Regime transitorio

Variabili di stato: { correnti in induttori; tensioni nei condensatori

Variabili algebriche

selezioniamo come variabile $u(t)$ e dopo passaggi algebrici ci troveremo:

$$a_n \frac{d^n u}{dt^n} + a_{n-1} \frac{d^{n-1} u}{dt^{n-1}} + \dots + a_1 \frac{du}{dt} + a_0 u = b$$

n: numero di induttori + condensatori (var. di stato)

Risoluzione dell'equazione differenziale

$u(t) = u_p(t) + u_t(t)$

Calcolo integrale particolare
ponendo $t \to \infty$

Calcolo integrale dell'eq. omogenea
(circuito dove sono stati rimossi i generatori)

$a_n p^n + a_{n-1} p^{n-1} + \dots + a_1 p + a_0 = 0$

$p_1, p_2, p_3, \dots$

$u_t(t) = K_1 e^{p_1 t} + K_2 e^{p_2 t} + \dots + K_n e^{p_n t}$

$p_1, p_2, \dots, p_n$ devono essere ≤ 0 se reali e se complessi $\mathrm{Re}(p_1), \mathrm{Re}(p_1), \dots, \mathrm{Re}(p_n) \leq 0$

$$u(t=0^-) = u(t=0^+) \Rightarrow K$$

Se: $p_K = p_{K+1} = p_{K+2} = p_0$

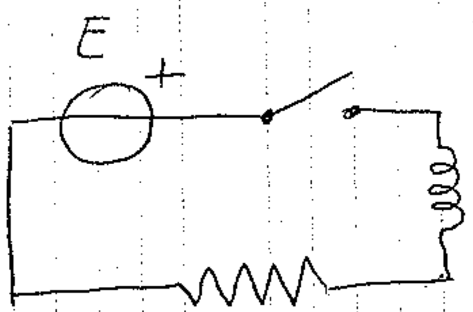
$$a_K e^{p_0 t} + a_{K+1} \cdot t e^{p_0 t} + a_{K+2} t^2 e^{p_0 t}$$

Consideriamo

e^{pt} $\quad p<0 \quad$ $\tau = \frac{1}{|\mathrm{Re}(p)|}$ costante di tempo

$$e^{p \frac{1}{|p|}} = e^{-1} = \frac{1}{e}$$

$\Delta t = (5 \div 6)\tau$ terminato il transitorio



in $t=0$ chiudiamo l'interruttore

$E = Ri + L \frac{di}{dt}$ equazione diff. da risolvere

$$i(t) = i_p(t) + i_t(t)$$

$$i_p(t) = \frac{E}{R}$$

Trovo $i_t(t)$

$$R i_t + L\frac{di_t}{dt} = 0$$

$$R + Lp = 0 \qquad p = -\frac{R}{L}$$

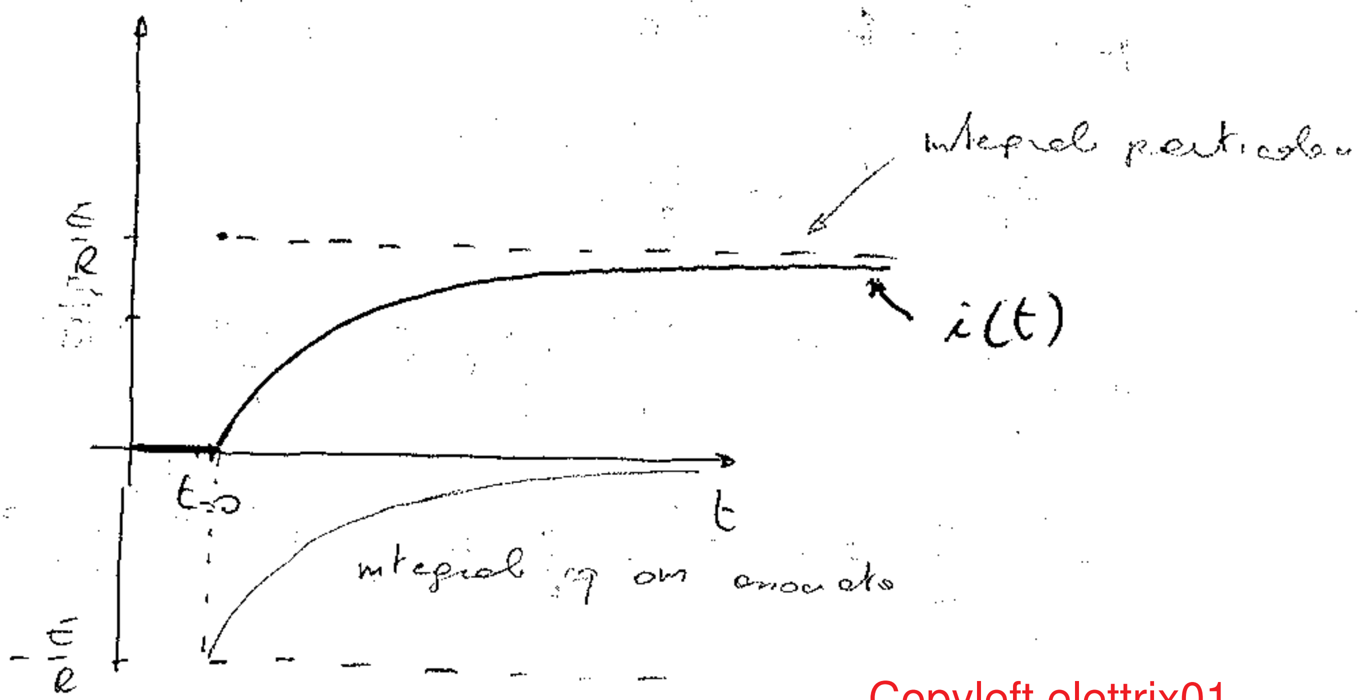
$$i_t = K_1 e^{-\frac{R}{L}t}$$

$$i(t) = \frac{\mathcal{E}}{R} + K_1 e^{-\frac{R}{L}t}$$

$$\left.\begin{array}{l} t = 0^- \Rightarrow i = 0 \\ t = 0^+ \Rightarrow \frac{\mathcal{E}}{R} + K_1 e^{-\frac{R}{L}t} = \frac{\mathcal{E}}{R} + K_1 \end{array}\right\} \Rightarrow$$

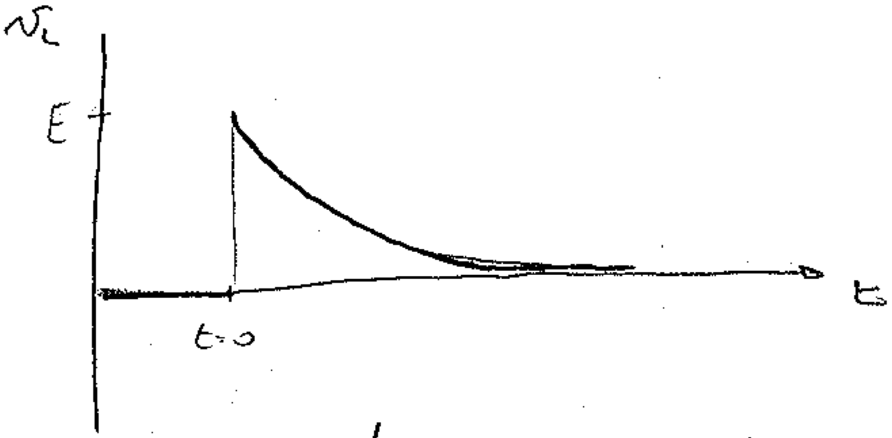
$$\Rightarrow \quad \frac{\mathcal{E}}{R} = -K_1 \qquad K_1 = -\frac{\mathcal{E}}{R}$$

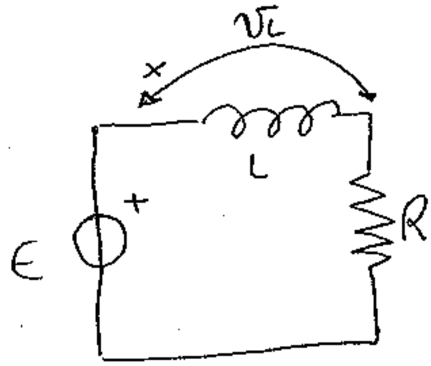
$$i_t(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$



$$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left(\frac{E}{R}-\frac{E}{R}e^{-\frac{R}{L}t}\right) =$$

$$= \cancel{L}\left(+\frac{E}{\cancel{R}}\cdot\frac{\cancel{R}}{\cancel{L}}e^{-\frac{R}{L}t}\right) = E\,e^{-\frac{R}{L}t}$$





$$\tau = \frac{1}{|\mathrm{Re}(\lambda)|} = \frac{L}{R}$$

durata ~~tanto~~ transitorio è tanto

> tanto L > R

durata tanto < tanto R grande rispetto a L

$$\Delta t = (5 \div 6)\frac{L}{R}$$

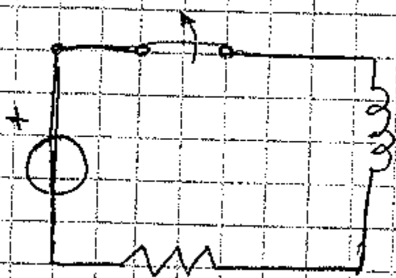
$$p_L = v_L \cdot i = E\,e^{-\frac{R}{L}t}\left(\frac{E}{R}-\frac{E}{R}e^{-\frac{R}{L}t}\right) =$$

$$= \frac{E^2}{R}e^{-\frac{R}{L}t} - \frac{E^2}{R}e^{-2\frac{R}{L}t}$$

$$w_L = \int_{0^+}^{\infty} p_L\,dt = \int_{0^+}^{\infty} \frac{E^2}{R}e^{-\frac{R}{L}t} - \frac{E^2}{R}e^{-2\frac{R}{L}t}\,dt$$

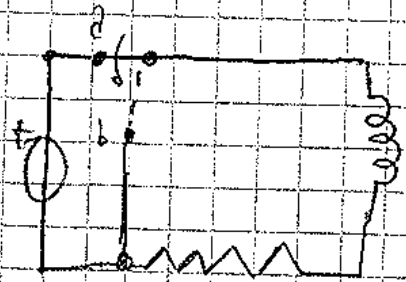
$$= \left[-\frac{E^2}{R}\frac{L}{R}e^{-\frac{R}{L}t} + \frac{1}{2}\frac{E^2}{R}\cdot\frac{L}{R}e^{-2\frac{R}{L}t}\right]_{0^+}^{\infty} =$$

$$\rightarrow \frac{E^2}{R^2} L + \frac{1}{2} \frac{E^2}{R^2} L = \frac{1}{2} I^2 L$$



Al tempo $t=0$ apro l'interruttore

Non posso studiare un circuito di questo genere



All'istante $t=0$ commuto il circuito da a a b
il circuito in cui ho commutato è il circuito in cui si svolge il transitorio.

$$0 = Ri + L \frac{di}{dt}$$

$$i_p = 0$$

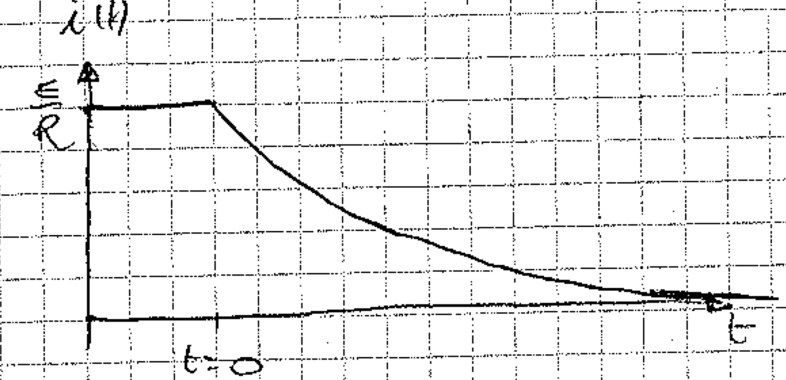
$$i(t) = i_t(t)$$

$$R + Lp = 0 \qquad p = -\frac{R}{L}$$

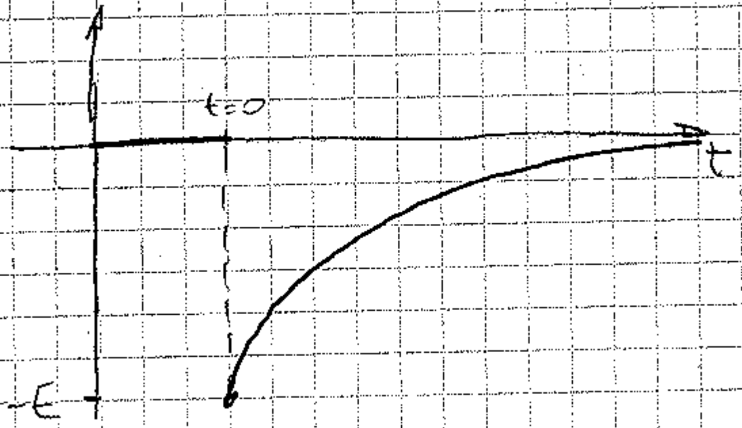
$$i(t) = K e^{-\frac{R}{L}t}$$

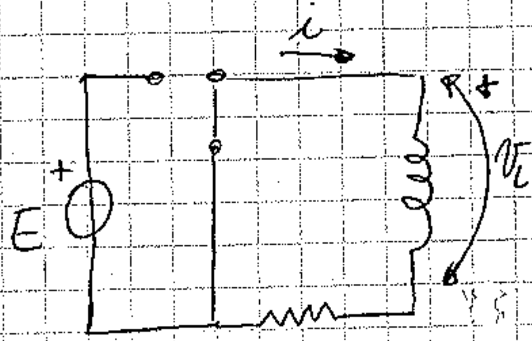
$$i(t=0^-) = i(t=0^+)$$

$$\frac{E}{R} = k \Rightarrow \boxed{i(t) = \frac{E}{R} e^{-\frac{R}{L}t}}$$

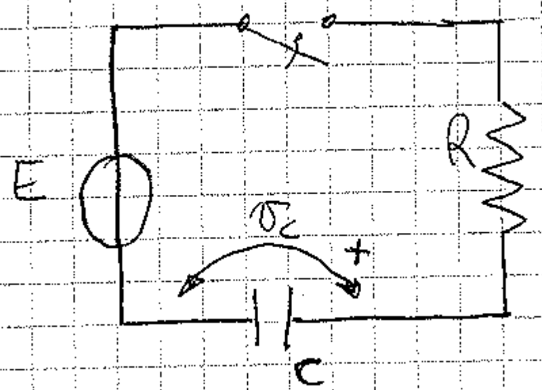


$$v_L = L\frac{di}{dt} = \cancel{L}\frac{E}{R}\left(-\frac{R}{\cancel{L}}\right) e^{-\frac{R}{L}t} = -E e^{-\frac{R}{L}t}$$





CIRCUITO RC



in $t=0$ il circuito viene chiuso

$v_c(t=0^-) = 0$

$E = Ri + v_c$ $\qquad i(t) = C\frac{dv_c}{dt}$

$$E = RC \frac{dv_c}{dt} + v_c$$

$$v_{cp} = E$$

$$RC \frac{dv_{ct}}{dt} + v_{ct} = 0$$

$$RC\,p + 1 = 0$$

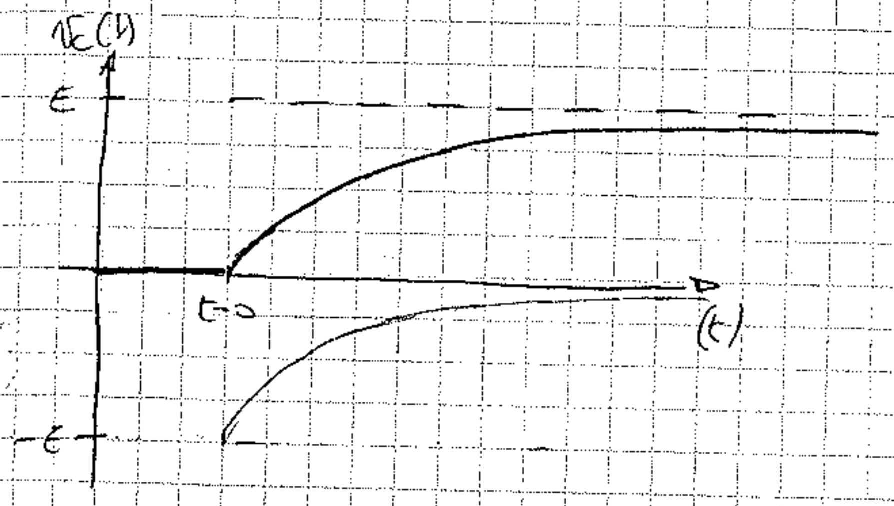
$$p = -\frac{1}{RC}$$

$$v_{ct} = K e^{-\frac{t}{RC}} \Rightarrow v_c = E + K e^{-\frac{t}{RC}}$$

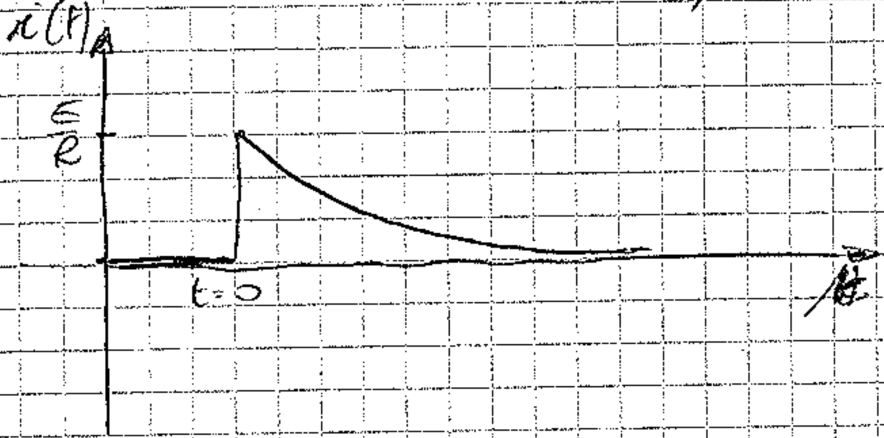
$$v_c(t=0^-) = v_c(t=0^+)$$

$$0 = E + K e^{\frac{-t}{RC}} \Rightarrow K = -E$$

$$v_c = E\left(1 - e^{\frac{-t}{RC}}\right)$$



$$i(t) = C \frac{dv_C}{dt} = \cancel{C}\left(+\frac{E}{R\cancel{C}}\right) e^{-\frac{t}{RC}} = -\frac{E}{R} e^{-\frac{t}{RC}}$$



$$\eta = \frac{\mathcal{E}_C}{\mathcal{E}_E} \qquad \mathcal{E}_C = \frac{1}{2} C E^2$$

$$\mathcal{E}_E = \int_0^\infty p_E \, dt = \int_0^\infty E i \, dt = E \underbrace{\int_0^\infty i \, dt}_{q(t \to \infty)} =$$

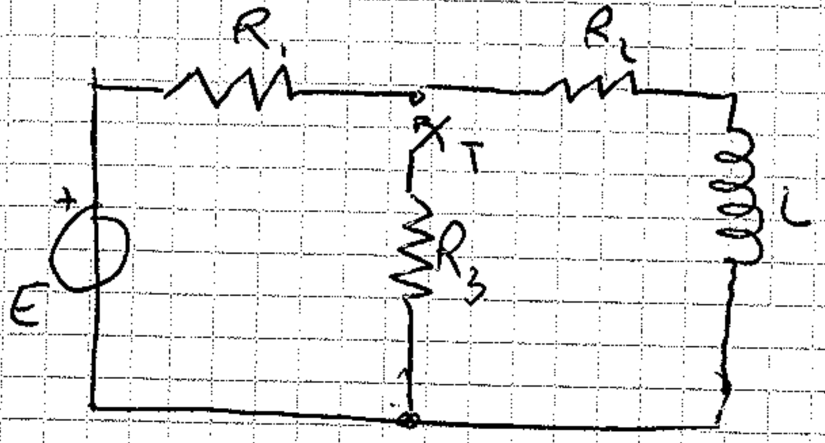
$$E q(t \to \infty) = E^2 C$$

$$\eta = \frac{\frac{1}{2} \cancel{C} \cancel{E^2}}{\cancel{C} \cancel{E^2}} = \frac{1}{2}$$

Se volete caricare un condensatore si può mettere qualsiasi resistenza ma l'energia immagazzinata sarà sempre metà rispetto a quella fornita dal generatore.

La resistenza può variare la costante di tempo

$$\tau = RC$$



$E = 240\ V$

$R_1 = R_2 = R_3 = 20\,\Omega$

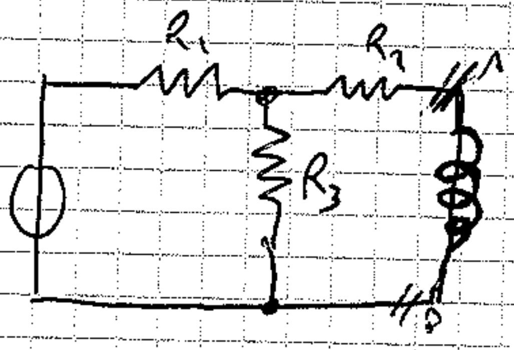
$L = 120\ mH$

A $t=0$ T si chiude e a $t_1 = 10\ ms$ T viene riaperto

Per $t = 0^-$ regime stazionario

$$i(t=0^-) = E/(R_1+R_2) = 6\ A$$

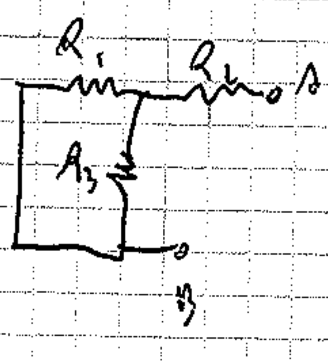
$t=0$



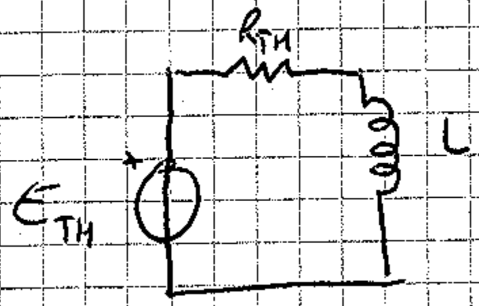
Possiamo applicare il teorema di Thevenin perché ci sono solo resistenze

$$E_{TH} = E\frac{R_3}{R_1+R_3} = 120\ V$$

$$R_{TH} = R_2 + \left(\frac{R_1 R_3}{R_1+R_3}\right) = 30\ \Omega$$



il transitorio si svolge in



$$i_p = \frac{E_{TH}}{R_{TH}} = 4\ A$$

$$0 = R_{th} I_t + L \frac{di_t}{dt}$$

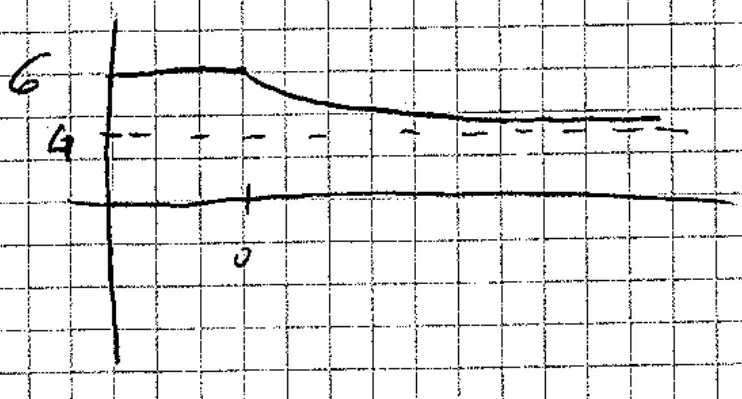
$$0 = R_{TH} + Lp \qquad p = -\frac{R_{TH}}{L}$$

$$i(t) = K e^{-\frac{R_{TH}}{L} t} + 4 = K e^{-250 t} + 4$$

$$i(t=0^-) = i(t=0^+)$$

$$6 - 4 = K e \qquad K = 2$$

$$i(t) = 4 + 2 e^{-250 t}$$



$$\tau = \frac{L}{R_{TH}} = 4\ ms$$

$$\Delta t = 5 \div 6\ \tau =$$

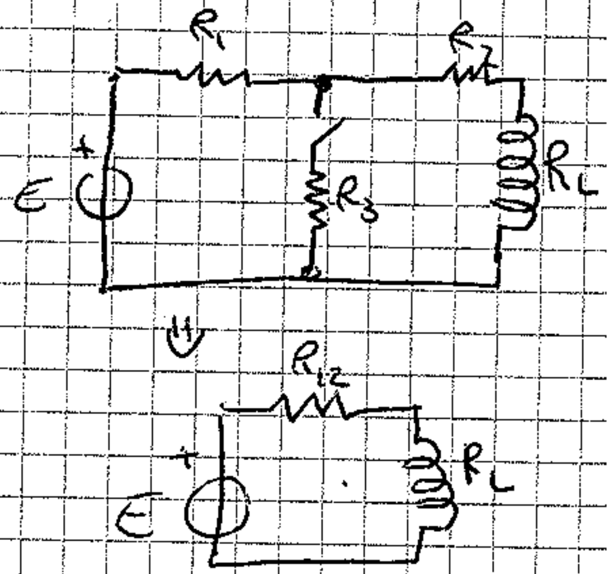
$$20 \div 24\ ms$$

$$i(t_1) = i(t = 10\,ms) = 4 + 2e^{-2,5} = 4,16\ A$$

$t' = t - t_1 \Rightarrow T$ viene aperto a $t = t_1$, cioè

$t' = 0$

$$i(t' = 0^-) = 4,16\ A$$



$$E = (R_1 + R_2)i' + L\frac{di'}{dt'}$$

$$i'_p = \frac{E}{R_1 + R_2} = 6\ A$$

$$(R_1 + R_2)i'(t') + L\frac{di'(t')}{dt'} = $$

$$R_1 + R_2 + L p' = 0 \qquad p' = -\frac{R_1 + R_2}{L} = -333,\bar{3}\ s^{-1}$$

$$i'(t') = 6 + K' e^{-333,\bar{3}\,t'}$$

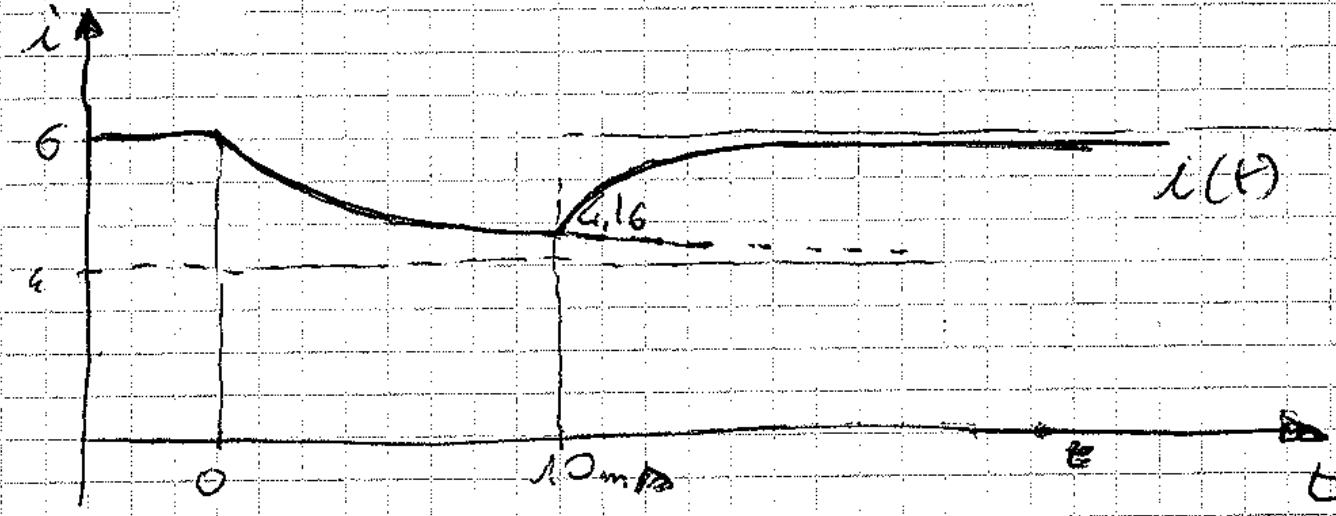
$$i'(t' = 0^-) = i(t' = 0^+)$$

$$\Downarrow$$

$$4,16 = 6 + K' \Rightarrow K' = -1,84$$

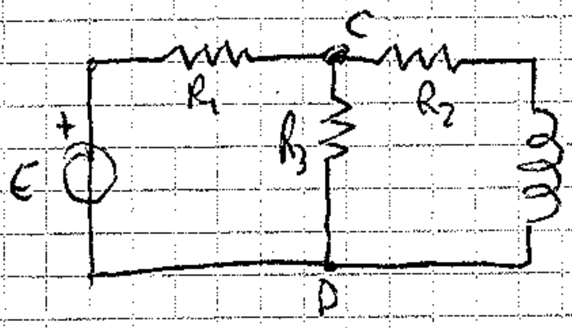
$$i(t') = 6 - 1,84\, e^{-333,\bar{3}\,t'}$$

$$\tau' = 3\ ms$$



Calcolo corrente fornita dal generatore

$t \leq 10\,ms$



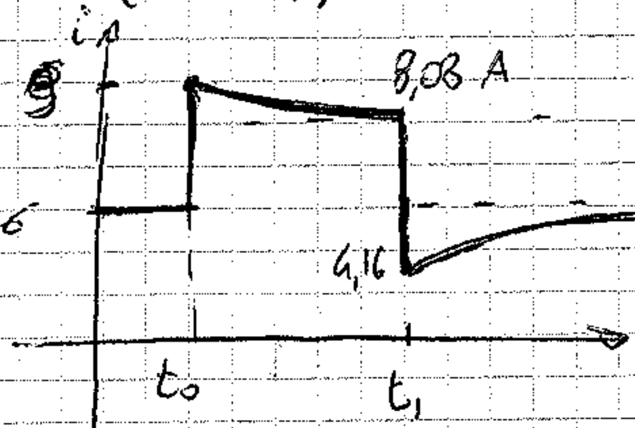
$$v_{CD} = L\frac{di}{dt} + R_2 I$$

$$v_{CD} = -L\,2 \cdot 250\, e^{-250t} + R_2(4+2e^{-250t})$$

$$= -60e^{-250t} + 80 + 40e^{-250t} =$$

$$= -20e^{-250t} + 80$$

$$I_E = (E - V_{CD})/R_1 = \frac{\cancel{160}^{\,80} + \cancel{20}^{\,10}e^{-250t}}{\cancel{20}} = 8 + 1e^{-250t}$$



$t \geq 10\,ms$ $\quad i_E(t) = i(t)$

Osservazioni: tutte le soluzioni sono nella forma $u(t) = A + Be^{-t/\tau}$

$t = 0^+ \qquad u = A + B$

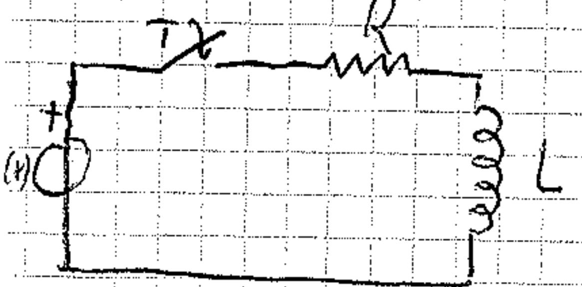
$t \to \infty \qquad u = A$

$$u(t=0^+) = u(t=0^-)$$

$$\begin{cases} B = u(t=0^-) - u(t \to \infty) \\ A = u(t \to \infty) \end{cases}$$

$$u(t) = u(t \to \infty) + \left(u(t=0^-) - u(t \to \infty)\right) e^{-\frac{t}{\tau}}$$

Non ha validità generale ma solo se
- regime stazionario
- una sola variabile di stato
- non devono esserci transitori in corso.



t=0 chiudo T

$$e(t) = \sqrt{2}\, E \sin(\omega t + \alpha)$$

$$e(t) = R i + L \frac{di}{dt}$$

$$0 = R i_t + L \frac{di}{dt} \quad \text{omog. ass.}$$

$$i_t = K e^{-\frac{R}{L} t}$$

$X_L = \omega L \qquad \bar{I} =$

$\bar{Z} = R + j X_L = Z e^{j\varphi} \qquad \bar{E} = E e^{j\alpha}$

$$\bar{I}_p = \frac{\bar{E}}{\bar{Z}} = \frac{E e^{j\alpha}}{Z e^{j\varphi}} = \frac{E}{Z} e^{j(\alpha-\varphi)}$$

$$i_p(t) = \sqrt{2}\,\frac{E}{Z} \sin(\omega t + \alpha - \varphi)$$

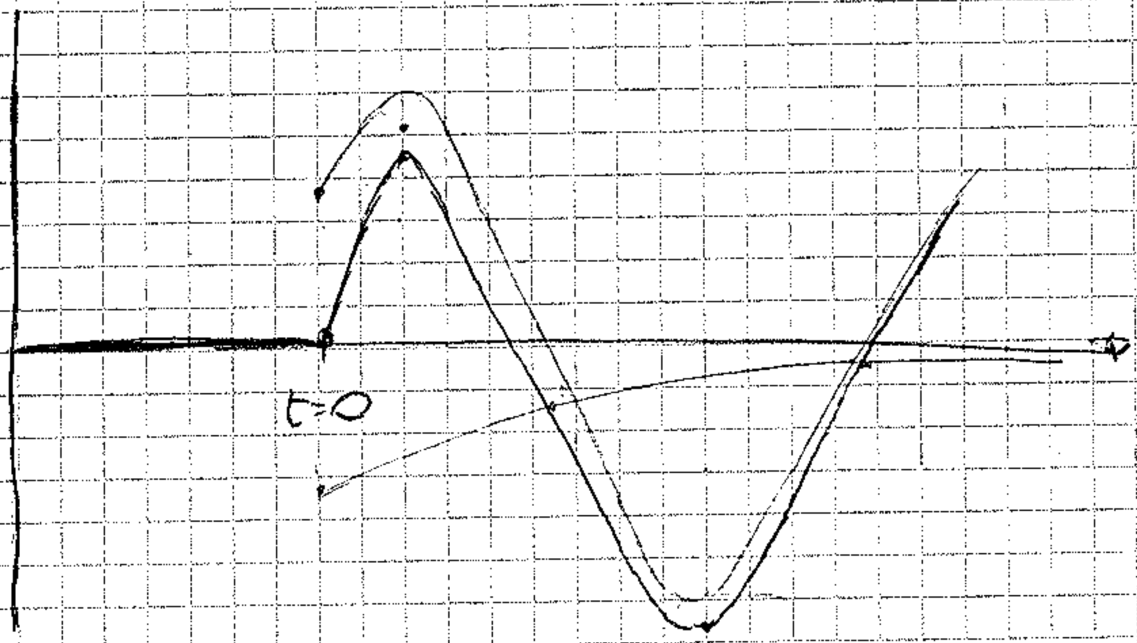
$$i(t) = \sqrt{2}\,\frac{E}{Z} \sin(\omega t + \alpha - \varphi) + K e^{-\frac{R}{L}t}$$

$$i(t=0^-) = i(t=0^+)$$

$$0 = \sqrt{2}\,\frac{E}{Z} \sin(\alpha - \varphi) + K$$

$$K = -\sqrt{2}\,\frac{E}{Z} \sin(\alpha - \varphi)$$

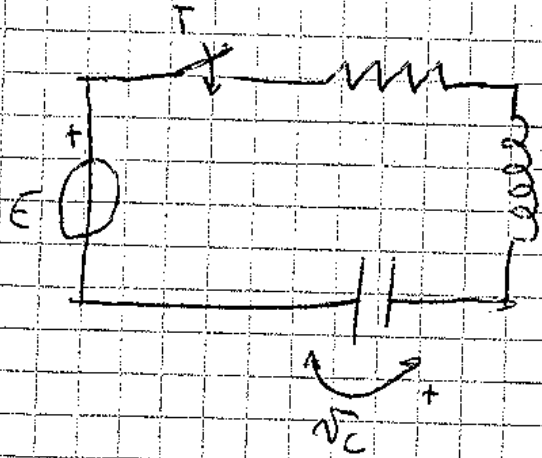
$$i(t) = \sqrt{2}\,\frac{E}{Z} \left(\sin(\omega t + \alpha - \varphi) - \sin(\alpha - \varphi) e^{-\frac{R}{L}t} \right)$$



valore di picco > valore picco funzione

puramente sinusoidale.

CIRCUITO RLC



T chiuso all'istante $t=0$

$v_C(t=0^-) = V_0 < E$

$$E = Ri + L\frac{di}{dt} + v_C$$

$$i = C\frac{dv_C}{dt}$$

$$\boxed{E = RC\frac{dv_C}{dt} + LC\frac{d^2v_C}{dt^2} + v_C}$$

$$0 = CR\frac{di}{dt} + CL\frac{d^2i}{dt^2} + \underbrace{C\frac{dv_C}{dt}}_{i}$$

$$\boxed{LC\frac{d^2i}{dt^2} + CR\frac{di}{dt} + i = 0}$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$$p^2 + \frac{R}{L}p + \frac{1}{LC} = 0$$

$$\Delta = \left(\frac{R}{L}\right)^2 - \frac{4}{LC} \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

$\boxed{\text{Se } \Delta > 0}$

$$p = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

entrambe le radici sono negative

$$p_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$i_t = K_1 e^{p_1 t} + K_2 e^{p_2 t}$$

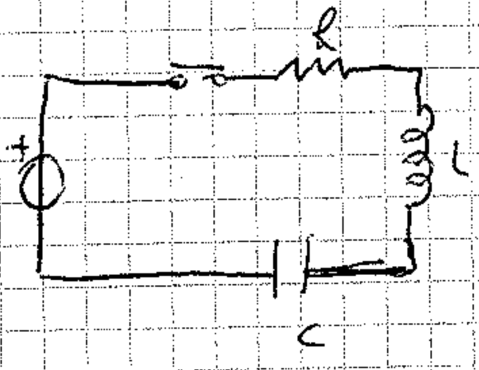
$$p_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\tau_1 = \frac{1}{|p_1|} \qquad \tau_2 = \frac{1}{|p_2|}$$

$\tau_1 > \tau_2$ $\qquad$ τ_1 determina durata dell'intero transitorio

$$\begin{cases} i(t=0^-) = i(t=0^+) \\ v_C(t=0^-) = v_C(t=0^+) \end{cases}$$

$$\begin{cases} 0 = K_1 + K_2 \\ p_1 K_1 + p_2 K_2 = \frac{E - V_0}{L} \end{cases}$$



In $t=0^+$

$$E = \cancel{R\,i(t=0^+)} + L\frac{di(t=0^+)}{dt} + v_c(t=0^+)$$

$$i(t=0^+)=0 \qquad v_c(t=0^+) \to V_0$$

$$E = L\left.\frac{di}{dt}\right|_{t=0^+} + V_0$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = \frac{E-V_0}{L}$$

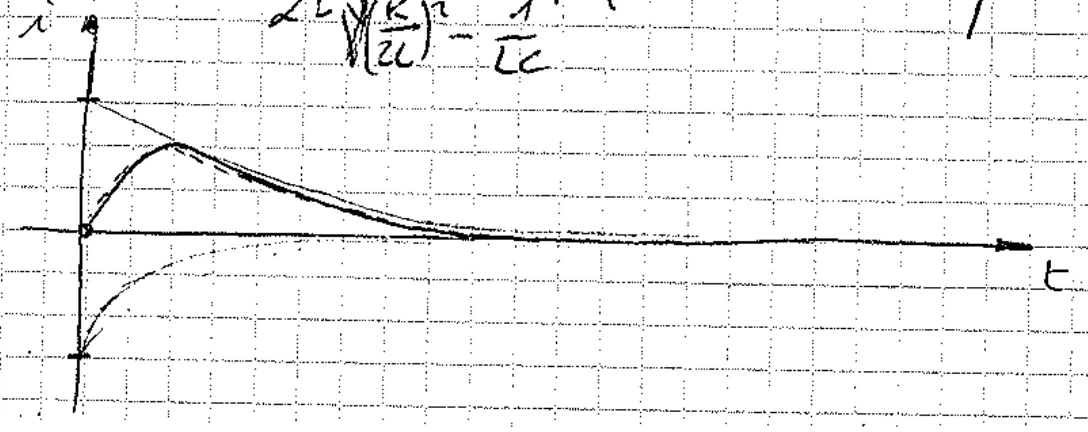
$$i(t) = K_1 e^{p_1 t} + K_2 e^{p_2 t}$$

$$p_1 K_1 e^{p_1 (t)}\Big|_{t=0^+} + K_2\, p_2\, e^{p_2 t}\Big|_{t=0^+} = \frac{E-V_0}{L}$$

$$p_1 K_1 + K_2 p_2 = \frac{E-V_0}{L}$$

$$\begin{cases} K_1 + K_2 = 0 \\ p_1 K_1 + p_2 K_2 = \dfrac{E-V_0}{L} \end{cases}$$

$$i(t) = \frac{E-V_0}{2L\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}\left(e^{p_1 t} - e^{p_2 t}\right)$$



La corrente ha lo scopo di caricare il condensatore.

Se $V_0 > E$ l'andamento della corrente sarebbe rovesciato rispetto a quello di $V_0 < E$, il condensatore si scarica sul generatore.

Se $V_0 = E$ non ci sarebbe transitorio.

Se $\Delta = 0$

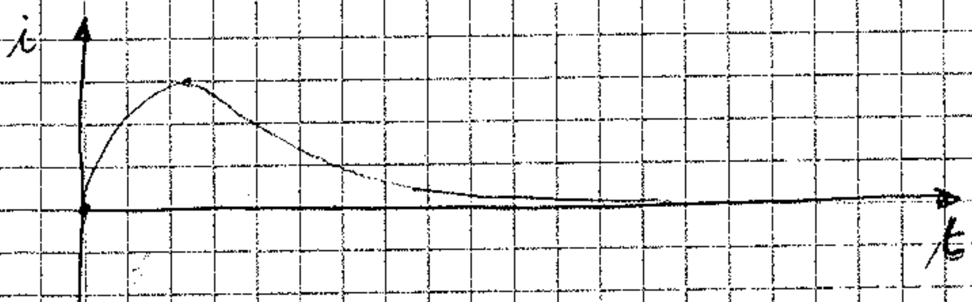
$$\mu_1 = \mu_2 = -\frac{R}{L \cdot 2} = \mu$$

$$i(t) = K_1 e^{\mu_1 t} + K_2 t e^{\mu_1 t}$$

$$\begin{cases} i(t=0^+) = i(t=0^-) \\ \left.\dfrac{di}{dt}\right|_{t=0^+} = \dfrac{E - V_0}{L} \end{cases}$$

$$\begin{cases} K_1 = 0 \\ \left[K_1 e^{\mu t} + \mu K_2 t e^{\mu t}\right]_{t=0^+} = \dfrac{E - V_0}{L} = \dfrac{E - V_0}{L} \end{cases}$$

$$\Rightarrow \begin{cases} K_1 = 0 \\ K_2 = \dfrac{E - V_0}{L} \end{cases} \Rightarrow i(t) = \frac{E - V_0}{L} t e^{-\frac{R}{2L} t}$$



Se $\Delta < 0$

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} < 0$$

definiamo $\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

$$p_1 = -\frac{R}{2L} + j\omega_0$$

$$p_2 = -\frac{R}{2L} - j\omega_0$$

$$i(t) = K_1 e^{p_1(t)} + K_2 e^{p_2 t}$$

$$\underset{i(t=0^-)}{0} = K_1 + \underset{i(t=0^+)}{K_2} \Rightarrow K_2 = -K_1$$

$$i(t) = K_1 \left(e^{p_1(t)} - e^{p_2(t)}\right) =$$

$$= K_1 \left(e^{-\frac{R}{2L}t} e^{j\omega_0 t} - e^{-\frac{R}{2L}t} e^{-j\omega_0 t}\right) =$$

$$= K e^{-\frac{R}{2L}t} \underbrace{\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)}_{2j\sin(\omega_0 t)} =$$

$$= j2K_1 e^{-\frac{R}{2L}t} \sin(\omega_0 t)$$

Troviamo K_1 ...

$$\left.\frac{di}{dt}\right|_{t=0^+} = \frac{E - V_0}{L}$$

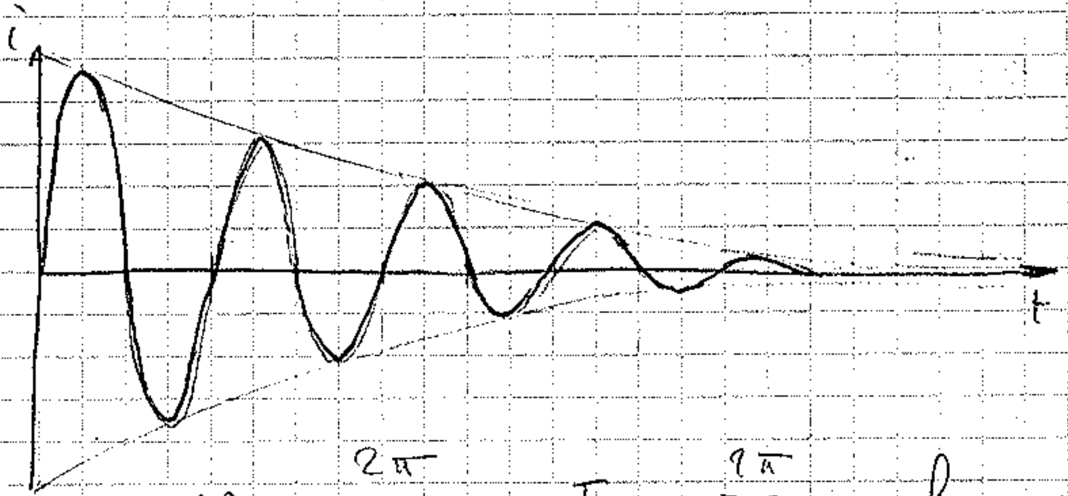
$$-j2K_1\frac{R}{2L}e^{-\frac{R}{2L}t}\sin(\omega_0 t) + j2K_1\omega_0 e^{-\frac{R}{2L}t}\cos(\omega_0 t)$$

$$\overset{t=0^+}{=} \cancel{-j2K_1\frac{R}{2L}}\cdot 0 + j2K_1\omega_0$$

$$jK_1 = \frac{E - V_0}{j2\cancel{K_1}\omega_0 L}$$

$$i(t) = \cancel{j2K_1}e^{-\frac{R}{2L}t}\cdot\frac{E - V_0}{\cancel{j2}\omega_0 L}\sin(\omega_0 t) =$$

$$= \frac{E - V_0}{\omega_0 L}e^{-\frac{R}{2L}t}\sin(\omega_0 t) \longleftarrow$$ funzione sinusoidale



$$\omega_0 = \frac{2\pi}{T_0} \qquad T_0 = \frac{2\pi}{\omega_0} \qquad f_0 = \frac{\omega_0}{2\pi}$$

$$\tau = \frac{2L}{R}$$