23-02-2010

FISICA — TERMODINAMICA
ELETTROMAGNETISMO
MECCANICA — CINEMATICA studio sotto il moto
STATICA
DINAMICA studio onde le forze

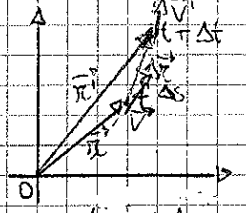
Sistema reale

Modello fisico

Modello matematico: sistema di equazioni algebriche e differenziali

CINEMATICA

CINEMATICA DEL PUNTO



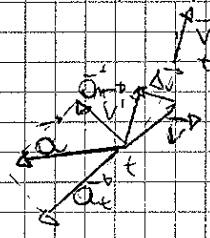
$\vec{r}' = \vec{r} + \Delta\vec{r}$

$\frac{\Delta\vec{r}}{\Delta t} = \vec{V}_m$ VELOCITÀ MEDIA DEL PUNTO

$\Delta s \neq \Delta r$

$\lim_{\Delta t \to 0} \vec{V}_m = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \vec{V}$ VELOCITÀ ISTANTANEA tangente alla traiettoria

$\vec{V}' = \vec{V} + \Delta\vec{V}$

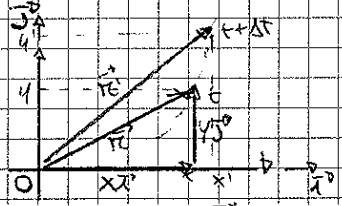


$\frac{\Delta\vec{V}}{\Delta t} = \vec{a}_m$ ACCELERAZIONE MEDIA

$\lim_{\Delta t \to 0} \vec{a}_m = \lim_{\Delta t \to 0} \frac{\Delta\vec{V}}{\Delta t} = \frac{d\vec{V}}{dt} = \vec{a}$ ACCELERAZIONE ISTANTANEA

$\vec{a} = \frac{d\vec{V}}{dt} = \dot{\vec{V}}$

COORDINATE CARTESIANE



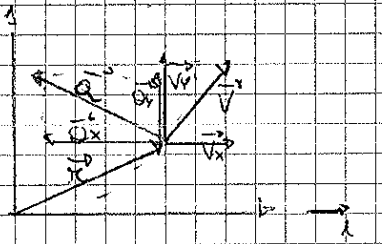
$x = x(t)$
$y = y(t)$ 2 GRADI DI LIBERTÀ 2gdl

$\vec{r} = x\vec{\imath} + y\vec{\jmath}$

$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{\imath} + x\frac{d\vec{\imath}}{dt} + \frac{dy}{dt}\vec{\jmath} + y\frac{d\vec{\jmath}}{dt}$

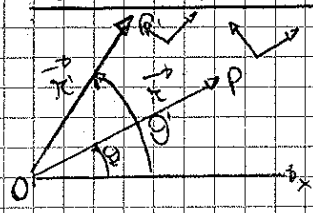
$\vec{V} = \dot{x}\vec{\imath} + \dot{y}\vec{\jmath} = \vec{V}_x + \vec{V}_y$

$\vec{a} = \frac{d\vec{V}}{dt} = \ddot{x}\vec{\imath} + \ddot{y}\vec{\jmath}$ $\quad \vec{a} = \vec{a}_x + \vec{a}_y$



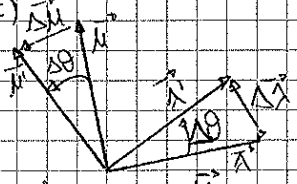
$|\vec{V}| = \sqrt{V_x^2 + V_y^2}$

COORDINATE POLARI



$r(t)$
$\theta(t)$

1, 2 versori sono legati al punto → RUOTANO



$\vec{r} = r\vec{\lambda}$ $\quad \vec{V} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\vec{\lambda} + r\frac{d\vec{\lambda}}{dt}$

$$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \dot{\theta} \qquad \text{VELOCITÀ ANGOLARE}$$

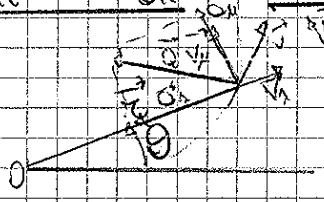
$$\Delta\theta = \dot{\theta} \cdot \Delta t$$

$$\frac{d\vec{\lambda}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\lambda}}{\Delta t} = \dot{\theta}\vec{\mu} \qquad \frac{\Delta\vec{\lambda}}{\Delta t} \cong \dot{\theta}\vec{\mu}$$

$$\Delta\vec{\lambda} \cong |\vec{\lambda}| \cdot \Delta\theta = 1 \cdot \dot{\theta} \cdot \Delta t\, \vec{\mu}$$

$$\boxed{\begin{aligned} \frac{d\vec{\lambda}}{dt} &= \dot{\theta}\vec{\mu} \\ \frac{d\vec{\mu}}{dt} &= -\dot{\theta}\vec{\lambda} \end{aligned}}$$

$$\vec{v} = \frac{dr}{dt}\vec{\lambda} + r\frac{d\vec{\lambda}}{dt} = \dot{r}\vec{\lambda} + r\dot{\theta}\vec{\mu} = \vec{V}_\lambda + \vec{V}_\mu$$



$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{r}\vec{\lambda} + \dot{r}\frac{d\vec{\lambda}}{dt} + \dot{r}\dot{\theta}\vec{\mu} + r\ddot{\theta}\vec{\mu} + r\dot{\theta}\frac{d\vec{\mu}}{dt} =$$

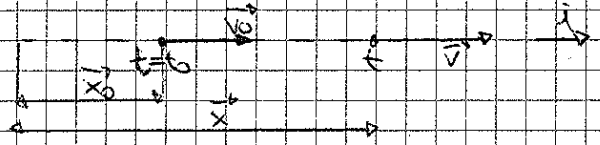
$$= \ddot{r}\vec{\lambda} + \dot{r}\dot{\theta}\vec{\mu} + \dot{r}\dot{\theta}\vec{\mu} + r\ddot{\theta}\vec{\mu} + r\dot{\theta}^2(-\vec{\lambda}) =$$

$$= (\ddot{r} - r\dot{\theta}^2)\vec{\lambda} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{\mu} =$$

$$= \vec{a}_\lambda + \vec{a}_\mu$$

21-02-2010

MOTO RETTILINEO



$\vec{x}_0 = x_0\vec{i}$
$\vec{x} = x\vec{i}$
$\vec{v} = v\vec{i}$
$\vec{a} = a\vec{i}$

x AVANTI O
v AVANTI O
a AVANTI O

MOTO RETTILINEO UNIFORME

v = costante $\qquad a = \frac{dv}{dt} = 0$

$$\boxed{v = \frac{dx}{dt}} \qquad \text{EQUAZIONE DEL MOTO}$$

$$dx = v \cdot dt \qquad \int_{x_0}^{x} dx = v\int_{t_0}^{t} dt \qquad x - x_0 = v(t - t_0)$$

$$\boxed{x(t) = x_0 + v(t - t_0)} \qquad \text{LEGGE DEL MOTO}$$

MOTO RETTILINEO UNIFORMEMENTE ACCELERATO

$$a = \text{cost} = \frac{dv}{dt}$$

$$\boxed{a = \frac{dv}{dt}} \qquad \text{EQUAZIONE DEL MOTO}$$

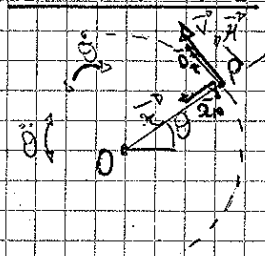
$$dv = a \cdot dt \qquad \int_{v_0}^{v} dv = a\int_{t_0}^{t} dt \qquad v - v_0 = a(t - t_0)$$

$$\boxed{v(t) = v_0 + a(t - t_0)} \qquad \text{LEGGE DEL MOTO (DELLE VELOCITÀ)}$$

$$v(t) = \frac{dx}{dt} \qquad \int_{x_0}^{x} dx = \int_{t_0}^{t} v\,dt \qquad x - x_0 = \int_{t_0}^{t} [v_0 + a(t - t_0)]\,dt = v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

$$\boxed{x(t) = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2} \qquad \text{LEGGE DEL MOTO (DELLO SPOSTAMENTO)}$$

MOTO CIRCOLARE



$\vartheta(t)$

$\frac{d\vartheta}{dt} = \dot{\vartheta}$ velocità angolare

$\frac{d^2\vartheta}{dt^2} = \ddot{\vartheta}$ accelerazione angolare

$\vec{r} = r\,\vec{\lambda}$ $\quad r = \text{costante} \rightsquigarrow \dot{r} = \frac{dr}{dt} = 0 \quad \ddot{r} = \frac{d^2r}{dt^2} = 0$

$\vec{V} = \frac{d\vec{r}}{dt} = r\frac{d\vec{\lambda}}{dt} = r\dot{\vartheta}\vec{\mu}$

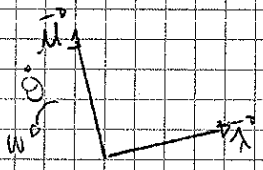
$\vec{a} = \frac{d\vec{v}}{dt} = r(\ddot{\vartheta}\vec{\mu} + \dot{\vartheta}\frac{d\vec{\mu}}{dt}) = r\ddot{\vartheta}\vec{\mu} - r\dot{\vartheta}^2\vec{\lambda} = \vec{a}_t + \vec{a}_n$ accelerazione centripeta

MOTO CIRCOLARE UNIFORME

$\dot{\vartheta} = \text{costante} \rightsquigarrow \ddot{\vartheta} = 0$

$\vec{a} = 0\,\vec{\mu} - r\dot{\vartheta}^2\vec{\lambda} = -r\dot{\vartheta}^2\vec{\lambda}$ c'è solo accelerazione centripeta

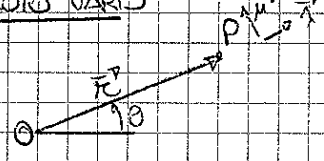
$\frac{d\vartheta}{dt} = \dot{\vartheta} = \omega$ $\quad \frac{d^2\vartheta}{dt^2} = \ddot{\vartheta} = \dot{\omega}$ $\quad \vec{\omega}$ regola della mano destra



$\odot\,\vec{\omega}$

$\frac{d\vec{\lambda}}{dt} = \vec{\omega} \wedge \vec{\lambda} = \omega\vec{\mu}$

MOTO VARIO

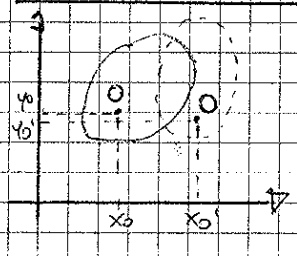


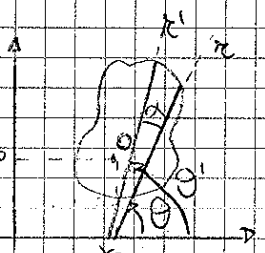
$\left.\begin{matrix} r(t) \\ \vartheta(t) \end{matrix}\right\} \geq 2\,gdl$

Se $\vartheta = cost \rightsquigarrow$ MOTO RETTILINEO
Se $r = cost \rightsquigarrow$ MOTO CIRCOLARE
$\left.\right\} \leq 1\,gdl$

CINEMATICA DEL CORPO RIGIDO

CORPO ESTESO RIGIDO





$x_0(t)$
$y_0(t)$
$\vartheta(t)$ COORDINATA ANGOLARE

SONO NECESSARIE 3 COORDINATE
3 GDL

$\frac{d\vartheta}{dt} = \dot{\vartheta} = \omega$ $\quad \frac{d^2\vartheta}{dt^2} = \ddot{\vartheta} = \dot{\omega}$

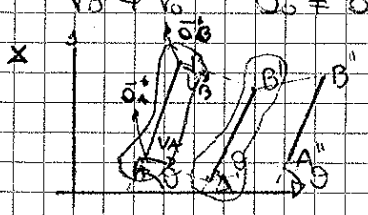
$\dot{\alpha} = \text{costante}$
$\vartheta' \neq \vartheta$
$\vartheta' = \vartheta + \alpha$

$\frac{d\vartheta'}{dt} = \frac{d\vartheta}{dt} + \frac{d\alpha}{dt} = \omega$

$\frac{d^2\vartheta'}{dt^2} = \frac{d^2\vartheta}{dt^2} = \dot{\omega}$

$\rightsquigarrow$ $\dot{\vartheta}$ e $\ddot{\vartheta}$ sono indipendenti dalla retta r che scelgo mentre x_0 e y_0 cambiano a seconda del punto scelto

$\vec{V}_O \neq \vec{V}_{O'}$ $\quad \vec{a}_O \neq \vec{a}_{O'}$



solo traslazione

$\vartheta = \text{costante}$

studiamo la traslazione del segmento $\times$ studiare quello del corpo

MOTO TRASLATORIO