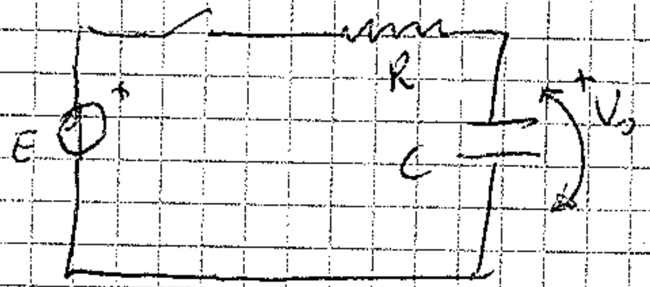
Trasformate di Laplace

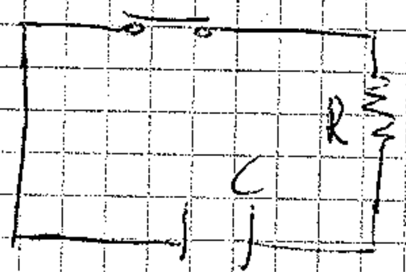
Definizione

~~Risposta con ingresso 0~~

Risposta con ~~stato~~ ingresso 0 evoluzione di una grandezza elettrica quando non sono + presenti generatori



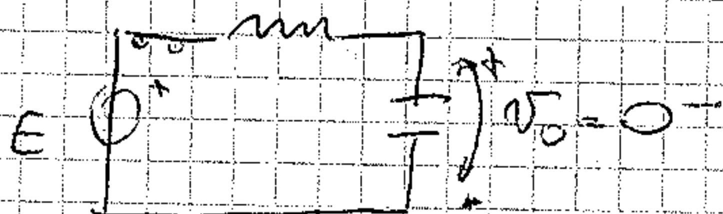
Risposta con ingresso 0 si ha questo



Risposta con ingresso 0 viene talvolta chiamata evoluzione libera

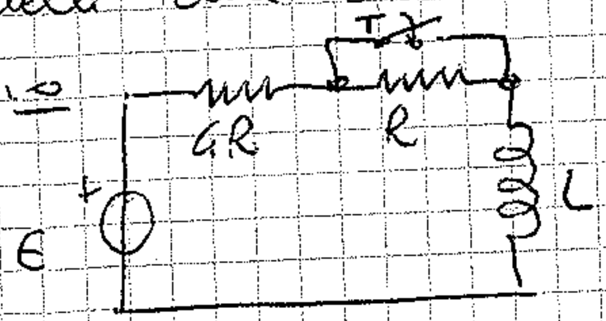
Risposta con stato 0

Evoluzione forzata



Evoluzione reale è data dalla somma delle due evoluzioni.

Esempio



T chiuso in $t=0$ studiare il transitorio

$$i(t=0^-) = \frac{E}{5R}$$

$$E = 4Ri + L\frac{di}{dt}$$

$$i_p = \frac{E}{4R} \qquad i_t = K_1 e^{-\frac{4R}{L}t}$$

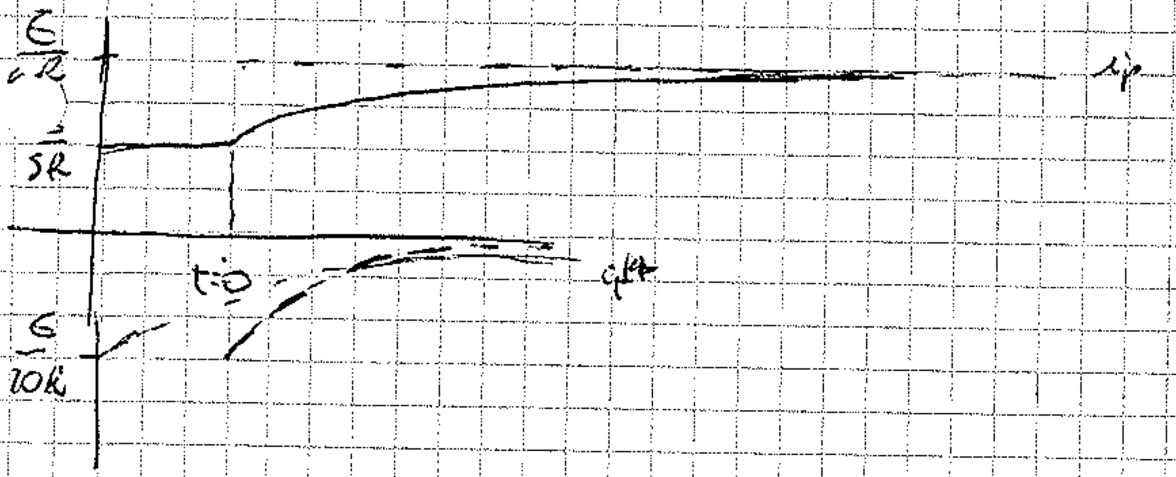
$$4R + Lp = 0 \qquad p = -\frac{4R}{L}$$

$$i(t) = \frac{E}{4R} + K_1 e^{-\frac{4R}{L}t}$$

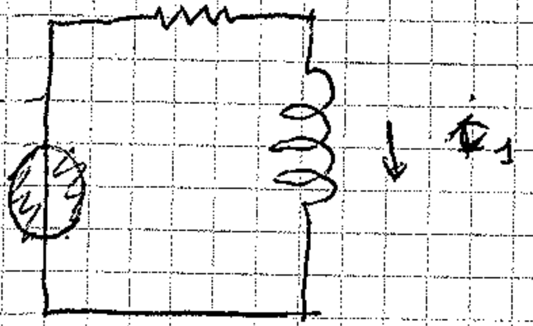
$$i(t=0^-) = i(t=0^-)$$

$$\frac{E}{5R} = \frac{E}{4R} + K \Rightarrow K = \frac{E}{5R} - \frac{E}{4R} = -\frac{E}{20R}$$

$$i(t) = \frac{E}{4R} + \frac{E}{20R} e^{-\frac{4R}{L}t}$$



Risoluzione utilizzando risposte con ingr. 0 e con stato 0

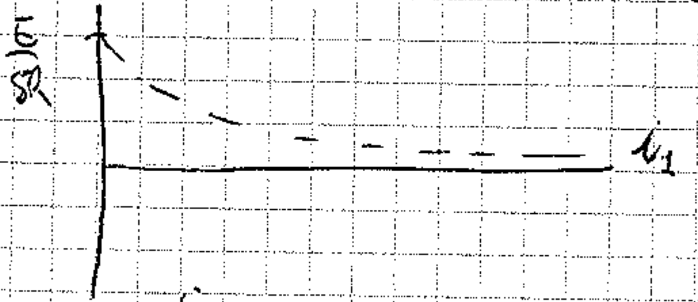


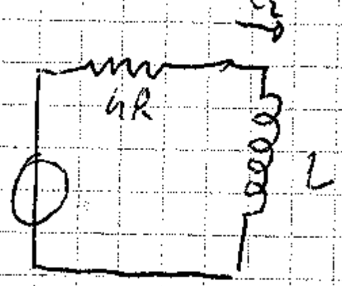
$$i_1(t=0^-) = \frac{E}{5R}$$

$$0 = 4R\, i_1 + L\frac{di_1}{dt}$$

risp. con ~~stato~~ ingresso 0

$$\hookrightarrow i_1 = K_1 e^{-\frac{4R}{L}t} = +\frac{E}{5R} e^{-\frac{4R}{L}t}$$





$$i_2(t=0^-) = 0$$

$$E = 4R\, i_2 + L\frac{di_2}{dt}$$

$$i_{2p} = \frac{E}{4R}$$

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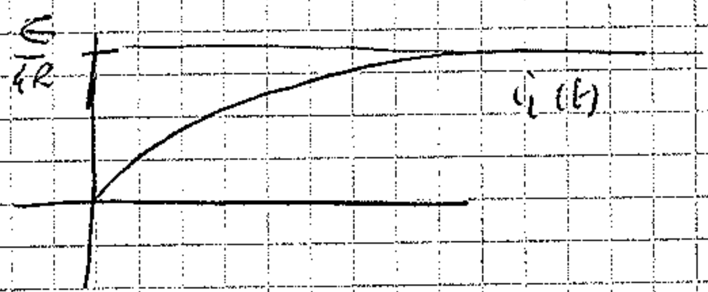
$$4R + L_2\, p \Rightarrow p = -\frac{4R}{L}$$

$$i_{2t} = K_2\, e^{-\frac{4R}{L}t}$$

$$i_2(t) = \frac{E}{4R} + K_2\, e^{-\frac{4R}{L}t}$$

$$0 = \frac{E}{4R} + K_2 \rightarrow K_2 = -\frac{E}{4R}$$

$$i_2(t) = \frac{E}{4R} - \frac{E}{4R}\, e^{-\frac{4R}{L}t}$$ risposta allo stato 0



$$i(t) = i_1(t) + i_2(t) = \frac{E}{4R} + \left(\frac{E}{5R} - \frac{E}{4R}\right) e^{-\frac{4R}{L}t}$$

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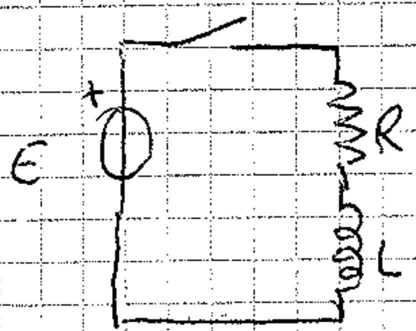
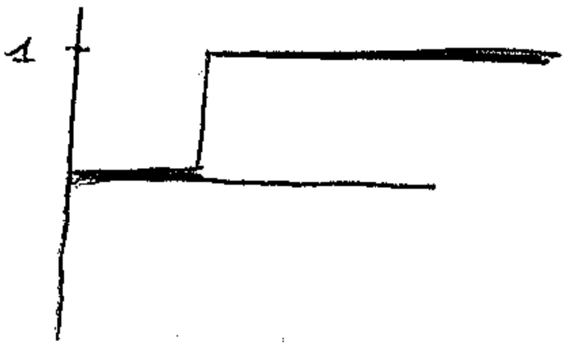


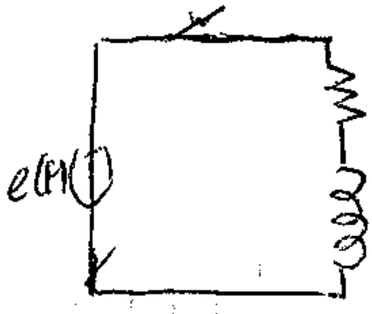
Funzione a gradino unitario



$$\begin{cases} u(t)=0 & \forall t<0 \\ u(t)=1 & \forall t>0 \\ u(t=0^-)=0 \\ u(t=0^+)=1 \end{cases}$$

$$e(t) = E\,u(t)$$

Risposta al gradino (solo se utilizzatori scarichi all'istante $t=0^-$)

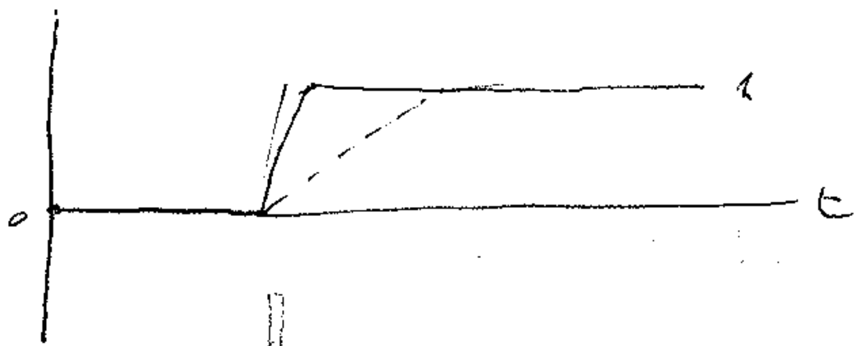


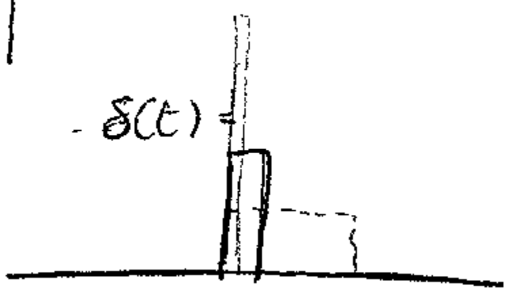
$$e(t) = E\,u(t)$$

$$i(t) = \frac{E}{R}\,u(t) + \frac{E}{R}\,e^{-\frac{R}{L}t}\,u(t)$$

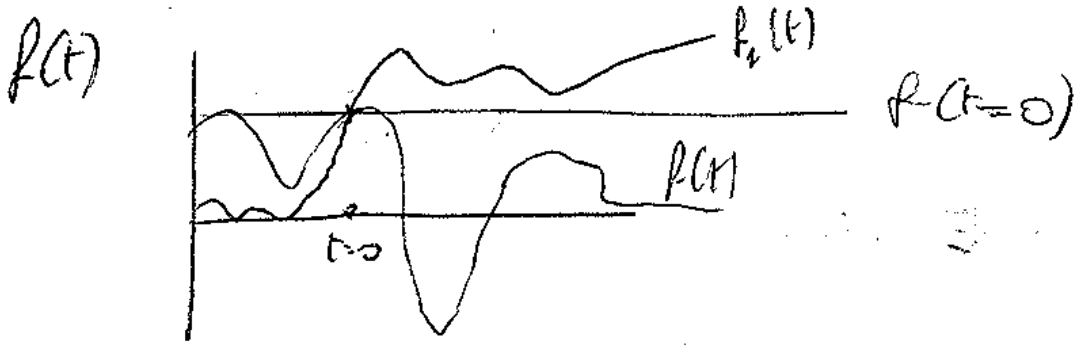
Funzione impulso unitario $\delta(t)$

$$\delta(t) = \frac{d\,u(t)}{dt}$$





$$\delta(t) = \begin{cases} 0 & \forall\, t < 0 \\ 0 & \forall\, t > 0 \end{cases}$$



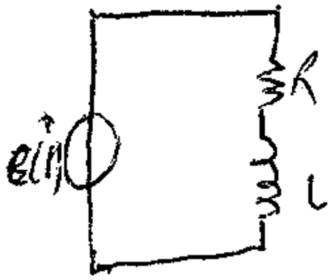
$$\int_{-\infty}^{+\infty} f(t)\,\delta(t)\,dt = f(t=0)$$

$$f(t) = f(t=0) = f_0$$

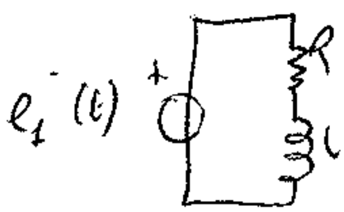
$$\int_{-\infty}^{+\infty} f(t)\,\delta(t)\,dt = \int_{-\infty}^{+\infty} f_0\,\delta(t)\,dt = f_0 \int_{-\infty}^{+\infty} \delta(t)\,dt =$$

$$f_0 \left[u(t)\right]_{-\infty}^{+\infty} = f_0 = f(t=0)$$

Esempio:



$$e(t) = E\,\delta(t)$$



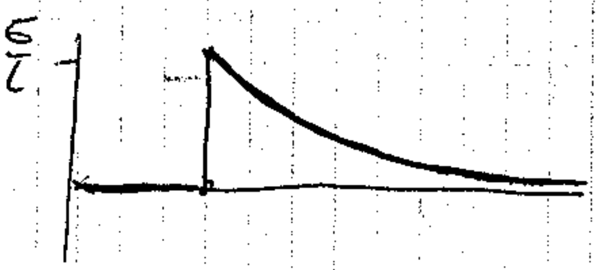
$$e_1(t) = E\,u(t)$$

$$i_1 = \frac{\epsilon}{R}u(t) - \frac{\epsilon}{R}e^{-\frac{R}{L}t}u(t)$$

$$i(t) = \frac{di_1(t)}{dt} = \frac{\epsilon}{R}\delta(t) + \frac{\epsilon}{R}\frac{R}{L}e^{-\frac{R}{L}t}u(t) - \frac{\epsilon}{R}e^{-\frac{R}{L}t}\delta(t) =$$

$$= \cancel{\frac{\epsilon}{R}\delta(t)} + \frac{\epsilon}{L}e^{-\frac{R}{L}t}u(t) \underbrace{- \frac{\epsilon}{R}e^{-\frac{R}{L}t}\delta(t)}_{-\frac{\epsilon}{R}\cancel{\delta(t)}} =$$

$$= \frac{\epsilon}{L}e^{-\frac{R}{L}t}u(t)$$



Impulso **non** è grandezza fisica.

$$e(t) = \epsilon\,\delta(t)$$

$$Ri = \epsilon\frac{R}{L}e^{-\frac{R}{L}t}u(t)$$

$$L\frac{di}{dt} = -\epsilon\frac{R}{L}e^{-\frac{R}{L}t}u(t) + \epsilon e^{-\frac{R}{L}t}\delta(t) =$$

$$= -\frac{\epsilon R}{L}e^{-\frac{R}{L}t}u(t) + \epsilon\,\delta(t)$$

legge delle tensioni è verificata.

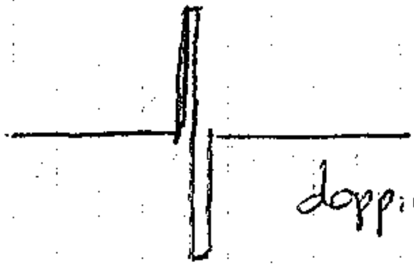
si possono utilizzare le risposte al gradino $(a(t))$ e all'impulso $\left(h(t) = \frac{d\,a(t)}{dt}\right)$.

Di solito

$$a(t) = \left(1 - e^{-\frac{t}{\tau}}\right) u(t)$$

$$h(t) = \cancel{\;} + \frac{1}{\tau} e^{-\frac{t}{\tau}}$$

$$\delta'(t) = \frac{d\,\delta(t)}{dt}$$

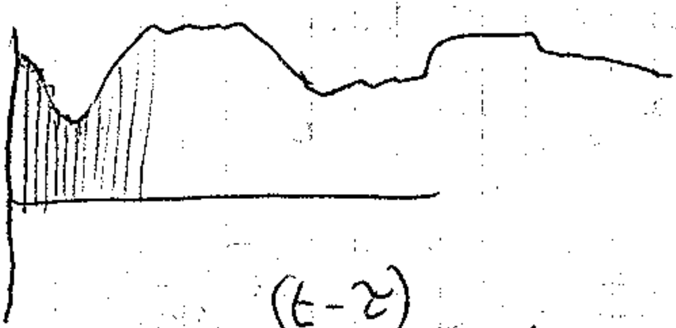


doppietto

$$\int_{+\infty}^{-\infty} \delta'(t) f(t)\,dt = \underbrace{\Big[\delta(t) f(t)\Big]_{-\infty}^{+\infty}}_{=0} - \int_{-\infty}^{+\infty} \delta(t) f'(t)\,dt$$

$$= -f'(t=0)$$

$f(t)$



$$\boxed{f(t) = \int_{-\infty}^{+\infty} f(\tau)\, \delta(\underset{(t-\tau)}{\cancel{\;}})\, d\tau}$$

risposta alla funzione $f(t)$

$$y(t) = \underbrace{\int_{-\infty}^{+\infty} f(\tau)\,h(t-\tau)\,d\tau}_{\text{integrale di convoluzione}} = f * h$$

(se tra 0 e $+\infty$ è di Duhamel)

Proprietà:

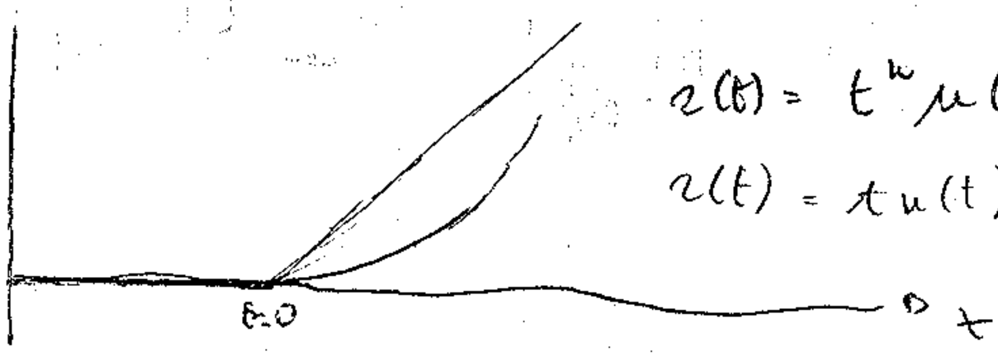
$$f * g = g * f$$

$$f * (g + \ell) = f * g + f * \ell$$

$$f * (g * \ell) = (f * g) * \ell = f * g * \ell$$

$$\frac{d}{dt}(f * g) = f * \frac{dg}{dt} = \frac{df}{dt} * g$$

Funzioni a rampa o monomiale

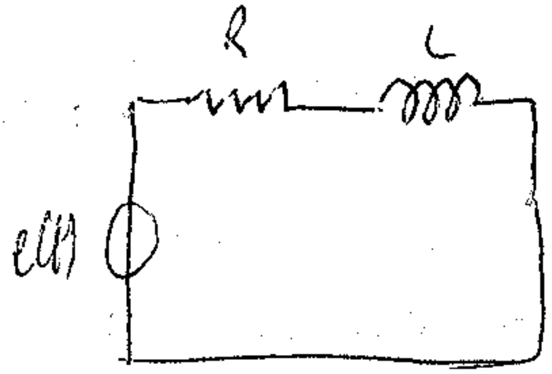


$$r(t) = t^n u(t)$$

$$r(t) = t\,u(t) = \int_{-\infty}^{+\infty} u(t)\,dt$$

Chiamo $b(t)$ la risposta alla rampa

$$b(t) = \int_{-\infty}^{+\infty} a(t)\,dt$$



$$e(t) = E\, t\, u(t)$$

$$i(t) = 0$$

funzione gradino $\frac{\text{tensione}}{\text{tempo}}$

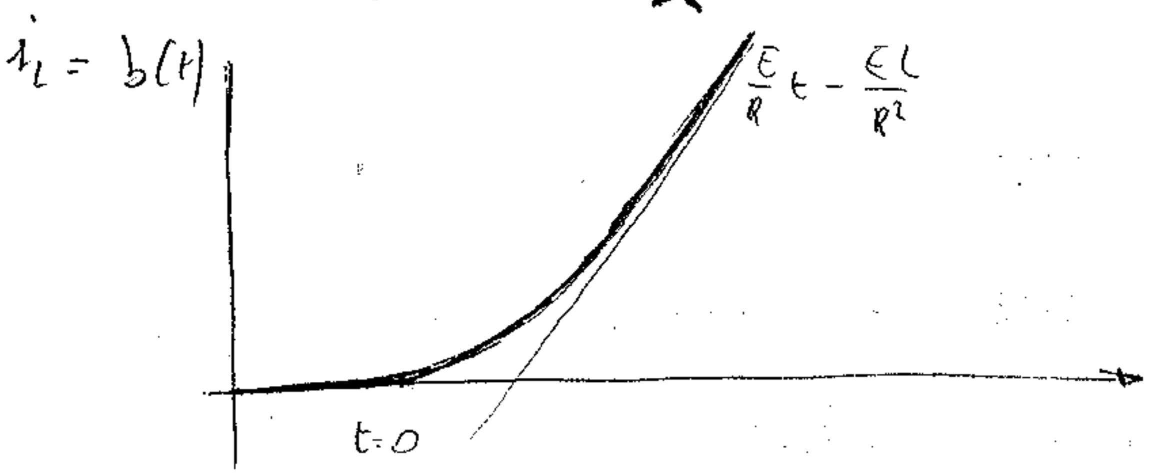
$$a(t) = \left(\frac{E}{R} - \frac{E}{R} e^{-\frac{R}{L}t}\right) u(t)$$

$$b(t) = \frac{E}{R} t\, u(t) - \frac{E}{R}\left(-\frac{L}{R}\right) e^{-\frac{R}{L}t} u(t) + C\, u(t)$$

$$b(t=0^-) = b(t=0^+)$$

$$0 = \frac{EL}{R^2} + C \Rightarrow C = -\frac{EL}{R^2}$$

$$b(t) = \frac{E}{R} t\, u(t) + \frac{EL}{R^2} e^{-\frac{R}{L}t} - \frac{EL}{R^2} u(t)$$



Consideriamo

$$\frac{E}{R}\left(t - \frac{L}{R}\right) u(t)$$

TRASFORMATE di LAPLACE

Data $f(t)$ definiamo trasformata di Laplace $F(s) = \mathcal{L}(f(t)) = \int_{-\infty}^{+\infty} f(t)\, e^{-st}\, dt$

dove $s = \sigma + j\omega$ è detta pulsazione complessa.

$$F(s) = \mathcal{L}(f(t)) = \int_{0^-}^{+\infty} f(t)\, e^{-st}\, dt$$

0^- → per beccare eventuali funzioni impulsive

<u>Proprietà:</u>

- $\mathcal{L}(a_1 f_1(t) + a_2 f_2(t) + \dots + a_n f_n(t)) = a_1 \mathcal{L}(f_1(t)) + \dots + a_n \mathcal{L}(f_n(t))$.
- unicità: $\mathcal{L}(f(t)) = \mathcal{L}(g(t)) \Downarrow f(t) = g(t)$

$$\mathcal{L}(u(t)) = \int_{0^-}^{+\infty} u(t)\, e^{-st}\, dt = \int_{0^+}^{+\infty} e^{-st}\, dt =$$

$$= -\left.\frac{e^{-st}}{s}\right|_{0^+}^{+\infty} = \frac{1}{s}$$

$$\int_{0^+}^{+\infty} e^{-st} = \int_{0^+}^{+\infty} e^{-\sigma t} e^{-j\omega t} = \int_{0^+}^{+\infty} e^{-\sigma t} \cos\omega t\, dt -$$

$$j \int_{0^+}^{+\infty} e^{-\sigma t} \sin\omega t\, dt$$

$$-\mathcal{L}(\delta(t)) = \int_{0^-}^{+\infty} \delta(t)\, e^{-st}\, dt =$$

$$\left[e^{-st}\right]_{t=0} = 1$$

$$-\mathcal{L}(e^{at}) = \int_{0^-}^{+\infty} e^{at} e^{-st}\, dt = \int_{0^-}^{+\infty} e^{-(s-a)t}\, dt =$$

$$= \left[-\frac{e^{-(s-a)t}}{(s-a)}\right]_{0}^{+\infty} = \frac{1}{s-a}$$

$$-\mathcal{L}(\cos(\omega t)) = \mathcal{L}\left(\frac{1}{2} e^{j\omega t} + \frac{1}{2} e^{-j\omega t}\right) =$$

$$= \frac{1}{2}\mathcal{L}(e^{j\omega t}) + \frac{1}{2}\mathcal{L}(e^{-j\omega t}) = \frac{1}{2}\frac{1}{s-j\omega} + \frac{1}{2}\frac{1}{s+j\omega} =$$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}s}{s^2-(j\omega)^2}\right) = \frac{s}{s^2+\omega^2}$$

$$-\mathcal{L}(\sin \omega t) = \mathcal{L}\left(\frac{1}{2j} e^{j\omega t} - \frac{1}{2j} e^{-j\omega t}\right) = \frac{\omega}{s^2+\omega^2}$$

$$f(t-t_0)\, u(t-t_0)$$

$$-\mathcal{L}(f(t-t_0)\, u(t-t_0))$$

$$\int_{0^-}^{+\infty} f(t-t_0)\, u(t-t_0)\, e^{-st}\, dt = \int_{t_0^-}^{+\infty} f(t-t_0)\, e^{-st}\, dt \; -$$

$\tau = t - t_0$

$$\int_{0^-}^{+\infty} f(\tau)\, e^{-s(\tau+t_0)}\, d\tau = \int_{0^-}^{+\infty} f(\tau)\, e^{-s\tau} e^{-st_0}\, d\tau$$

$$= e^{-st_0} \underbrace{\int_{0^-}^{+\infty} f(\tau)\, e^{-s\tau}\, d\tau}_{\text{trasformata di Laplace di } f(t)} = e^{-st_0} \mathcal{L}(f(t))$$

$$- \mathcal{L}\left(\frac{df}{dt}\right) = \int_{0^-}^{+\infty} \frac{df}{dt}\, e^{-st}\, dt =$$

$$f(t)\, e^{-st}\Big|_{0^-}^{+\infty} - \int_{0^-}^{+\infty} f(t)(-s)\, e^{-st}\, dt =$$

$$= -f(t=0^-) + s\int_{0^-}^{+\infty} f(t)\, e^{-st}\, dt =$$

$$= s\mathcal{L}(f(t)) - f(t=0^-)$$

$$- \mathcal{L}(\sin \omega t) = \frac{\omega}{s^2+\omega^2}$$

$$\mathcal{L}\left(\frac{d \sin(\omega t)}{dt}\right) = \frac{s\omega}{s^2+\omega^2} - \sin(0) =$$

$$\omega\, \mathcal{L}(\cos \omega t)$$

• $\mathcal{L}\left(\frac{d^2f}{dt}\right) = \mathcal{L}\left(\frac{dg}{dt}\right) = s\mathcal{L}(g(t)) - g(t=0^-) =$

$g = \left(\frac{df}{dt}\right)$

$= s\mathcal{L}\left(\frac{df}{dt}\right) - \frac{df}{dt}(t=0^-) = s^2\mathcal{L}(f(t)) - s f(t=0^-) - \frac{df}{dt}(t=0^-)$

• $\mathcal{L}\left(\frac{d\delta(t)}{dt}\right) = s\mathcal{L}(\delta(t)) - \delta(t=0^-) =$

s

$\mathcal{L}\left(\frac{d^n\delta(t)}{dt^n}\right) = s^n$

• $\mathcal{L}\left(f_i = \int_0^t f(\tau)\,d\tau\right) = \int_{0^-}^{+\infty}\left(\int_0^t f(\tau)\,d\tau\right) e^{-st}\,dt =$

$= \left[\int_0^t f(\tau)\,d\tau\left(-\frac{e^{-st}}{s}\right)\right]_{0^-}^{+\infty} - \int_{0^-}^{+\infty} f(t)\left(-\frac{e^{-st}}{s}\right)dt$

$= 0 + \frac{1}{s}\mathcal{L}(f(t))$

• $\mathcal{L}(te^{-\alpha t}) =$

$= \int_{0^-}^{+\infty} te^{-\alpha t}e^{-st}\,dt = \int_{0^-}^{+\infty} t\, e^{-(s+\alpha)t}\,dt$

- $\mathcal{L}(t\,u(t)) = \mathcal{L}\left(\int_0^t u(\tau)\,d\tau\right) =$

$$= \frac{1}{s}\mathcal{L}(u(t)) = \frac{1}{s}\cdot\frac{1}{s} = \frac{1}{s^2}$$

- $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$
- $\mathcal{L}(t e^{-\alpha t}) = \int_0^{+\infty} t e^{-\alpha t} e^{-st}\,dt =$

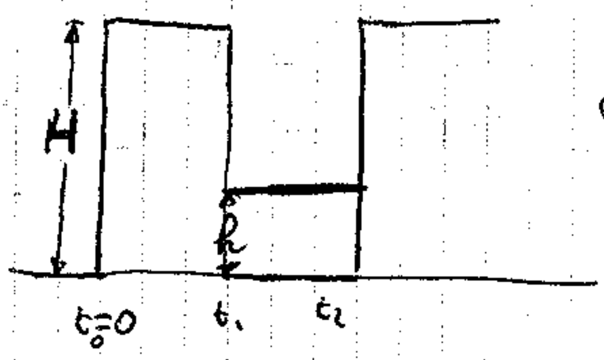
$$= \int_{0^-}^{+\infty} t e^{-(s+\alpha)t} = \int_{0^-}^{+\infty} t e^{-\beta t}\,dt =$$

$\beta = s+\alpha$

$$= \frac{1}{\beta} = \frac{1}{s+\alpha}$$

- $g(t) = e^{\gamma t} f(t) \Rightarrow G(s) = F(s-\gamma)$

Funzioni definite a tratti



$$f = \begin{cases} H\,u(t) & t<t_1 \\ h\,u(t-t_1) & t_1<t<t_2 \end{cases}$$

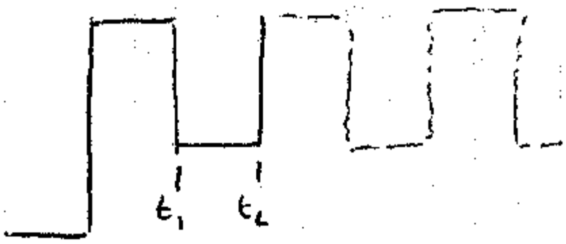
$$f = H\,u(t) + h\,u(t-t_1) - H\,u(t-t') + H\,u(t-t_2) - h\,u(t-t_2')$$

Posso vedere f come:

$$f = f(t)u(t) + [f_1(t) - f(t)]u(t-t_1) + [f_2(t) - f_1(t)]u(t-t_2) + \dots$$

$$F(s) = \frac{H}{s} + \frac{h-H}{s}e^{-st_1} + \frac{H-h}{s}e^{-st_2}$$

Si possono rendere periodiche delle funzioni definite a tratti:



$$f_A = H u(t) + (h-H)u(t-t_1)$$

$$\mathcal{L}(f_A) = \frac{H}{s} + \frac{h-H}{s}e^{-st_1}$$

$$\frac{F_A(s)}{1-e^{-st_2}} = G_A(t)$$ funzione periodica che ha andamento di quel tipo in ogni periodo.

$$f(t) \xrightarrow{\mathcal{L}} (F(s)) \rightarrow G(s) \xrightarrow{\mathcal{L}^{-1}} g(t)$$

Funzioni da antitrasformare

$$F(s) = \frac{P_0(s)}{Q_0(s)} + \frac{P_1(s)}{Q_1(s)} e^{-st_1} + \frac{P_2(s)}{Q_2(s)} e^{-st_2}$$

$$\mathcal{L}^{-1}\left(\frac{P_1(s)}{Q_1(s)} e^{-st}\right) \Rightarrow \mathcal{L}^{-1}\left(\frac{P_1(s)}{Q_1(s)}\right)$$

$$\overset{\shortparallel}{W_1}\ (t-t_1)\mu(t-t_1)$$

$$\mathcal{L}^{-1}\left(\frac{P_0(s)}{Q_0(s)}\right)$$

Considero 2 casi - Siano n_q e n_p i gradi dei polinomi Q e P

Se $\underline{n_p \geq n_q}$ la frazione è impropria

$$\frac{P_0(s)}{Q_0(s)} = N(s) + \left.\frac{R(s)}{Q_0(s)}\right\} \text{frazioni propria}$$

$$s^n = \frac{d^{n-1}\delta}{dt^{n-1}}$$

Se $n_p < n_q$ frazione propria.

$$\frac{R(s)}{Q(s)} \qquad n_R < n_q \quad \text{grado di } R < \text{grado di } Q$$

$$\frac{R(s)}{(s-q_1)(s-q_2)(\dots)(s-q_{n_q})}$$

Supponiamo che tutte le radici siano distinte e a molteplicità 1

$$\frac{R(s)}{Q(s)} = \sum_{n=1}^{n_q} \frac{K_i}{(s-q_i)} =$$ decomposizione in fratti semplici

$$= \underbrace{\frac{K_1}{(s-q_1)}}_{K_1 e^{+tq_1}} + \frac{K_2}{(s-q_2)} + \dots$$

Dobbiamo ricavare K

moltiplico $\frac{R(s)}{q(s)} \cdot (s-q_j) =$

$$= \frac{K_1 (s-q_j)}{(s-q_1)} + \dots + \frac{K_j \cancel{(s-q_j)}}{\cancel{(s-q_j)}} + \dots + \frac{K_{n_q} (s-q_j)}{(s-q_{n_q})}$$

Se attribuisco a s il valore q_j

$$(s-q_j) \frac{R(s)}{Q(s)} = K_j$$

Esempio 1

$$\frac{s-4}{(s+1)(s+2)(s+3)} = \frac{K_1}{(s+1)} + \frac{K_2}{(s+2)} + \frac{K_3}{(s+3)}$$

$$K_1 = \left. \frac{(s-4)\cancel{(s+1)}}{\cancel{(s+1)}(s+2)(s+3)} \right|_{s=-1} = -\frac{5}{2}$$

$$K_2 = \left. \frac{(s-4)\cancel{(s+2)}}{(s+1)\cancel{(s+2)}(s+3)} \right|_{s=-2} = 6$$

$$K_3 = \frac{(s-4)\cancel{(s+3)}}{(s+1)(s+2)\cancel{(s+3)}}\Bigg|_{s=-3} = \frac{-7}{2} =$$

$$\mathcal{L}^{-1}\left(\frac{R(s)}{Q(s)}\right) = -\frac{5}{2}e^{-t} + 6e^{-2t} - \frac{7}{2}e^{-3t}$$

Radici con molteplicità > 1

$$\frac{R(s)}{Q(s)} = \frac{R(s)}{(s-q)^3} = \underbrace{\frac{K_1}{(s-q)}}_{e^{qt}} + \underbrace{\frac{K_2}{(s-q)^2}}_{te^{qt}} + \underbrace{\frac{K_3}{(s-q)^3}}_{t^2e^{qt}}$$

$$(s-q)^3\frac{R(s)}{Q(s)} = K_1(s-q)^2 + K_2(s-q) + K_3$$

$$(s-q)^3\frac{R(s)}{Q(s)}\Bigg|_{s=q} = K_3$$

$$\frac{d}{ds}\left((s-q)^3\frac{R(s)}{Q(s)}\right)\Bigg|_{s=q} = 2K_1(s-q) + K_2 \to K_2$$

$$\frac{d^2}{ds^2}\left((s-q)^3\frac{R(s)}{Q_s}\right)\Bigg|_{s=q} = 2K_1$$

Esempio: $Q(s) = (s+1)(s+4)^2$

$R(s) = (s+3)$

$$\frac{R(s)}{Q(s)} = \frac{(s+3)}{(s+1)(s+4)^2} = \frac{K_1}{(s+1)} + \frac{K_2}{(s+4)} + \frac{K_3}{(s+4)^2}$$

$$(s+1)\frac{R(s)}{Q(s)} = \left.\frac{s+3}{(s+4)^2}\right|_{s=-1} = \frac{2}{9} = K_1$$

$$\left.(s+4)^2 \frac{R(s)}{Q(s)}\right|_{s=-4} = \left.\frac{(s+3)\cancel{(s+4)^2}}{(s+1)\cancel{(s+4)^2}}\right|_{s=-4} = +\frac{1}{3} = K_3$$

$$\left.\frac{d}{ds}\left((s+4)^2\frac{R(s)}{Q(s)}\right)\right|_{s=-4} = \left.\frac{(s+1)-(s+3)}{(s+1)^2}\right|_{s=-4} = -\frac{2}{9} = K_2$$

$$\mathcal{L}^{-1}\left(\frac{R(s)}{Q(s)}\right) = \frac{2}{9}e^{-t} - \frac{2}{9}e^{-4t} + \frac{1}{3}te^{-4t}$$

<u>Se radici complesse</u>

Se $\alpha + j\beta$ è radice allora anche $\alpha - j\beta$ è radice

$$K_1 = (s-\alpha-j\beta)\left.\frac{R(s)}{Q(s)}\right|_{s=+\alpha+j\beta}$$

$$K_2 = (s-\alpha+j\beta)\left.\frac{R(s)}{Q(s)}\right|_{s=\alpha-j\beta}$$

K_1 e K_2 sono complessi e coniugati

$$K_1 = K e^{j\gamma} \qquad K_2 = K e^{-j\gamma}$$

$$\frac{K_1}{(s-\alpha-j\beta)} + \frac{K_2}{(s-\alpha+j\beta)} = \frac{R(s)}{Q(s)}$$

$$\mathcal{L}^{-1}\left(\frac{R(s)}{Q(s)}\right) = |K| e^{j\gamma} e^{(\alpha+j\beta)t} + |K| e^{-j\gamma} e^{(\alpha-j\beta)t}$$

$$= |K| e^{\alpha t}\left[e^{j(\beta t+\gamma)} + e^{-j(\beta t+\gamma)}\right] =$$

(80)

$$= 2|K|\, e^{\alpha t} \cos(\beta t + \gamma)$$

Trasformate di Laplace per:

- generatori ideali

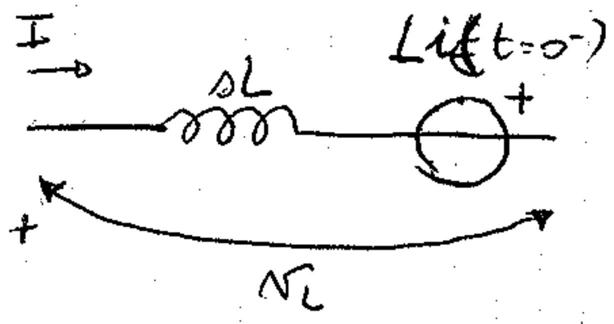
$$e(t) \Rightarrow E(s)$$
$$i_0(t) \Rightarrow I_0(s)$$

- resistori

$$e(t) = R\, i(t) \Rightarrow E(s) = R\, I(s)$$

- induttore ideale

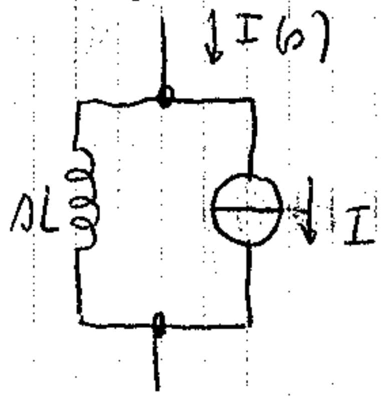
$$v_L = L\frac{di}{dt} \Rightarrow V_L(s) = sL\, I(s) - \underbrace{L\,(i(t=0))}_{\text{non ha la grandezza di una tensione ma di un flusso magnetico}}$$



s ha le dimensioni della frequenza

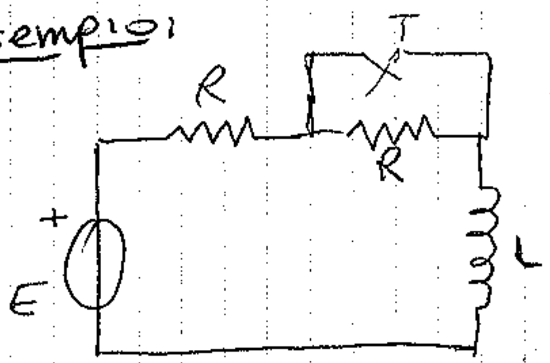
I è una carica

Passaggio al generatore ideale di corrente



$$I = \frac{L\, i(t=0^-)}{sL} = \frac{i(t=0^-)}{s}$$

Esempio:



t=0 chiusura interruttore

$$i(t=0^-) = \frac{E}{2R}$$

$$E = Ri + L\frac{di}{dt}$$

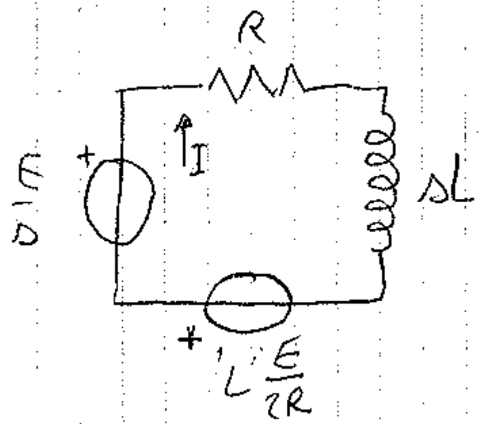
Dominio del tempo

$$i_p = \frac{E}{R} \qquad K = -\frac{E}{2R}$$

$$i_t = K e^{-\frac{R}{L}t}$$

$$i(t) = \frac{E}{R} - \frac{E}{2R} e^{-\frac{R}{L}t}$$

Dominio di Laplace



$$I = \frac{\frac{E}{s} + L\frac{E}{2R}}{R+sL} =$$

$$= \frac{E + sL\frac{E}{2R}}{s(R+sL)}$$

$$= \frac{E\left(\frac{1}{L} + \frac{s}{2R}\right)}{s\left(\frac{R}{L} + s\right)}$$

$$I = \frac{K_1}{s} + \frac{K_2}{\left(s + \frac{R}{L}\right)}$$

$$K_1 = \left.\frac{E\left(\frac{1}{L} + \frac{s}{2R}\right)}{\left(\frac{R}{L} + s\right)}\right|_{s=0} = \frac{\frac{E}{\cancel{L}}}{\frac{R}{\cancel{L}}} = \frac{E}{R}$$

$$K_2 = \left.\frac{E\left(\frac{1}{L} + \frac{s}{2R}\right)}{s}\right|_{s=-\frac{R}{L}} = \frac{E\left(\frac{1}{L} + \frac{-R/L}{2\cancel{R}}\right)}{-\frac{R}{L}} =$$

$$= \frac{E/2L}{-R/L} = -\frac{E}{2R}$$

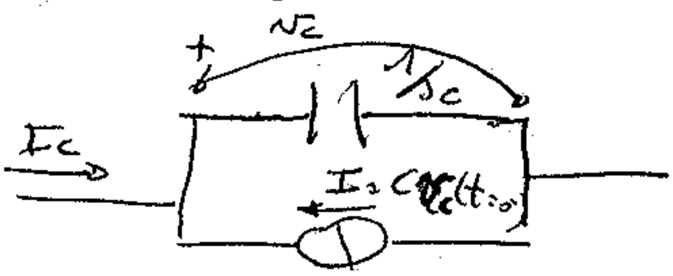
$$I = \frac{E/R}{s} - \frac{E/2R}{\left(s + \frac{R}{L}\right)}$$

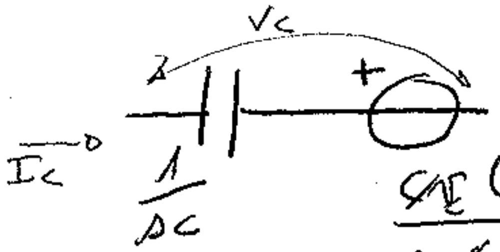
$$i(t) = \frac{E}{R}\, u(t) + \frac{E}{2R}\, e^{-\frac{R}{L}t}\, u(t)$$

– condensatore ideale

$$i_c(t) = C\,\frac{dv_c}{dt}$$

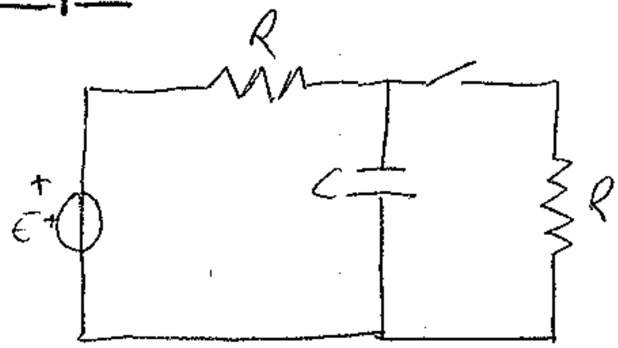
$$I_c(s) = sC\,V_c(s) - C v_c(t = 0^-)$$



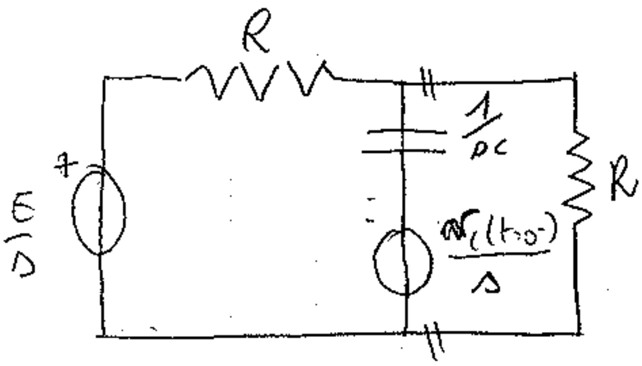


$$\frac{\cancel{C}\, v_C(t=0)}{s\,\cancel{C}} = \frac{v_C(t=0^-)}{s}$$

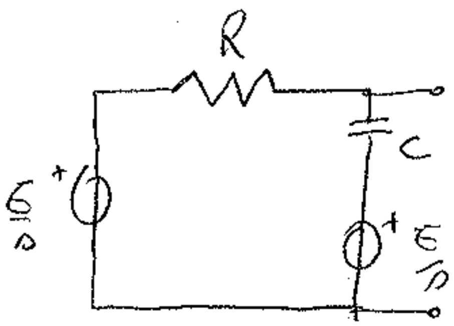
Esempio



$$v_C(t=0^-) = E$$

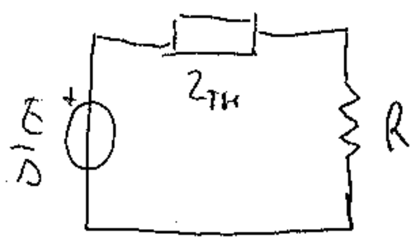


Calcolo equivalente di Thevenin



$$E_{TH} = \frac{E}{s}$$

$$Z_{TH} = \frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}} = \frac{R}{sRC + 1}$$



$$V_C = \frac{E}{s}\,\frac{R}{R + \frac{R}{sRC+1}} =$$

$$= \frac{E}{s}\,\frac{\cancel{R}(sRC+1)}{\cancel{R}(sRC+2)} =$$

$$= \frac{E(sRC+1)}{s(sRC+2)} = \frac{E\left(s + \frac{1}{RC}\right)}{s\left(s + \frac{2}{RC}\right)}$$

$$V_c = \frac{K_1}{s} + \frac{K_2}{s + \frac{2}{RC}}$$

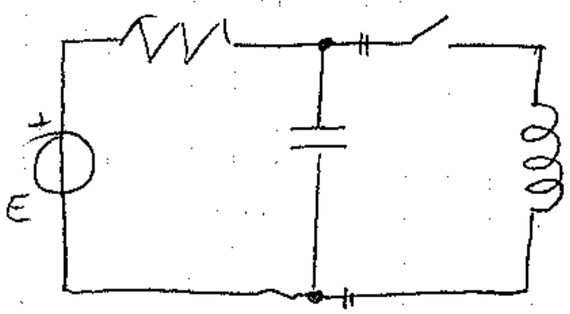
$$K_1 = \left. \frac{\cancel{s}\, E\left(s + \frac{1}{RC}\right)}{\cancel{s}\left(s + \frac{2}{RC}\right)} \right|_{s=0} = \frac{E}{2}$$

$$K_2 = \left. \frac{E\left(s + \frac{1}{RC}\right)}{s} \right|_{s=-\frac{2}{RC}} = \frac{E}{2}$$

$$V_c = \frac{E/2}{s} + \frac{E/2}{s + \frac{2}{RC}}$$

$$v_c(t) = \frac{E}{2} u(t) + \frac{E}{2} e^{-\frac{2}{RC} t} u(t)$$

Esempio



$$E_{TH} = \frac{E}{s}$$

$$Z_{TH} = \frac{R}{sRC + 1}$$

$$I(s) = \frac{E/s}{sL + \frac{R}{sRC+1}} = \frac{E/s\,(sRC+1)}{sL(sRC+1) + R} =$$

$$= E \frac{\frac{s}{L} + \frac{1}{RLC}}{s\left(s^2 + \frac{1}{RC} s + \frac{1}{LC}\right)}$$

$$p_1 = 0$$

$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$$

$$p_{2,3} = \frac{1}{2}\left(-\frac{1}{RC}\right) \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}$$

Supponiamo che il discriminante sia 0

$$I(s) = \frac{E\left(\frac{s}{L} + \frac{1}{RLC}\right)}{s\left(s + \frac{1}{2RC}\right)^2} = \frac{K_1}{s} + \frac{K_2}{\left(s + \frac{1}{2RC}\right)^2} + \frac{K_3}{\left(s + \frac{1}{2RC}\right)}$$

$$K_1 = \left.\frac{E\left(\frac{s}{L} + \frac{1}{RLC}\right)}{\left(s + \frac{1}{2RC}\right)^2}\right|_{s=0} = E\frac{1}{R L \cancel{C}} \cdot 4R^{\cancel{2}}C^{\cancel{2}} = E\frac{4RC}{L}$$

$$K_2 = \left.\frac{E}{s}\left(\frac{s}{L} + \frac{1}{RLC}\right)\right|_{s=-\frac{1}{2RC}} = -\frac{E}{L}$$

$$K_3 = \left.\frac{d}{ds}\left(\frac{E}{s}\left(\frac{s}{L} + \frac{1}{RLC}\right)\right)\right|_{s=-\frac{1}{2RC}} = -\left.\frac{E}{RLCs^2}\right|_{s=-\frac{1}{RC2}} =$$

$$= -4E\frac{RC}{L}$$

$$I(s) = \frac{4ERC/L}{s} + \frac{-4ERC/L}{s + \frac{1}{2RC}} - \frac{E/L}{\left(s + \frac{1}{2RC}\right)^2}$$

$$i(t) = E\frac{4RC}{L}u(t) - E\frac{4RC}{L}e^{-\frac{t}{2RC}}u(t) - \frac{E}{L}t\,e^{-\frac{t}{2RC}}$$

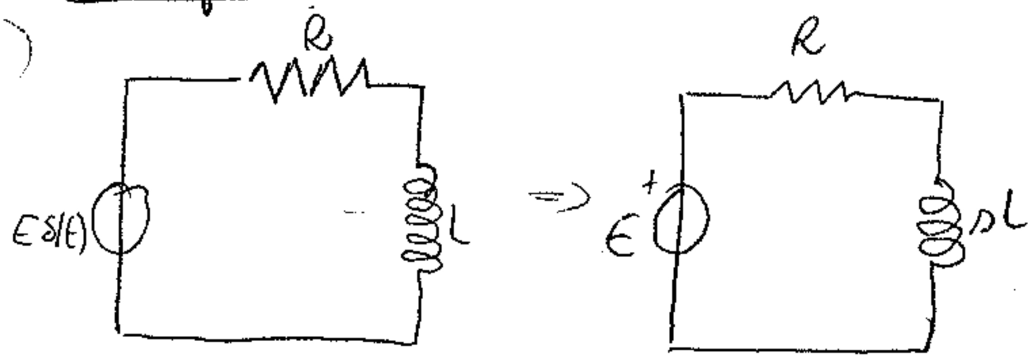
Poichè ho supposto $\Delta = 0$ ho che:

$$\frac{1}{R^2C^2} = \frac{4}{LC} \Rightarrow \frac{1}{R} = \frac{4RC}{L}$$

quindi

$$i(t) = \frac{E}{R}u(t) - \frac{E}{R}e^{-\frac{t}{2RC}}u(t) - \frac{E}{L}\,t\,e^{-\frac{t}{2RC}}$$

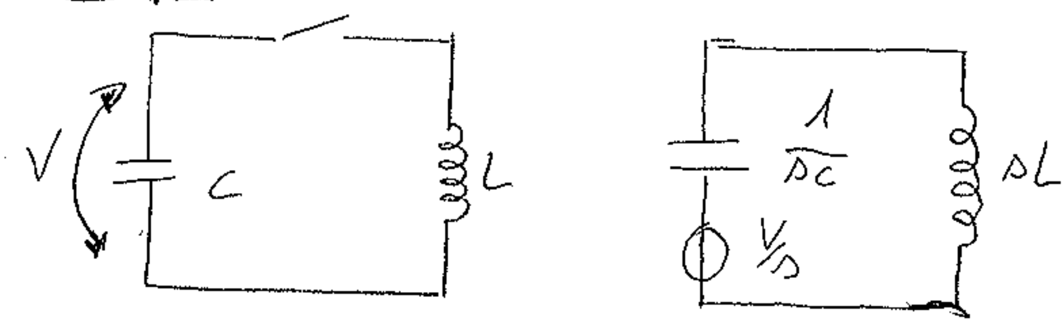
Esempio



$$I(s) = \frac{E}{R+sL} = \frac{E/L}{R/L + s}$$

$$i(t) = \frac{E}{L}e^{-\frac{R}{L}t}\,u(t)$$

Esempio



$$I(s) = \frac{V/s}{sL + \frac{1}{sC}} = \frac{V/L}{s^2 + \frac{1}{LC}} = \frac{V/L}{s^2 + \left(\sqrt{\frac{1}{LC}}\right)^2}$$

$$= \frac{\sqrt{\frac{1}{LC}}}{s^2 + \left(\sqrt{\frac{1}{LC}}\right)^2}\,\frac{V}{L}\sqrt{LC} = V\sqrt{\frac{C}{L}}\,\frac{\sqrt{\frac{1}{LC}}}{s^2 + \left(\sqrt{\frac{1}{LC}}\right)^2}$$

$$i(t) = V\sqrt{\frac{C}{L}}\, \sin\left(\sqrt{\frac{1}{LC}}\, t\right) u(t)$$

Andamento anomalo legato all'assenza di elementi dissipativi. (il condensatore si scarica, la corrente aumenta il contenuto energetico dell'induttore che si carica, quando è completamente carico il condensatore si ricarica e l'induttore si scarica).

$$g(t) \overset{\mathcal{L}}{\Longrightarrow} G(s)$$ $g(t)$ è una generica forzante

$$y(t) \Longrightarrow Y(s) = \mathcal{L}(y)$$ $y(t)$ risposta alla forzante

$$H(s) = \frac{Y(s)}{G(s)}$$ $H(s)$ funzione di rete

Se $\delta(t) = g(t)$

$$H(s) = \frac{\mathcal{L}(h)}{1} = \mathcal{L}(h)$$

$$y(t) = \int_{-\infty}^{+\infty} g(\tau)\, h(t-\tau)\, d\tau = g * h$$

$$\Uparrow$$

$$Y(s) = G(s) \cdot H(s)$$

Teorema di convoluzione.