{ex} + F_{ix} = 0 \\ \sum F_{ey} + F_{iy} = 0 \\ \cancel{\sum F_{ez} + F_{iz} = 0} \end{cases}$$

$$\sum \vec{M_e} + \vec{M_i} = 0 \quad \begin{cases} \times \\ \times \\ \sum M_{ez} + M_{iz} = 0 \end{cases}$$

- $\rightarrow x, \dot{x}, \ddot{x}$



- Diagramma di corpo libero dinamico di questo sistema



$$\begin{cases} \sum F_{ex} + F_{ix} = 0 \\ \sum F_{ey} + F_{iy} = 0 \\ \sum M_e + M_i = 0 \end{cases}$$

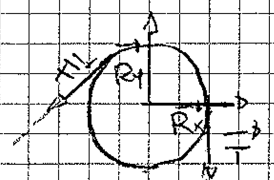
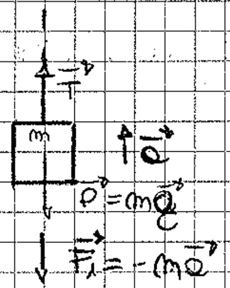
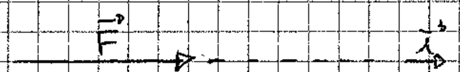


Diagramma di Corpo libero della sola puleggia



- 

$\vec{F} = F \cdot \vec{\imath} \qquad F \geq 0$

$|\vec{F}| = F \qquad \vec{\imath} =$ verso della forza

- 

$\vec{F} = F \, \vec{\imath} \qquad F \gtrless 0$

DIAGRAMMA DI FORZE DISTRIBUITE

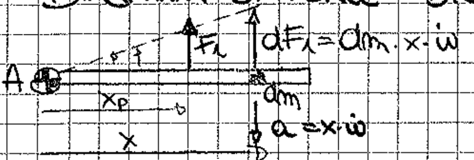
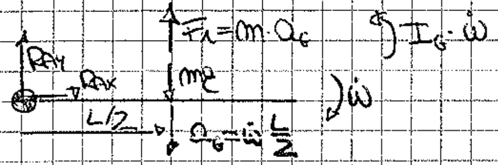


DIAGRAMMA DI FORZE DISTRIBUITE

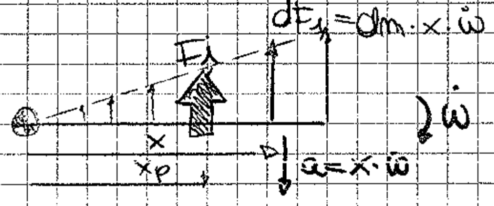
$$m = \rho \cdot L \qquad dm = \rho \cdot dx$$

$$F_i = \int dF_i = \int dm \cdot x \cdot \dot\omega = \dot\omega \rho \int_0^L x\, dx = \dot\omega \rho \frac{L^2}{2} = \frac{1}{2} \dot\omega m L \qquad a_G = \dot\omega \frac{L}{2}$$

$$F_i = m \cdot a_G$$



$$m = L \cdot \rho \qquad I_G = \frac{mL^2}{12}$$



Calcolo la risultante delle F_i:

$$F_i = \int_m dF_i = \int x \cdot \dot\omega \cdot \rho \, dx = \dot\omega \cdot \rho \int_0^L x\,dx = \dot\omega \cdot \rho \cdot \frac{L^2}{2} = (\rho L) \cdot \left(\dot\omega \frac{L}{2}\right) = m \cdot a_G$$ RISULTANTE DI FORZE D'INER

Questa risultante F_i deve essere applicata in un punto tale che abbia lo stesso effetto di tutte le forze inerziali

$$\underbrace{F_i \cdot x_p}_{\text{Momento della risultante delle forze d'inerzia}} = \underbrace{\int_m dF_i \cdot x}_{\text{Momento risultante delle forze d'inerzia}}$$

$$\int x \cdot dm \cdot \dot\omega \cdot x = \int \dot\omega x^2 \rho \, dx = \rho \dot\omega \int_0^L x^2 dx = \rho \cdot \dot\omega \cdot \frac{L^3}{3} = (\rho \cdot L)\left(\dot\omega \cdot \frac{L^2}{3}\right) =$$

$$= \dot\omega \left(m \frac{L^2}{3}\right) = \dot\omega \cdot I_A$$

$$F_i \, x_p = m \cdot a_G \cdot x_p = \dot\omega \cdot I_A$$

$$\cancel{m} \cdot x_p \cdot \cancel{\dot\omega} \frac{\cancel{L}}{2} = \cancel{\dot\omega} \, \cancel{m} \frac{L^{\cancel{2}}}{3} \qquad \boxed{x_p = \frac{2}{3} L}$$

RIDUZIONE DELLE FORZE D'INERZIA

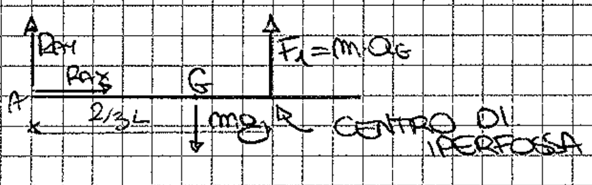
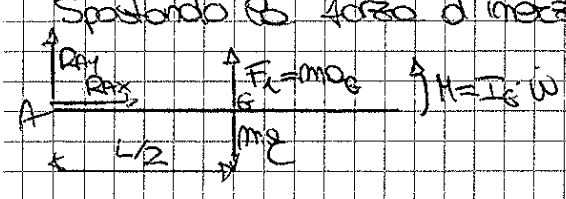


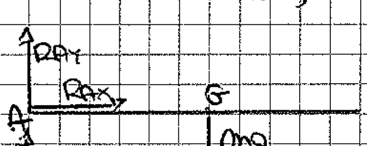
DIAGRAMMA DI CORPO LIBERO DEL SISTEMA

Spostando la forza d'inerzia nel baricentro:



$$M = I_G \dot\omega \qquad M = F_i \cdot \frac{L}{6} = m \cdot a_G \cdot \frac{L}{6} = m \frac{L}{6} \dot\omega \frac{L}{2} = \dot\omega \, m \frac{L^2}{12}$$

Posso anche ridurre le forze d'inerzia ad un punto fisso del corpo (il punto A in questo caso)



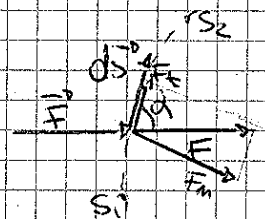
$$M_T = I_A \cdot \dot\omega$$

$$M_T = F_i \cdot \frac{2}{3} L = \frac{2}{3} L \, m \dot\omega \frac{L}{2} = \dot\omega m \frac{L^2}{3} = \dot\omega \cdot I_A$$

LAVORO

18-03-2010

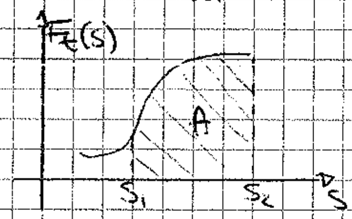
Concetto di lavoro di una forza

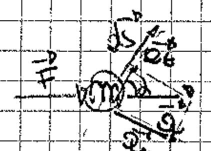


$$\underline{dL = \vec{F}\cdot\vec{ds} = F\cdot ds\cdot\cos\alpha = F_t\cdot ds}$$

- lavoro positivo: forza e spostamento concordi
- lavoro negativo: forza e spostamento discordi

$$L_{1-2} = \int_1^2 dL = \int_{S_1}^{S_2} F_t\cdot ds \;\rightarrow \text{area } A$$



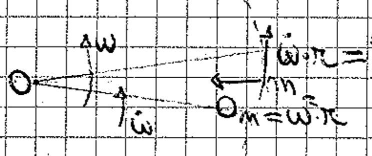


$$\vec{F} = m\cdot\vec{a}$$

$$dL = \vec{F}\cdot\vec{ds} = m\,a\cdot ds\cdot\cos\alpha = m\cdot a_t\cdot ds = \frac{dv}{dt}\cdot m\,ds = m\,v\cdot dv$$

$$L_{1-2} = \int_1^2 dL = m\int_{V_1}^{V_2} v\,dv = \frac{1}{2}m\left(V_2^2 - V_1^2\right) = \Delta E_c$$

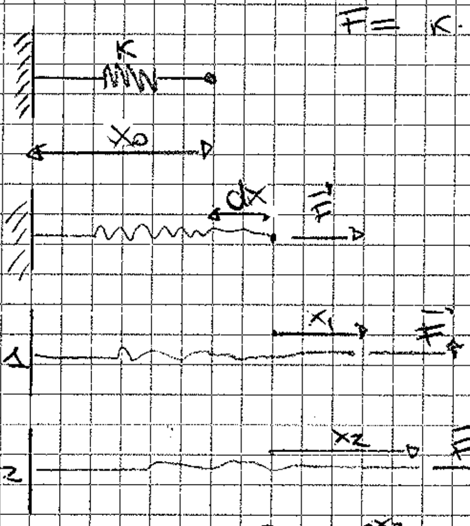
ENERGIA CINETICA



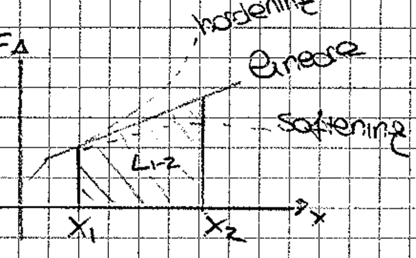
$$\underline{L_{1-2} = \Delta E_c = E_{c_2} - E_{c_1}}$$

CORPI ELASTICI – ENERGIA POTENZIALE ELASTICA

$$F = k\cdot\Delta x$$



$$dL = \vec{F}\cdot\vec{dx}$$

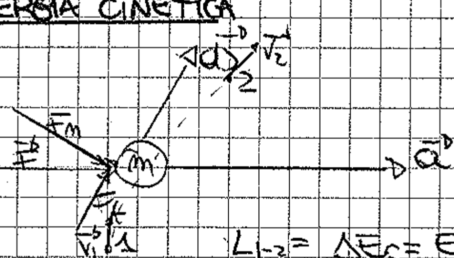


$$L_{1-2} = \int_1^2 dL = \int_{x_1}^{x_2} k\cdot x\cdot dx = \frac{1}{2}k\cdot\left(x_2^2 - x_1^2\right) = \Delta E_E = E_{E_2} - E_{E_1}$$

$$\underline{E_E = \frac{1}{2}kx^2}$$ ENERGIA POTENZIALE ELASTICA

ENERGIA CINETICA

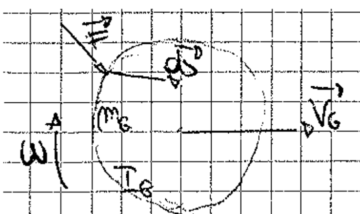
23-03-2010



$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$dL = \vec{F}\cdot\vec{ds} = F_t\cdot ds$$

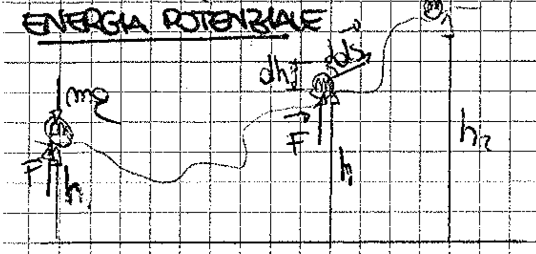
$$L_{1-2} = \Delta E_c = E_{c_2} - E_{c_1} = \frac{1}{2}m\left(V_2^2 - V_1^2\right)$$



$$L_{1-2} = \Delta E_C = E_{C_2} - E_{C_1}$$

$$E_C = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \cdot \omega^2$$

ENERGIA POTENZIALE



$$dL = \vec{F} \cdot d\vec{s} = F \cdot ds \cos\alpha = F \cdot dh = mg\,dh$$

$$L_{1-2} = mg \int_{h_1}^{h_2} dh = mg(h_2 - h_1)$$

$\underline{E_G = mgh}$ ENERGIA POTENZIALE GRAVITAZIONALE

$$L_i + L_e = \Delta E_T = \Delta E_C + \Delta E_E + \Delta E_G$$

Energia meccanica totale

$E_T = E_C + E_E + E_G$

$\Delta E_C \to \frac{1}{2} m (V_{G(2)}^2 - V_{G(1)}^2) + \frac{1}{2} I_G (\omega_2^2 - \omega_1^2)$

$\Delta E_E \to \frac{1}{2} K (x_2^2 - x_1^2)$

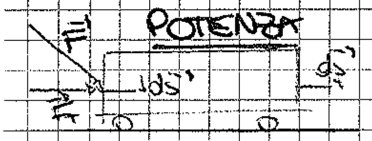
$\Delta E_G \to mgh_2 - mgh_1$

non considerare i pesi e le forze d'inerzia

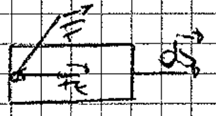
$$L_i + L_E = \Delta E_T \qquad E_T = E_C + E_E + E_G = \text{costante}$$

se $L_i + L_e = 0 \to \Delta E_T = 0$ Sistema CONSERVATIVO

POTENZA



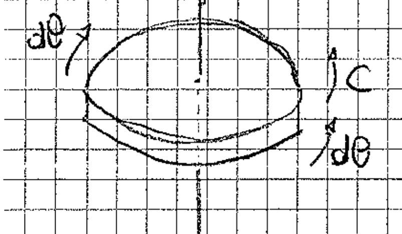
$dL = \vec{F} \cdot d\vec{s} = F_T \cdot ds > 0$ LAVORO MOTORE



$L < 0$ LAVORO FRENANTE

$$\frac{dL}{dt} = F_T \cdot \frac{ds}{dt} = F_T \cdot V$$

POTENZA SVILUPPATA
- > 0 POTENZA MOTRICE
- < 0 POTENZA FRENANTE (DISSIPATA)



$$dL = \vec{C} \cdot d\vec{\theta} = C \cdot d\theta \gtrless 0$$

$$\frac{dL}{dt} = C \frac{d\theta}{dt} = C\,\omega$$ POTENZA SVILUPPATA

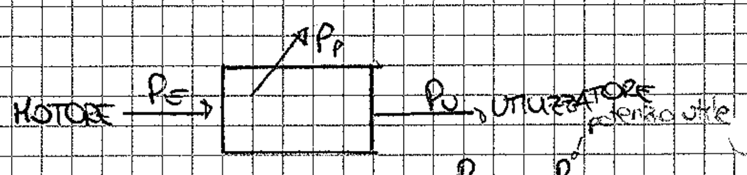
$F \cdot ds$

E entrante / P entrante → $(F \cdot V)$ $(C \cdot \omega)$

Sistema fisico meccanico

Energia uscente / Potenza uscente (UTILE)

Energia, Potenza PERSA

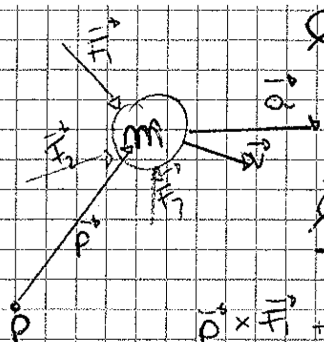


$$\eta : \text{RENDIMENTO} = \frac{P_U}{P_E} = \frac{P_U}{P_M}$$ P_M: potenza motrice

$$\eta = \frac{F_T \cdot V}{C\,\omega} \leq 1$$

QUANTITÀ DI MOTO

24-03-201



$$\sum \vec{F_e} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{Q}}{dt}$$

$$\vec{Q} = m\vec{V}$$

$$\boxed{\sum \vec{F_e} = \frac{d\vec{Q}}{dt}}$$ TEOREMA DELLA QUANTITÀ DI MOTO

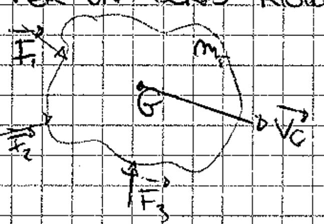
$$\vec{\rho} \times \vec{F_1} + \vec{\rho} \times \vec{F_2} + \dots = \vec{M_{e_P}}$$

$$\vec{\rho} \times m\vec{V} = \vec{K_P}$$ MOMENTO DELLA QUANTITÀ DI MOTO RISPETTO A P

se $P \equiv O$ (fisso $V_0 = 0$)

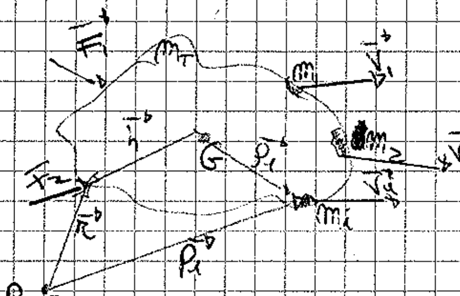
$$\boxed{\frac{d\vec{K_0}}{dt} = \vec{M_{e_0}}}$$ TEOREMA DEL MOMENTO DELLA QUANTITÀ DI MOTO

PER UN CORPO RIGIDO



$$\vec{Q} = m_T \cdot \vec{V_G}$$

$$\sum \vec{F_e} = \frac{d\vec{Q}}{dt}$$ Vale lo stesso relazione



$$\vec{Q_i} = m_i \cdot \vec{V_i}$$

$$\sum \vec{\rho_i} \wedge m_i \cdot \vec{V_i} = \sum \vec{K_{0_i}} = \vec{K_0}$$

$$\sum \vec{\rho_i} \wedge m_i \cdot \vec{V_i} = \sum \vec{K_{G_i}} = \vec{K_G}$$

MOMENTO RISULTANTE DELLA QUANTITÀ DI MOTO DEL CORPO RISPETTO A O FISSO / G BARICENTRO

$$\sum \vec{M_{e_0}} = \vec{r} \wedge \vec{F_2} + \dots$$

$$\sum \vec{M_{e_G}} = \vec{h} \wedge \vec{F_2} + \dots$$

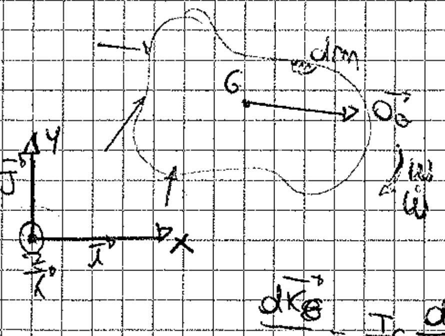
MOMENTO DELLE FORZE ESTERNE RISPETTO A O (G)

$$\boxed{\frac{d\vec{K_G}}{dt} = \vec{M_{e_G}}}$$

$$\begin{cases} \sum \vec{F_e} + \vec{F_i} = 0 \\ \sum \vec{M_e} + \vec{M_i} = 0 \end{cases}$$

$$\frac{d\vec{Q}}{dt} + \vec{F_i} = 0 \qquad \vec{F_i} = -\frac{d\vec{Q}}{dt} = -\frac{d(m_T\vec{V_G})}{dt} = -m_T\frac{d\vec{V_G}}{dt} = -m_T\,\vec{a_G}$$

$$\frac{d\vec{K_G}}{dt} + \vec{M_{i_G}} = 0 \qquad \vec{M_{i_G}} = -\frac{d\vec{K_G}}{dt} = -I_G \cdot \dot{\vec{\omega}}$$

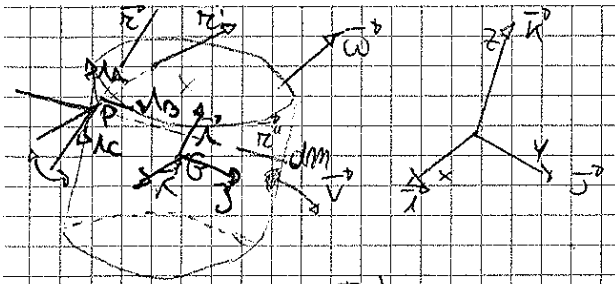


$$\vec{K_G} = I_G \cdot \vec{\omega}$$

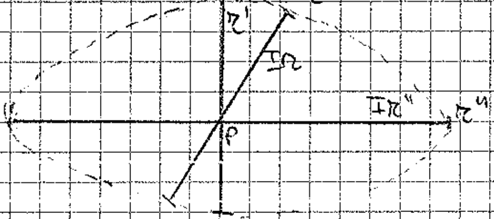
Moto nel piano x-y

$$\vec{\omega} = \omega(\pm\vec{k})$$

$$\frac{d\vec{K_G}}{dt} = I_G\frac{d\vec{\omega}}{dt} = I_G \cdot \dot{\vec{\omega}}$$



ELLISSOIDE D'INERZIA



I_A (MAX)

I_B (MIN)

I_C ($\perp I_A$ e I_B)

il punto P ha tre momenti principali d'inerzia

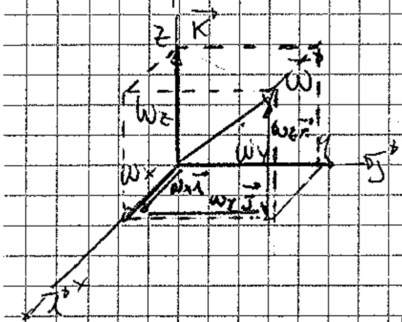
$\vec{\lambda}_A, \vec{\lambda}_B, \vec{\lambda}_C$ } TERNA PRINCIPALE D'INERZIA IN P

$\vec{i}, \vec{j}, \vec{k}$ } TERNA CENTRALE D'INERZIA — centrata di baricentro

• $\vec{K}_G = I_x \omega_x \vec{i} + I_y \omega_y \vec{j} + I_z \omega_z \vec{k}$

$\vec{i}, \vec{j}, \vec{k}$ terna centrale d'inerzia

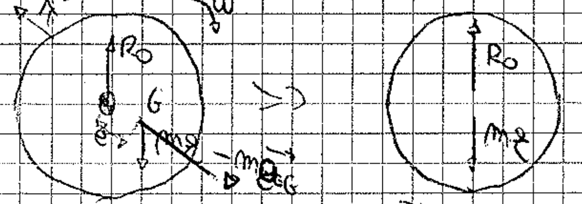
I_x, I_y, I_z momenti centrali d'inerzia



$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$

$$\vec{M}_{I_G} = -\frac{d\vec{K}_G}{dt} = -\left(I_x \dot{\omega}_x \vec{i} + I_y \dot{\omega}_y \vec{j} + I_z \dot{\omega}_z \vec{k} + I_x \omega_x \frac{d\vec{i}}{dt} + I_y \omega_y \frac{d\vec{j}}{dt} + I_z \omega_z \frac{d\vec{k}}{dt} \right)$$

Se $\vec{i}\,\vec{j}\,\vec{k}$ è solidale al corpo $\Rightarrow \vec{\omega}_T \equiv \vec{\omega}$ $\qquad \frac{d\vec{i}}{dt} = \vec{\omega}_T \wedge \vec{i}$



$\vec{a}_G = e\,\omega^2 \vec{\lambda}$

$\omega_x = \omega \cos\alpha$

$\omega_y = \omega \sin\alpha$

$\omega_z = 0$

$\vec{\omega} =$ costante

$\frac{d\vec{i}}{dt} = \vec{\omega} \wedge \vec{i}$

$\frac{d\vec{j}}{dt} = \vec{\omega} \wedge \vec{j}$

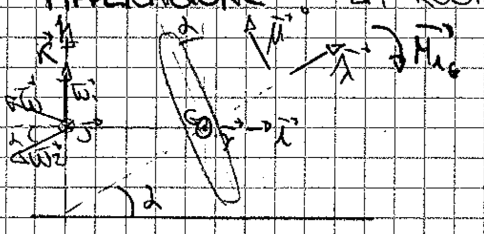
$$\frac{dK_G}{dt} = I_x \omega_x \cdot \omega \cdot \sin\alpha (-\vec{k}) + I_y \omega_y \omega \cos\alpha\, \vec{k} =$$

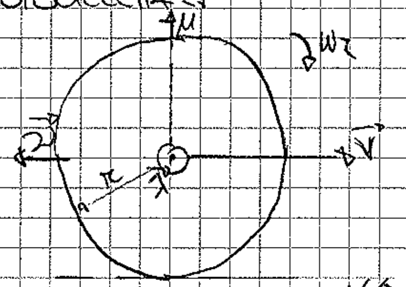
$$= I_x \cdot \omega^2 \cos\alpha \sin\alpha (-\vec{k}) + I_y \omega^2 \sin\alpha \cos\alpha\, \vec{k} =$$

$$= \omega^2 \sin\alpha \cos\alpha (I_x - I_y)(-\vec{k})$$

$$\vec{M}_{I_G} = \omega^2 \sin\alpha \cos\alpha\, (I_x - I_y)\, \vec{k}$$

APPLICAZIONE : LA RUOTA DI UNA MOTOCICLETTA

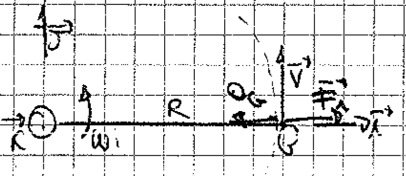




$V = \omega_1 \cdot R = \omega_2 \cdot r$ $\qquad \omega_2 = V/r$

$\omega_2 > \omega_1$

$\omega_1 = \frac{V}{R} \quad \omega_2 = \frac{V}{r}$



$\vec{\lambda}, \vec{\mu}, \vec{\nu}$ TERNA CENTRALE D'INERZIA

$\vec{K}_G = I_\lambda \cdot \omega_\lambda \vec{\lambda} + I_\mu \omega_\mu \vec{\mu} + I_\nu \omega_\nu \vec{\nu}$

Se la ruota è approssimabile ad un disco sottile $\quad I_\lambda \cong \frac{mr^2}{2} \quad I_\mu \cong I_\nu \cong \frac{mr^2}{4} \cong \frac{I_\lambda}{2}$

$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 = \omega_1 \vec{k} - \omega_2 \vec{\lambda}$

$\omega_\lambda = \vec{\omega} \cdot \vec{\lambda} = \omega_1 \sin\alpha - \omega_2$

$\omega_\mu = \vec{\omega} \cdot \vec{\mu} = \omega_1 \cos\alpha$

$\omega_\nu = \vec{\omega} \cdot \vec{\nu} = 0$

$\frac{d\vec{K}_G}{dt} = I_\lambda \omega_\lambda \frac{d\vec{\lambda}}{dt} + I_\mu \omega_\mu \frac{d\vec{\mu}}{dt} + \cancel{I_\nu \omega_\nu \frac{d\vec{\nu}}{dt}}$

$\frac{d\vec{\lambda}}{dt} = \vec{\omega}_T \wedge \vec{\lambda} = \omega_1 \vec{k} \wedge \vec{\lambda} = \omega_1 \cos\alpha (-\vec{\nu})$ $\qquad \frac{d\vec{\mu}}{dt} = \vec{\omega}_T \wedge \vec{\mu} = \omega_1 \vec{k} \wedge \vec{\mu} = \omega_1 \sin\alpha \, \vec{\nu}$

Questo è un corpo solido giroscopico → corpi a simmetria circolare
λ, μ, ν sarebbero versori d'inerzia anche se non ruotassero insieme alla ruota

$\vec{\omega}_T = \omega_1 \cdot \vec{k}$

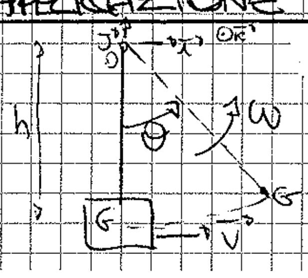
$$\vec{M}_G = -\frac{d\vec{K}_G}{dt} = -\left[I_\lambda (\omega_1 \sin\alpha - \omega_2) \omega_1 \cos\alpha (-\vec{\nu}) + \overset{\frac{I_\lambda}{2}}{I_\mu} \omega_1^2 \cos\alpha \sin\alpha \, \vec{\nu} \right] =$$
$$= (-\vec{\nu}) \left[I_\lambda \omega_1^2 \sin\alpha \cos\alpha \left(-\frac{1}{2}\right) + I_\lambda \omega_1 \omega_2 \cos\alpha \right] =$$
$$= (-\vec{\nu}) \left[I_\lambda \omega_1 \cos\alpha \underbrace{\left(\omega_2 - \frac{\omega_1 \sin\alpha}{2} \right)}_{>0} \right]$$
perché $\omega_2 > \omega_1$

$\vec{a}_G = R\omega_1^2 (-\vec{\lambda}) = \frac{V^2}{R}(-\vec{\lambda})$ $\qquad V = cost$, $\omega_1 = cost$, $\omega_2 = cost$

$\vec{F}_\lambda = m \frac{V^2}{R} \vec{\lambda}$

$\tan\alpha = \frac{mV^2}{Rmg} = \frac{V^2}{Rg}$

APPLICAZIONE 2 : ARRESTO DI UNA FUNIVIA



$L_i + L_e = \Delta E_{TOT} = 0$

$E_{TOT} = E_c + E_E + E_G = 0$

V, m, ρ_G, h $\quad \{ I_G = m \rho_G^2$

ω ? θ_{MAX} ? E_{persa} ?

- $\sum \vec{F_e} = \frac{d\vec{Q}}{dt} = 0 \rightarrow \vec{Q} = cost$

$\sum \vec{M_{e_G}} = \frac{d\vec{K_G}}{dt} = 0 \rightarrow \vec{K_G} = cost$

$\sum \vec{M_{e_O}} = 0 \rightarrow \vec{K_O} = cost \qquad \vec{K_{O(1)}} = \vec{OG} \wedge m\vec{v} = hmV \cdot \vec{k}$

$\vec{K_{O(1)}} = (\cancel{I_x \omega_x \vec{i}} + \cancel{I_y \omega_y \vec{j}} + I_z \omega_z \vec{k}) = I_O \omega \vec{k}$

$\vec{\omega} = \omega \vec{k} = \omega_z \vec{k} \qquad I_O = I_G + mh^2 = m(\rho_G^2 + h^2)$

$h \cancel{m} V (\cancel{\vec{k}}) = \cancel{m}(\rho_4^2 + h^2)\omega(\cancel{\vec{k}}) \qquad \omega = V \frac{h}{\rho_4^2 + h^2}$

- ① $E_{c_1} = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$

$v_G = h \cdot \omega$

$E_{c_1} = \frac{1}{2} m h^2 \omega^2 + \frac{1}{2} \omega^2 m \rho_4^2 = \frac{1}{2} m \omega^2 (h^2 + \rho_4^2)$

$E_{E_1} = 0 \qquad E_{G_1} = 0 \qquad \Rightarrow \qquad E_{TOT(1)} = E_{c(1)}$

② $E_{c(2)} = 0$
$E_{E(2)} = 0$
$E_{G(2)} = mg \cdot \Delta h = mgh(1 - \cos\theta_M)$

$\Downarrow$

$$E_{c(1)} = E_{c(2)}$$

$$\frac{1}{2} \cancel{m} \omega^2 (h^2 + \rho_G^2) = \cancel{m} gh (1 - \cos\theta_M)$$

$$= \frac{1}{2} \cancel{(h^2 + \rho_G^2)} \frac{V^2 h^2}{(h^2 + \rho_4^2)^{\cancel{2}}}$$

$$(1 - \cos\theta_M) = \frac{V^2 h}{2g(h^2 + \rho_4^2)}$$

- $E_{c(0)} = \frac{1}{2} m V^2$

$E_{G(0)} = 0 \qquad E_{E(0)} = 0$

$E_{c(1)} = \frac{1}{2} m \omega^2 (h^2 + \rho_4^2) = \frac{1}{2} m \cancel{(h^2 + \rho_4^2)} \frac{V^2 h^2}{(\rho_G^2 + h^2)^{\cancel{2}}}$

$E_{peso} = E_{c(0)} - E_{c(1)} = \frac{1}{2} m V^2 - \frac{1}{2} m V^2 \frac{h^2}{\rho_G^2 + h^2} = \frac{1}{2} m V^2 \left(1 - \frac{h^2}{\rho_G^2 + h^2}\right)$

$\rho_G = 0$ pendolo semplice $E_p = 0$

$h = 0$ $E_p = 100\%$ urto anelastico