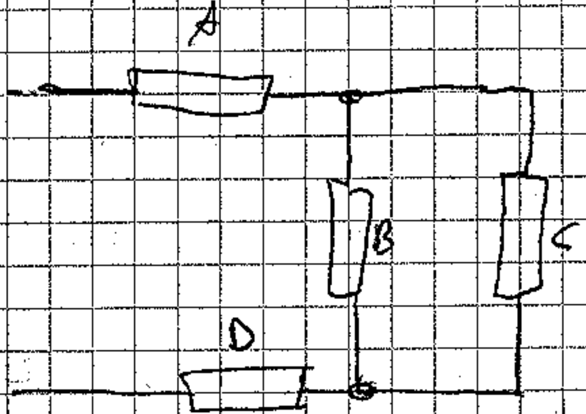
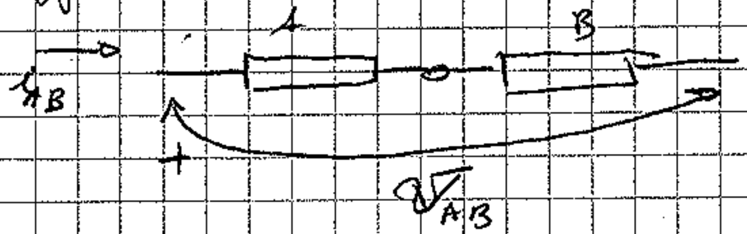
TOPOLOGIA DEI CIRCUITI ELETTRICI

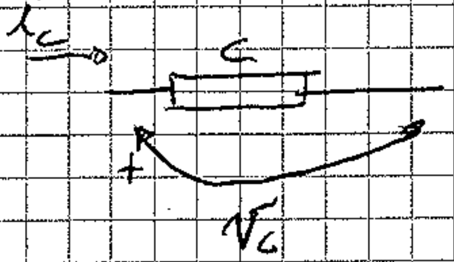
Connessione serie



Supponiamo



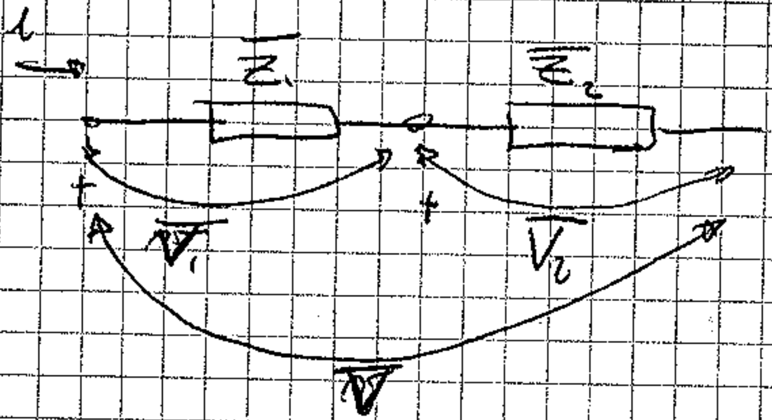
$\Downarrow$

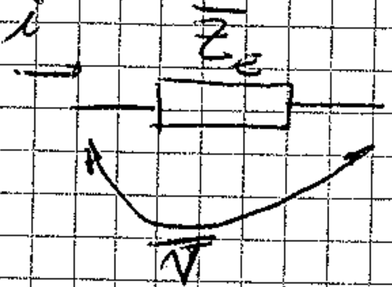


C è equivalente alla serie A+B se

$$v_C = v_{AB} \quad e \quad i_C = i_{AB}$$

BIPOLI PASSIVI e regime permanente $\left[\begin{smallmatrix}\text{staz} \\ \text{sinus.}\end{smallmatrix}\right.$





$$\bar V = \bar V_1 + \bar V_2 = \bar Z_1 \bar I + \bar Z_2 \bar I =$$

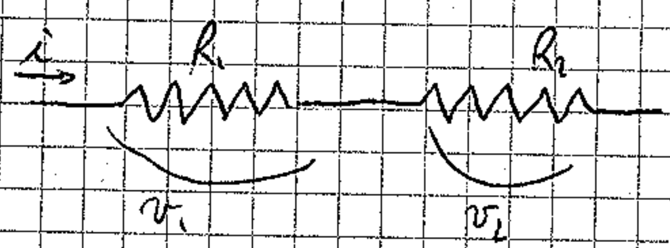
$$\bar I(\bar Z_1 + \bar Z_2)$$

$$\bar V = \bar Z_e \bar I$$

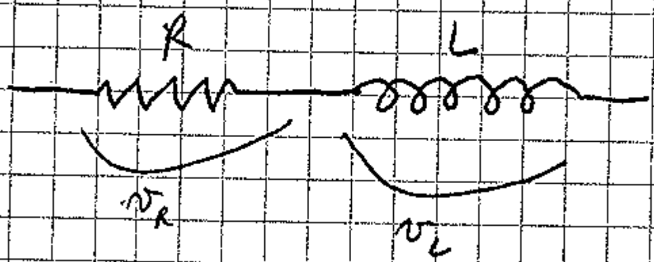
$$\bar Z_e = \bar Z_1 + \bar Z_2$$

$$\bar Z_e = \bar Z_1 + \bar Z_2 + \dots + \bar Z_n$$

Regime transitorio

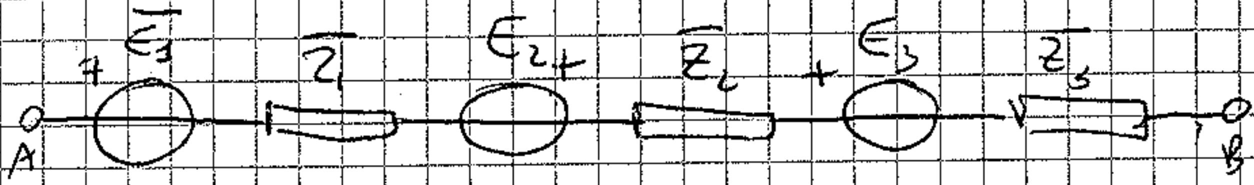


$$v = v_1 + v_2 = (R_1 + R_2)i$$

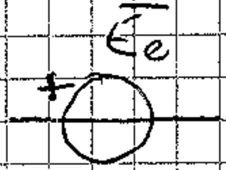


non posso trovare equivalente

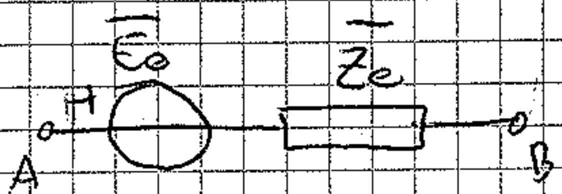
$$v = v_R + v_L = Ri + L\frac{di}{dt}$$

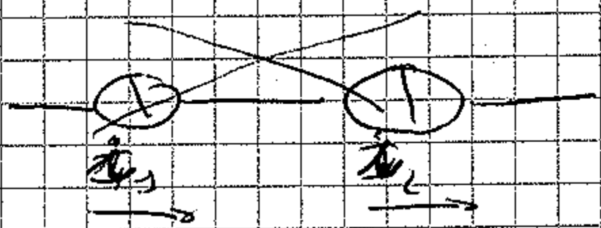


$$\overline{Z}_E = \overline{Z}_1 + \overline{Z}_2 + \overline{Z}_3$$



$$\overline{E}_e = +E_1 - E_2 + E_3$$



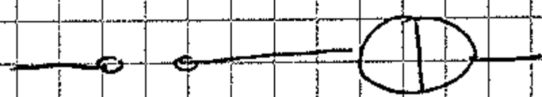


circuito patologico

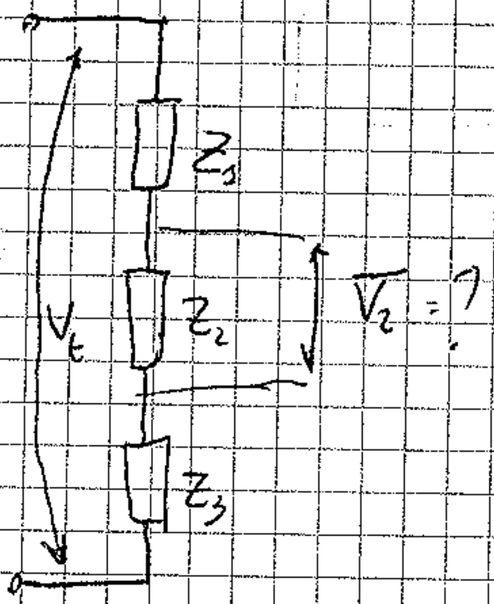
Casi particolari:

non cambia nulla

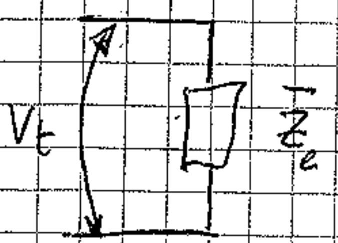
risultante è circuito aperto



situazione patologica



$$\overline{Z}_e = \overline{Z}_1 + \overline{Z}_2 + \overline{Z}_3$$

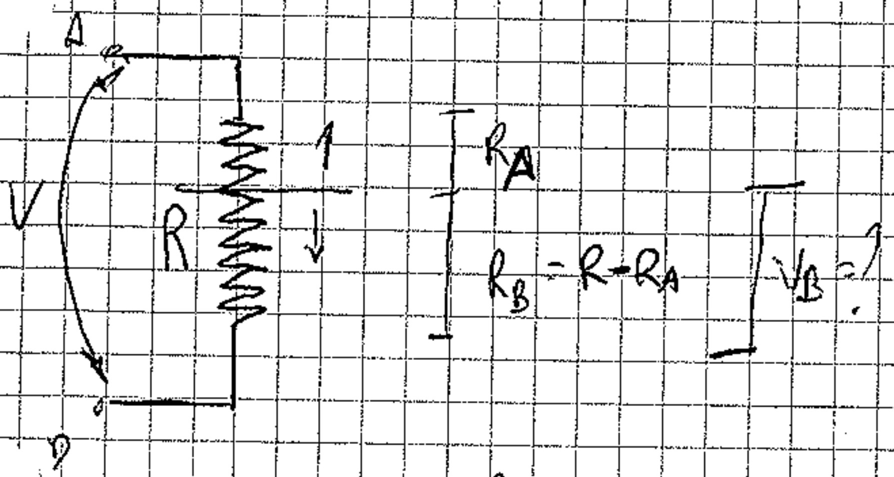


$$\overline{I} = \frac{\overline{V_t}}{\overline{Z}_e}$$

$$\overline{V_2} = \overline{Z}_2 \overline{I} = \overline{Z}_2 \frac{\overline{V_t}}{\overline{Z}_e} \Longrightarrow$$

$$\Longrightarrow \boxed{\overline{V_2} = \overline{V_t} \frac{\overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2 + \overline{Z}_3}}$$

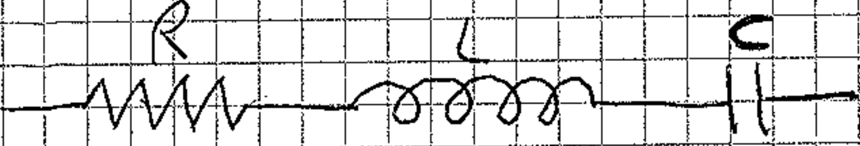
PARTITORE DI TENSIONE (reg. sinusoidali)



$$V_B = V \frac{R - R_A}{R}$$

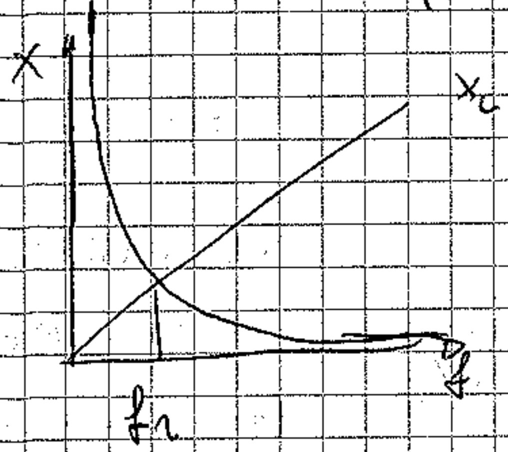
$$V_2 = V_t \frac{R_2}{R_1 + R_2 + R_3}$$ ← PARTITORE di Tensione

$\boxed{\text{regime stazionario}}$



$R \qquad X_L = 2\pi f L \qquad X_C = \frac{1}{2\pi f C}$

$$\bar{Z}_e = R + j(X_L - X_C) = R + j\left(2\pi f L - \frac{1}{2\pi f C}\right)$$



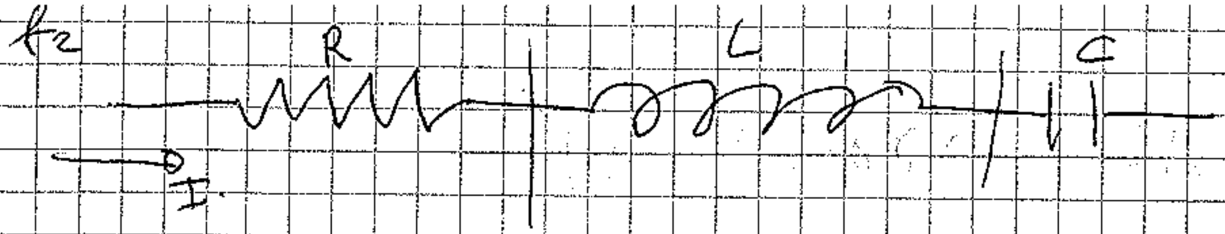
$$Z_e(f_r) = R$$

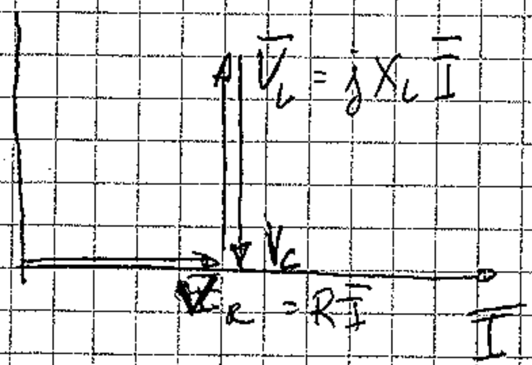
f_r è frequenza di risonanza

$X_L = X_C$ risonanza

$$2\pi f L = \frac{1}{2\pi f C} \Rightarrow 4\pi^2 f^2 LC = 1 \Rightarrow$$

$$\Rightarrow f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$





$V_C = -jX_C I$

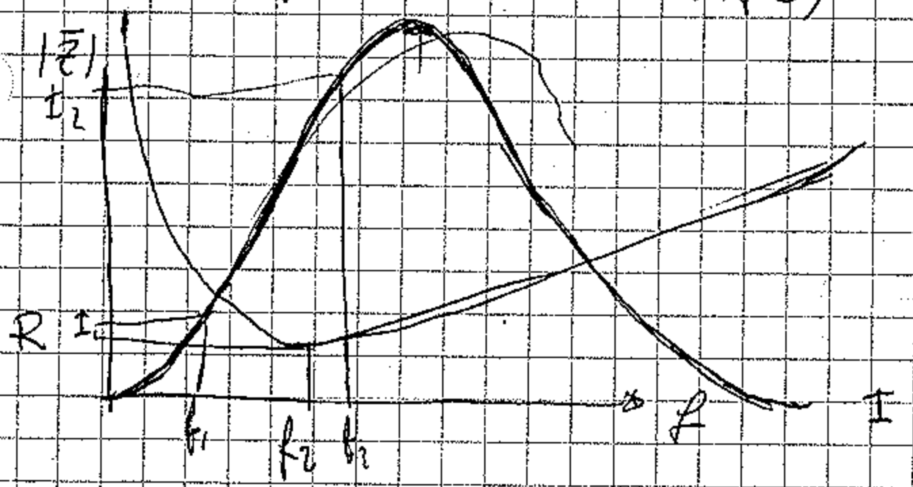
P reattiva totale = 0

P reattiva su singolo elemento c'è -

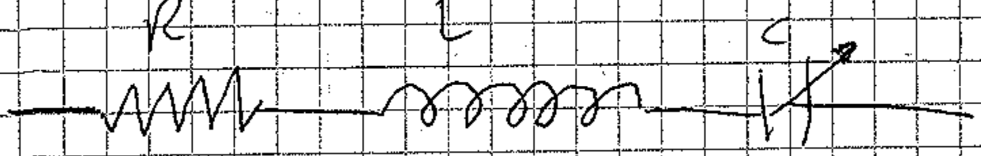
Mentre uno cede l'altro si carica.

La corrente in corrispondenza della risonanza può avere un picco.

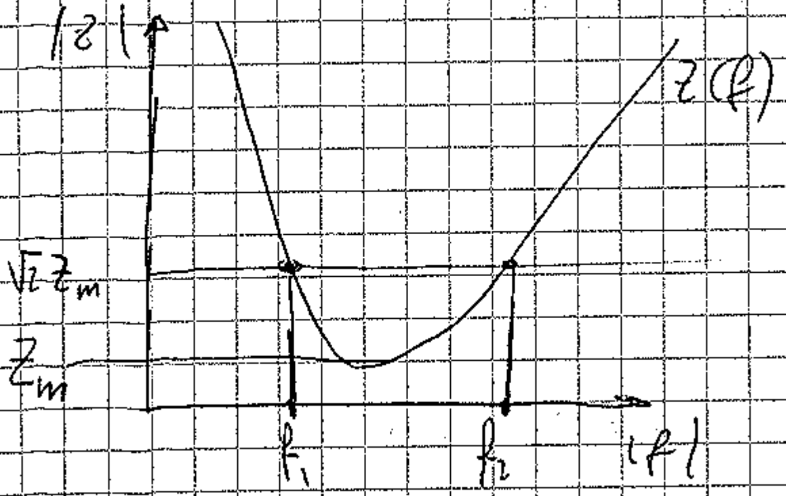
$$|\bar{Z}| = \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)}$$



Il circuito è selettivo rispetto alla frequenza

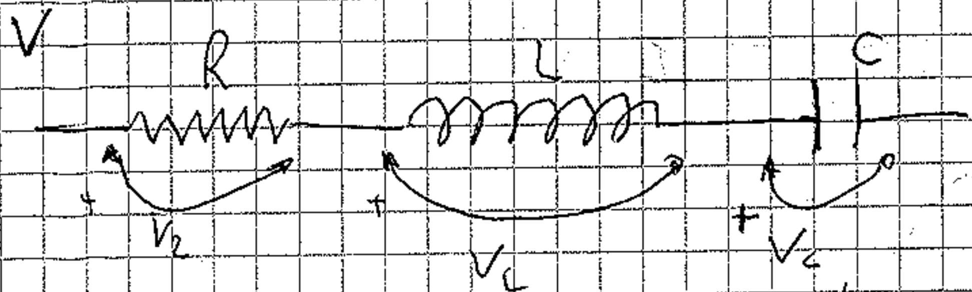


$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$



$f_2 - f_1$ larghezza di banda

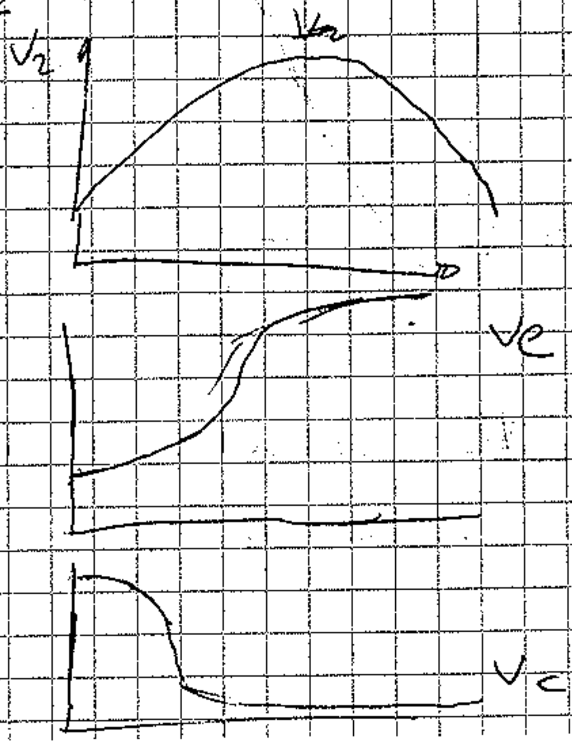
tanto più $(f_2 - f_1)$ è piccolo tanto più il circuito è selettivo



$$V_R = \frac{R}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}} V$$

$$V_L = V \frac{2\pi f L}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}$$

$$V_C = \frac{\frac{1}{2\pi f C}}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}$$



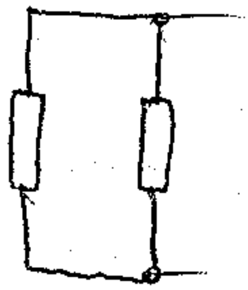
V_2 fa filtro passabanda (amplifica i segnali che stanno intorno ad un certo valore)

V_1 fa filtro passaalto amplifica le frequenze che stanno oltre un certo valore

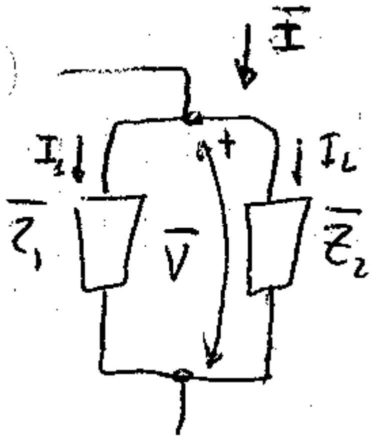
V_c fa filtro passa basso amplifica le frequenze che stanno prima di un ~~certo~~ valore.

PARALLELO (Derivazione)

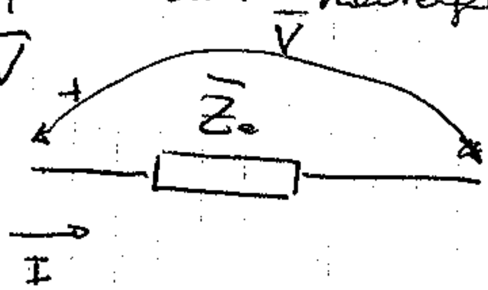
Due elementi sono detti in parallelo se la tensione tra i morsetti è la <u>medesima</u>



Certe volte è meno facilmente individuabile di quella in serie.



Obiettivo: trovare $\bar{Z}_e$ ~~equiv~~ che mantenga $\bar{I}$ e $\bar{V}$



Sappiamo che

$$\begin{cases} \bar{I} = \bar{I}_1 + \bar{I}_2 = \dfrac{\bar{V}}{\bar{Z}_1} + \dfrac{\bar{V}}{\bar{Z}_2} \\ \bar{I} = \dfrac{\bar{V}}{\bar{Z}_e} \end{cases}$$

$$\frac{1}{\bar{Z}_e} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2}$$

Caso generale

$$\frac{1}{\bar{Z}_e} = \frac{1}{\bar{Z}_1} + \dots + \frac{1}{\bar{Z}_n}$$

$$\bar{Y}_e = \bar{Y}_1 + \dots + \bar{Y}_n$$

Regime stazionario

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

Regime transitorio

si possono definire resistore equivalente, induttore equivalente, condensatore equivalente ma non mischiati.

Una resistenza eq. in parallelo è sempre < del minore delle resistenze in parallelo.

<u>Se elementi in parallelo è 2</u>

$$\frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} = \frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1 \cdot \bar{Z}_2} = \frac{1}{\bar{Z}_e}$$

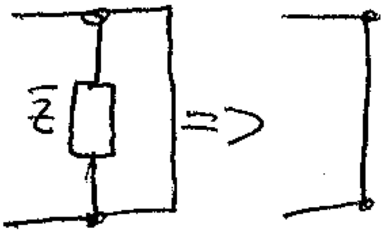
$$\bar{Z}_e = \frac{\bar{Z}_1 \cdot \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

Caso resistenze $\quad R_e = \frac{R_1 R_2}{R_1 + R_2}$

Se $\bar{Z}_1 = \bar{Z}_2 = \dots = \bar{Z}_n = \frac{1}{\bar{Z}_e} = \frac{n}{\bar{Z}_1} \Rightarrow$

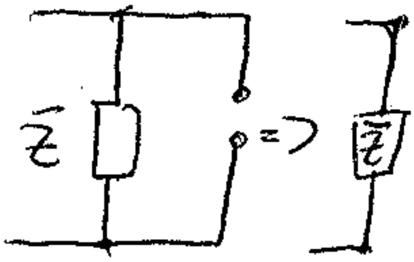
$$\bar{Z}_e = \frac{\bar{Z}_1}{n}$$

Casi particolari

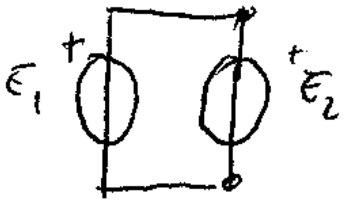


$\bar{Z}_{eq} = 0$

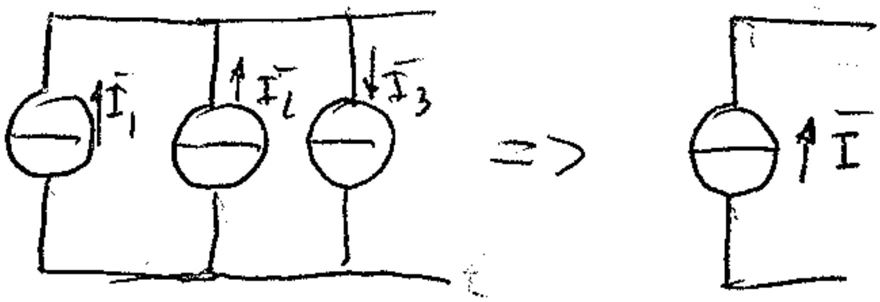
In questa situazione Z è by-passato (ha funzione puramente decorativa)



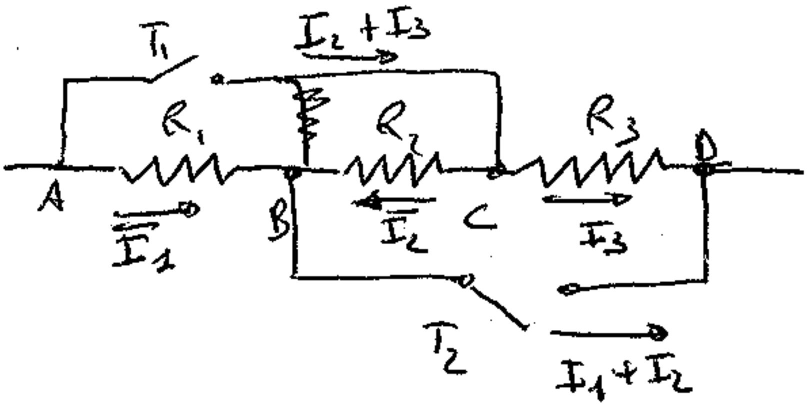
$\bar{Z}_{eq} = \bar{Z}$



Circuito patologico.
Unico caso accettabile è che $\bar{E}_1 = \bar{E}_2$



$\bar{I}_e = \bar{I}_1 + \bar{I}_2 - \bar{I}_3$



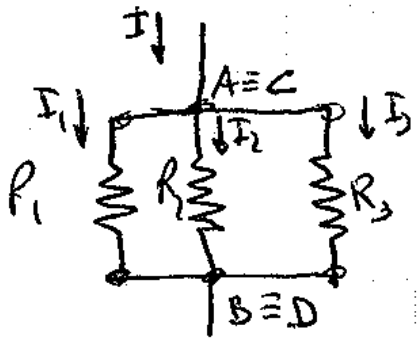
T_1 O , T_2 O

$$R_e = R_1 + R_2 + R_3$$

T_1 C , T_2 O $\qquad R_e = R_3$

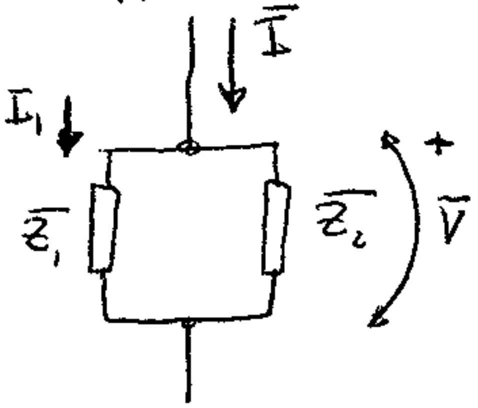
T_1 O , T_2 $\qquad R_e = R_1$

T_1 C , T_2 C



$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Supponiamo



$$\overline{Z}_e = \frac{\overline{Z}_1 \overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2}$$

$$\overline{V} = \frac{\overline{Z}_1 \overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2} \overline{I} = \overline{Z}_e \overline{I}$$

$$\overline{I}_1 = \frac{\overline{V}}{\overline{Z}_1} = \overline{I} \frac{\cancel{\overline{Z}_1} \overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2} \cdot \frac{1}{\cancel{\overline{Z}_1}} =$$

$$\boxed{\overline{I}_1 = \frac{\overline{Z}_2 \overline{I}}{\overline{Z}_1 + \overline{Z}_2}}$$ partitore di corrente

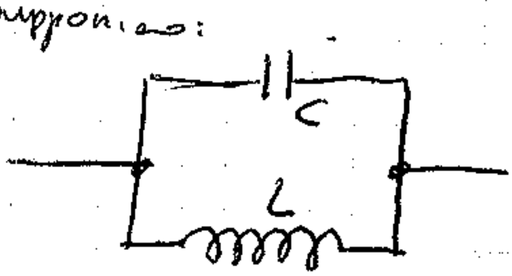
- Il partitore di corrente si può applicare se ci sono solo 2 elementi in parallelo

- Nel partitore di corrente compare l'impedenza dell'altro ramo

In regime stazionario il partitore di corrente ha la forma:

$$I_1 = I \frac{R_2}{R_1 + R_2}$$

Supponiamo:



$$X_L = 2\pi f L$$

$$X_C = \frac{1}{2\pi f C}$$

$$Z_e = \frac{jX_L(-jX_C)}{j(X_L - X_C)} = -j\left(\frac{X_L X_C}{X_L - X_C}\right) =$$

$$= -j \frac{2\pi f L \cdot \frac{1}{2\pi f C}}{2\pi f L - \frac{1}{2\pi f C}}$$

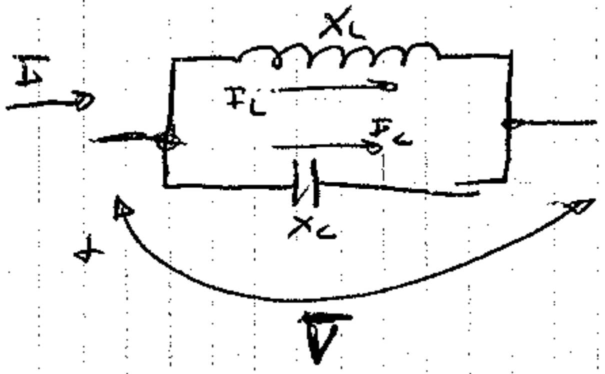
Se f è bassa $\Rightarrow X_C > X_L \Rightarrow$ comportamento induttivo ~~Z_e~~

f è alta $\Rightarrow X_L > X_C \Rightarrow$ comportamento capacitivo

Se $X_L = X_C \Rightarrow f_e = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ ~~quind~~

quando $X_L = X_C$ si hanno casi di "risonanza parallelo" o "antirisonanza" (risultante è circuito aperto).

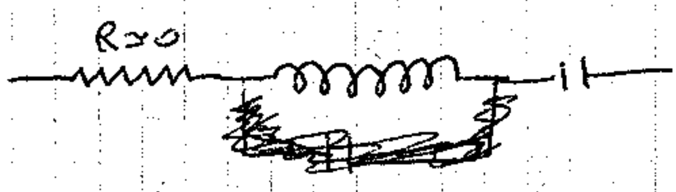
Nel caso in cui si verifichi l'antirisonanza:



$\bar{I} = 0$

$\bar{I}_L + \bar{I}_C = 0$

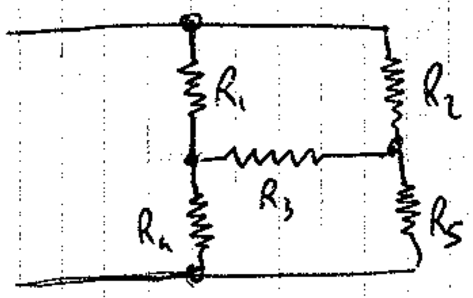
Supponiamo



$V = 2\pi f L I$

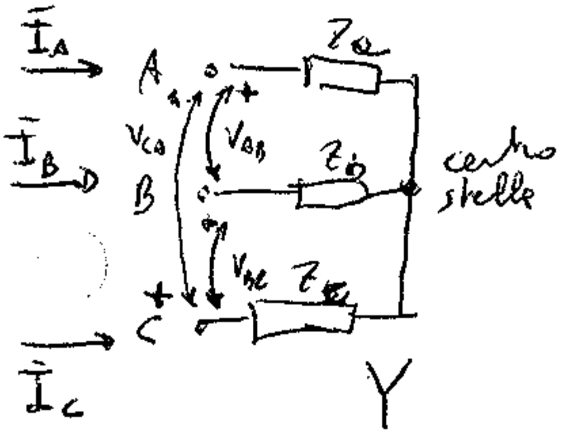
$Z_e = j(X_L - X_C)$ Serve per diminuire la tensione V

Con serie e parallelo si tratta ogni serie di circuito? NO

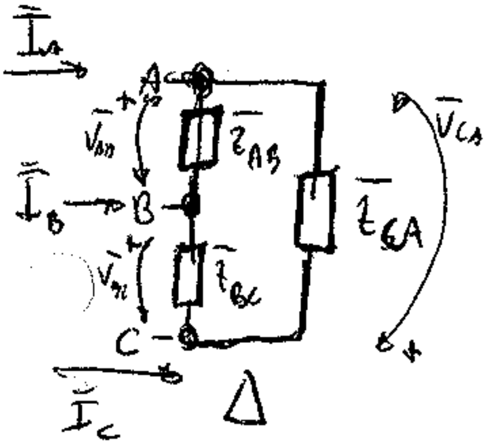


Posso sostituire un bipolo equivalente?

$Z_1 \quad Z_2 \quad Z_3$



Disposizione a stella (Y)

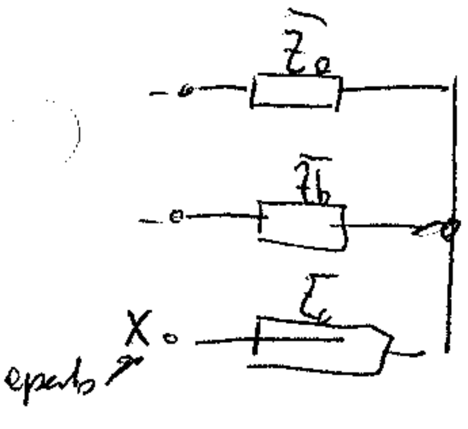


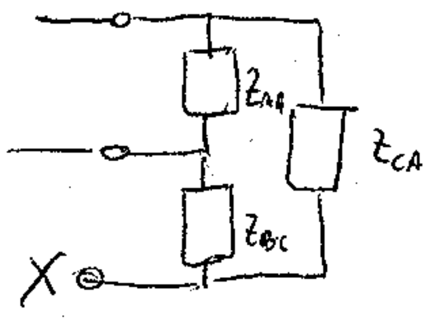
Disposizione a triangolo (Δ)

Due correnti sono indipendenti (x legge corrent.)

Due tensioni sono indipendenti (x legge tensioni)

Dobbiamo imporre 4 condizioni





$$\bar Z_a + \bar Z_b = \bar Z_{AB} \parallel (\bar Z_{CA} + \bar Z_{BC}) = \frac{\bar Z_{AB}(\bar Z_{CA} + \bar Z_{BC})}{\bar Z_{AB} + \bar Z_{BC} + \bar Z_{CA}}$$

Trasformazioni triangolo-stella:

$$\overline{Z}_a = \frac{\overline{Z}_{AB} \cdot \overline{Z}_{CA}}{\overline{Z}_{AB} + \overline{Z}_{BC} + \overline{Z}_{CA}}$$

$$\overline{Z}_b = \frac{\overline{Z}_{AB} \cdot \overline{Z}_{BC}}{\overline{Z}_{AB} + \overline{Z}_{BC} + \overline{Z}_{CA}}$$

$$\overline{Z}_c = \frac{\overline{Z}_{BC} \cdot \overline{Z}_{CA}}{\overline{Z}_{AB} + \overline{Z}_{BC} + \overline{Z}_{CA}}$$

Se:

$$\overline{Z}_{AB} = \overline{Z}_{BC} = \overline{Z}_{CA} = \overline{Z}_\Delta$$

$$\Downarrow$$

$$\overline{Z}_a = \overline{Z}_b = \overline{Z}_c = \overline{Z}_Y = \frac{\overline{Z}_\Delta}{3}$$

Trasformazioni stella-triangolo

$$\overline{Z}_{AB} = \overline{Z}_a + \overline{Z}_b + \frac{\overline{Z}_a \cdot \overline{Z}_b}{\overline{Z}_c}$$

$$\overline{Z}_{BC} = \overline{Z}_b + \overline{Z}_c + \frac{\overline{Z}_b \cdot \overline{Z}_c}{\overline{Z}_a}$$

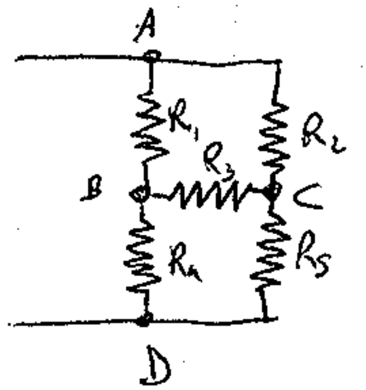
$$\overline{Z}_{CA} = \overline{Z}_c + \overline{Z}_a + \frac{\overline{Z}_a \cdot \overline{Z}_c}{\overline{Z}_b}$$

Se $\overline{Z}_a = \overline{Z}_b = \overline{Z}_c = \overline{Z}_Y$

$$\Downarrow$$

$$\overline{Z}_{AB} = \overline{Z}_{BC} = \overline{Z}_{CA} = \overline{Z}_\Delta = 3\,\overline{Z}_Y$$

Esercizio numerico

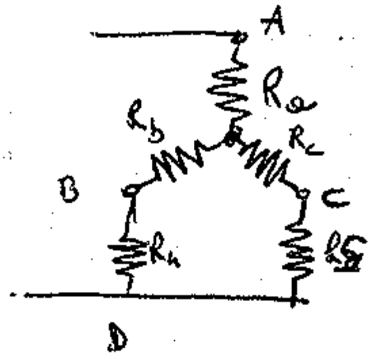


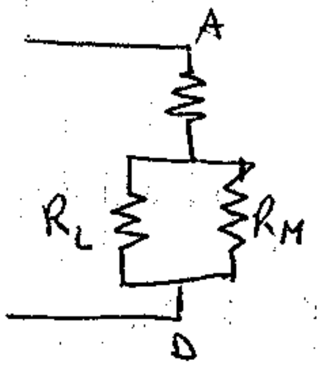
$R_1 = R_2 = R_3 = R_4 = R_5 = 3\,\Omega$

Re_{AD} ?

- Sostituisco ~~stella~~ triangolo con ~~triang~~ stella

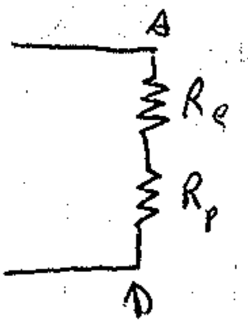
$R_a = R_b = R_c = 1\,\Omega$





$R_L = R_b + R_4 = 4\,\Omega$

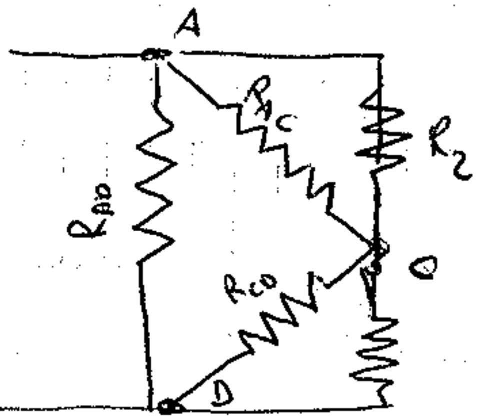
$R_M = R_c + R_5 = 4\,\Omega$



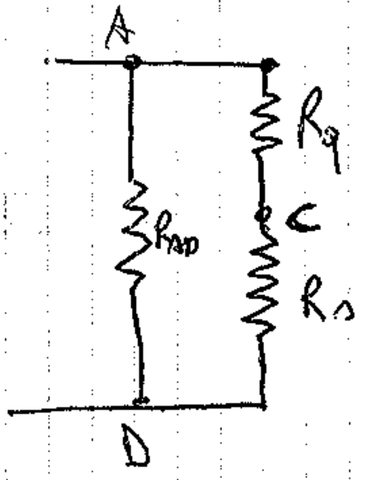
$R_p = \frac{R_L}{2} = 2\,\Omega$

$R_{AD} = 3\,\Omega$

- sostituisco stella con triangolo

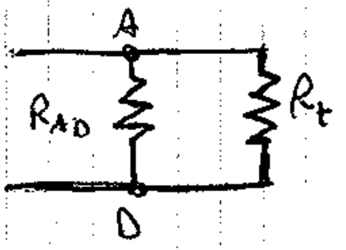


$R_{AD} = R_{AC} = R_{CD} = 9\,\Omega$



$$R_q = \frac{R_{AC}\cdot R_2}{R_{AC}+R_2} = \frac{9}{4}\,\Omega = 2,25\,\Omega$$

$$R_D = 2,25\,\Omega$$

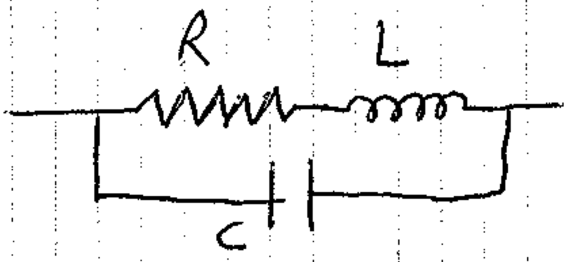


$$R_t = R_p + R_D = 4,5\,\Omega$$

$$R_{eAD} = \frac{4,5\cdot 9}{13,5}\,\Omega = \frac{40,5}{13,5}\,\Omega = 3\,\Omega$$

ELEMENTI REALI

Resistore reale



capacità parassita

Effetto prevalente è del resistore

$$R \gg X_L$$

$$X_C = \frac{1}{2\pi f C} \gg R$$

anche questa rappresentazione non è esaustiva dal punto di vista modellistico (non tiene conto che R cambia in base al riscaldamento, effetto pelle: la corrente tende a addensarsi sui contorni del conduttore questo effetto è maggiore, maggiore è la frequenza, ne segue che non viene usata tutta la sezione