$$R_{TH} = \frac{V_{CD}^{A}}{I} = 15\,\Omega$$

b) $$I_0 = \frac{200\,W}{50\,V} = 4\,A$$

$$V_{FG} = V_{CD} + R_0 I_0 = 50\,V + 60\,V = 110\,V$$

$$V_{FG} = E_A - R_A I_A \Rightarrow I_A = 0{,}5\,A$$

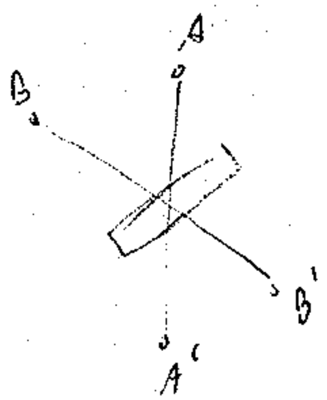
$$I_B + I_A = I_0 \Rightarrow I_B = 3{,}5\,A$$

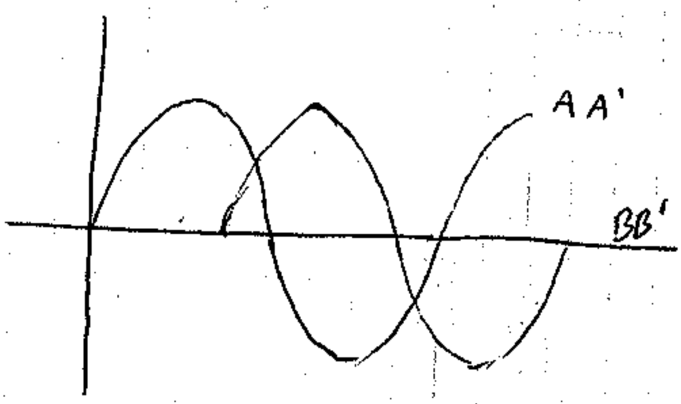
$$V_{FG} = E_B - R_B I_B \Rightarrow E_B = 320\,V$$

$$K_1 = \frac{E_B}{I_0} = 80\,\Omega$$

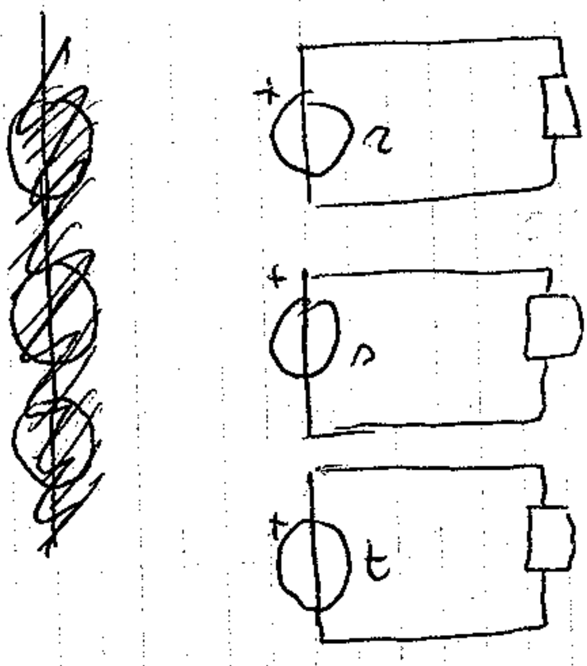
SISTEMI POLIFASE

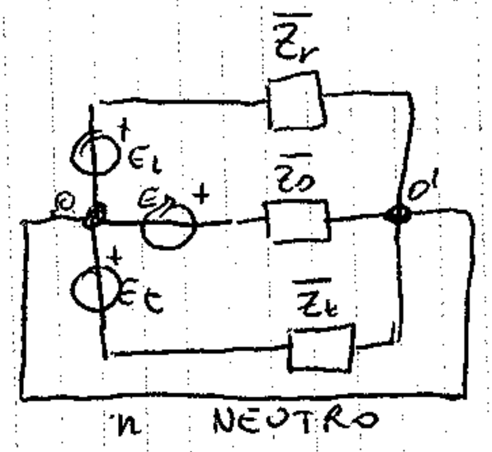
Serie di generatori in cui le tensioni sono sfasate di un certo angolo.





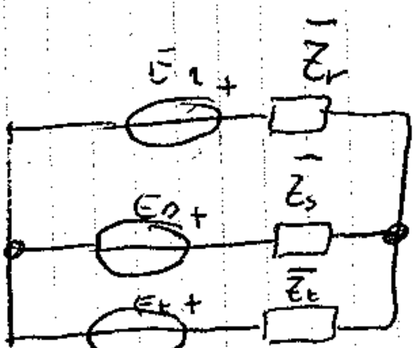
Sistema tri fase e sei fili





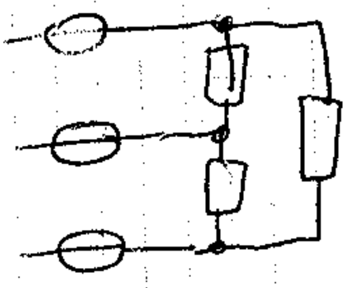
Sistema a 4 fili
(TRIFASE con NEUTRO)

$V_{00'}=0 \Rightarrow \bar I_r = \dfrac{\bar E_r}{\bar Z_r}$



Sistema tri fase senza neutro

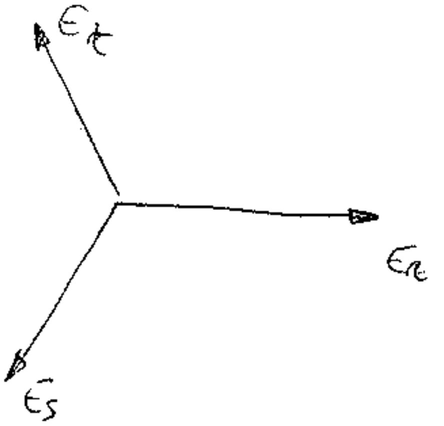
Risoluzione con Millman



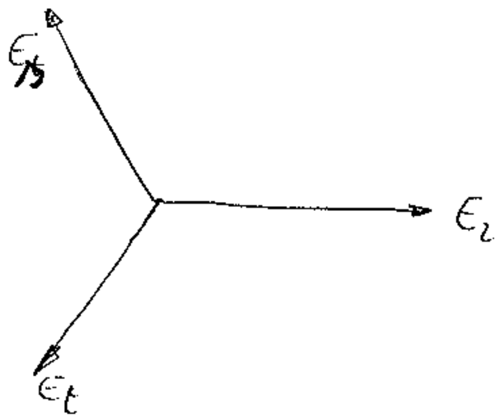
$\overline{E}_r, \quad \overline{E}_s, \quad \overline{E}_t$

Definiamo Terna Simmetrica una terna in cui $E_r = E_s = E_t$

$$E_r \hat{\ } E_s = E_s \hat{\ } E_t = E_t \hat{\ } E_r = \pm \frac{2}{3}\pi = \pm 120°$$



Terna diretta o destrogira



Terna inversa o levogira

$$\overline{Z}_r = \overline{Z}_s = \overline{Z}_t = \overline{Z}_u$$

Caso 1) Sistema 3 fili

Caso 2) 4 fili (neutro con imp. non trascurabile)

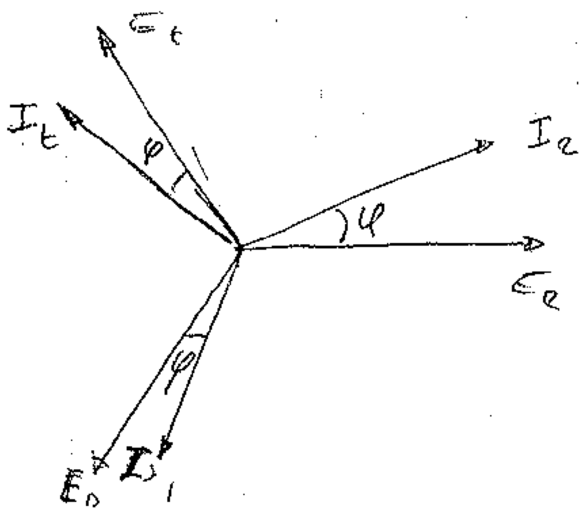
Caso 1)
$$V_{OO'} = \frac{\frac{\overline{E}_s}{\overline{Z}_s} + \frac{\overline{E}_r}{\overline{Z}_r} + \frac{\overline{E}_t}{\overline{Z}_t}}{\frac{1}{Z_r} + \frac{1}{Z_s} + \frac{1}{Z_t}} = \frac{\frac{\overline{E}_r + \overline{E}_s + \overline{E}_t}{\overline{Z}_u}}{\frac{3}{Z_u}} = 0$$

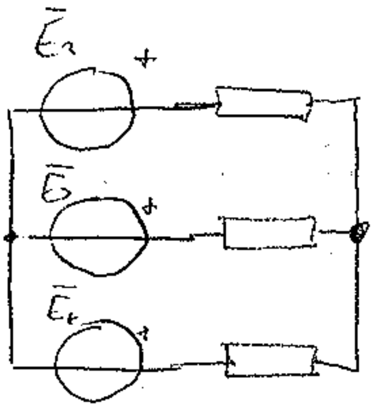
Stesso caso di quando si ha un conduttore di neutro a resistenza nulla

$$\overline{I}_r = \frac{\overline{E}_r}{\overline{Z}_r} = \frac{E_r}{\overline{Z}_u} \quad \overline{I}_s = \frac{\overline{E}_s}{\overline{Z}_s} = \frac{E_s}{\overline{Z}_u} \quad \overline{I}_t = \frac{\overline{E}_t}{\overline{Z}_t} = \frac{\overline{E}_t}{Z_u}$$

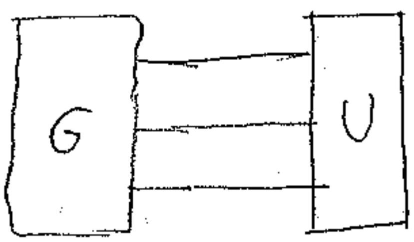
Sistema equilibrato

Caso 2) $V_{00'} = 0$ Nel conduttore di neutro non passa corrente. Il Neutro è inutile considerarlo





Per misure il centro-stella è raggiungibile



Caso reale il centrostella non è raggiungibile e non è detto che esiste.

Come faccio a misurare le grandezze?

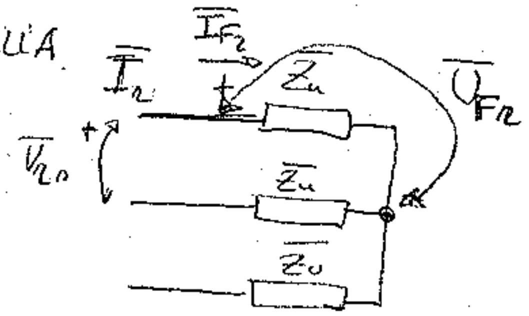
Per corrente non ho problemi

Per tensione posso misurare solo quella tra due fasi (tensione fase - fase o concatenata)

GRANDEZZE DI LINEA

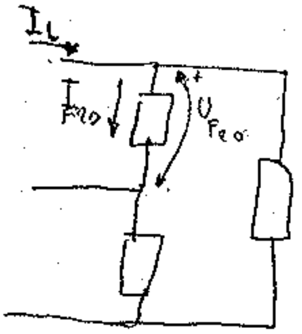
Sistemi simmetrici equilibrati

STELLA



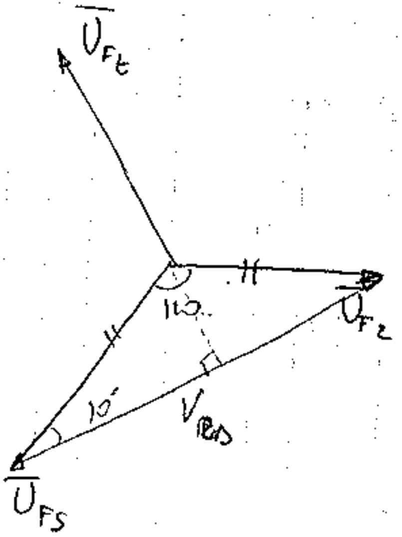
Conosco $\bar I_r$ e $\bar V_{rs}$
voglio trovare $\bar U_{Fr}$ e $\bar I_{Fr}$
(GRANDEZZE di LINEA)

TRIANGOLO



Caso stella (Y)

$\bar I_r = \bar I_{Fr}$ $\bar I_s = \bar I_{Fs}$ $\bar I_t = \bar I_{Ft}$



$$\bar V_{rs} = \bar U_{Fr} - \bar U_{Fs}$$

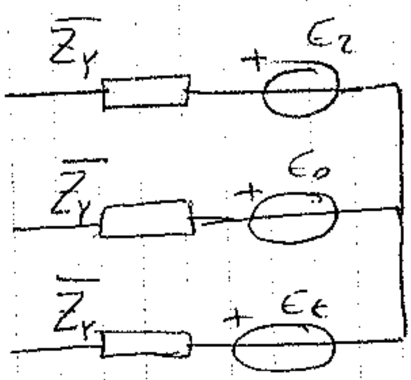
$\bar V_{rs}$ è in anticipo di 30° rispetto U_{Fr}

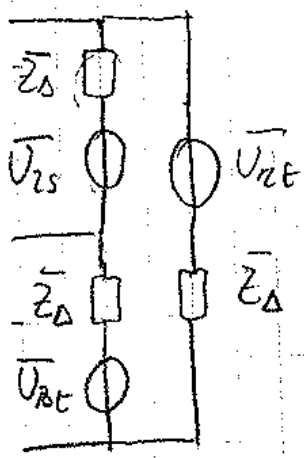
$$V_{rs} = \sqrt{3}\, U_{Fr}$$

Caso triangolo (Δ)

$$\overline{V}_{rs} = \overline{U}_{Frs} \qquad \overline{V}_{rt} = \overline{U}_{Frt} \qquad \overline{V}_{st} = \overline{U}_{Fst}$$

$$I_L = \sqrt{3}\, I_F$$





$Y \rightarrow \Delta$

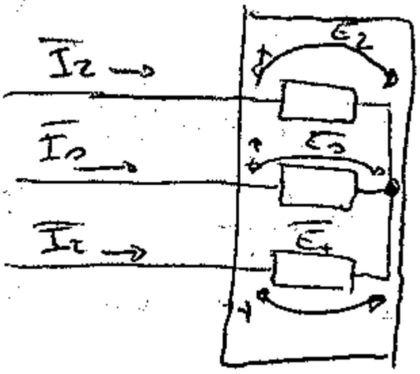
$$\overline{U}_{rs} = \overline{E}_r - \overline{E}_s$$
$$\overline{U}_{st} = \overline{E}_s - \overline{E}_t$$
$$\overline{U}_{rt} = \overline{E}_t - \overline{E}_r$$
$$\overline{Z}_\Delta = 3\,\overline{Z}_Y$$

$\Delta \rightarrow Y$

$$\overline{E}_r = \frac{1}{3}\left(\overline{U}_{rs} - \overline{U}_{tr}\right)$$
$$\overline{E}_s = \frac{1}{3}\left(\overline{U}_{st} - \overline{U}_{rs}\right)$$
$$\overline{E}_t = \frac{1}{3}\left(\overline{U}_{tr} - \overline{U}_{st}\right)$$
$$Z_Y = \frac{1}{3} Z_\Delta$$

Configurazione a triangolo non usata perché c'è un circuito chiuso e si può avere circolazione di corrente.

Supponiamo di avere un sistema simmetrico ed equilibrato



$$\bar{S}_T^* = \bar{E}_r \bar{I}_r^* + \bar{E}_s \bar{I}_s^* + \bar{E}_t \bar{I}_t^* =$$

$$= E_r I_r \cos\varphi_r + j E_r I_r \sin\varphi_r + E_s I_s \cos\varphi_s + j E_s I_s \sin\varphi_s + E_t I_t \cos\varphi_t + j E_t I_t \sin\varphi_t =$$

$$\begin{cases} E_r = E_s = E_t = E \\ I_r = I_s = I_t = I \\ \varphi_r = \varphi_s = \varphi_t = \varphi \end{cases}$$

$$\bar{S}_T^* = 3EI\cos\varphi + j3EI\sin\varphi$$

$$\begin{cases} P_T = 3EI\cos\varphi \\ Q_T = 3EI\sin\varphi \\ S = 3EI \end{cases}$$

$$V = \sqrt{3}E$$

$$\begin{cases} P_T = \sqrt{3}VI\cos\varphi \\ Q_T = \sqrt{3}VI\sin\varphi \\ S = \sqrt{3}VI \end{cases}$$

N.B.

$\boxed{\varphi}$ è angolo di sfasamento tra tensioni di fase a stella e correnti di linea

Vogliamo calcolare la potenza istantanea in un sistema trifase simmetrico ed equilibrato

$$p_T = p_r + p_s + p_t$$

$e_r = \sqrt{2} E \sin \omega t$

$e_s = \sqrt{2} E \sin(\omega t - 120°)$

$e_t = \sqrt{2} E \sin(\omega t - 240°)$

$i_r = \sqrt{2} I \sin(\omega t - \varphi)$

$i_s = \sqrt{2} I \sin(\omega t - 120° - \varphi)$

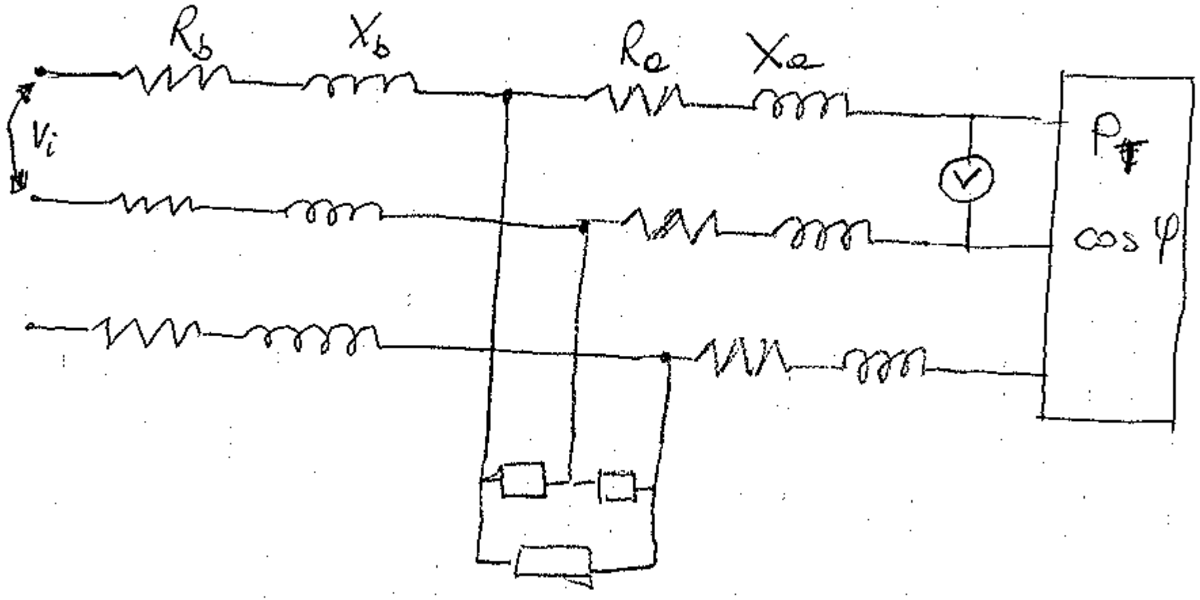
$i_t = \sqrt{2} I \sin(\omega t - 240° - \varphi)$

$$p_t = e_r i_r + e_s i_s + e_t i_t = 2EI \sin(\omega t) \sin(\omega t - \varphi) +$$
$$+ 2EI \sin(\omega t - 120°) \sin(\omega t - 120° - \varphi) +$$
$$+ 2EI \sin(\omega t - 240°) \sin(\omega t - 240° - \varphi) =$$
$$EI \left(\cos\varphi - \cos(2\omega t - \varphi) + \cos\varphi - \cos(2\omega t - \varphi - 240°) + \cos\varphi - \cos(2\omega t - \varphi - \tfrac{8}{3}\pi) \right) =$$
$$= 3EI \cos\varphi - EI \cdot 0 = 3EI \cos\varphi = P_T$$

la potenza istantanea è costante istante per istante.

Metodo di riduzione al monofase equivalente. (56)

Bisogna avere un collegamento a stella



$\overline{Z}_1 = R_1 + j X_1$

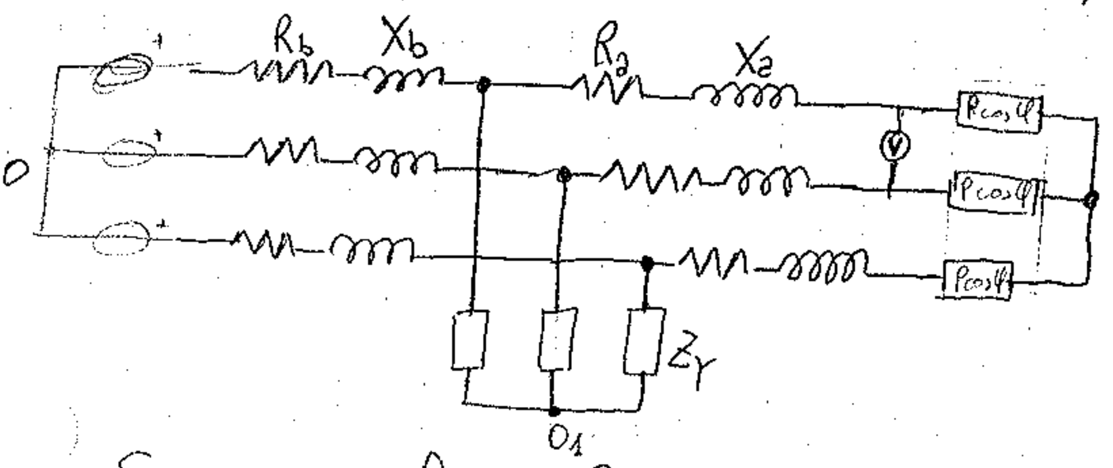
Conosco $R_a, X_a, R_b, X_b, \overline{Z}_1, P_T, \cos\varphi, V$

Vogliamo trovare V_i

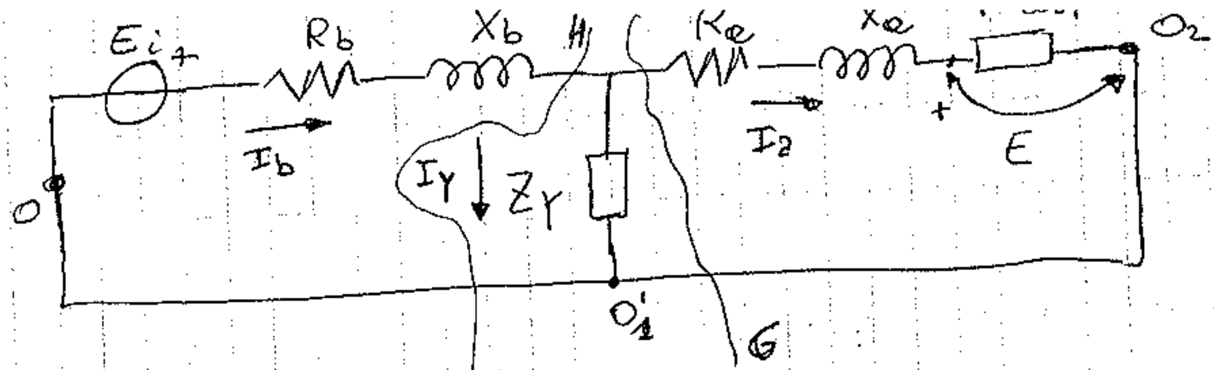
Trasformiamo il triangolo $Z_\lambda = \frac{Z_1}{3}$

$E = \frac{V\sqrt{3}}{3}$ $\qquad P = \frac{P_T}{3}$

Dati da riportare al monofase equivalente



Seguiamo fase R



Usiamo Boucherot

$P = E\, I_a \cos\varphi \Rightarrow I_a = \dfrac{P}{E\cos\varphi}$

$P_a = R_a I_a^2 \qquad Q_a = X_a I_a^2 \qquad Q = P \operatorname{tg}\varphi = E I \operatorname{sen}\varphi$

$P_G = P_a + P \qquad Q_G = Q_a + Q \qquad S_G = \sqrt{Q_G^2 + P_G^2}$

$E_G = \dfrac{S_G}{I_a} \qquad I_Y = \dfrac{E_G}{|Z_Y|}$

$P_Y = \operatorname{Re}(Z_Y)\, I_Y^2 \qquad Q_Y = \operatorname{Im}(Z_Y)\, I_Y^2$

$P_H = P_G + P_Y \qquad Q_H = Q_G + Q_Y \qquad S_H = \sqrt{P_H^2 + Q_H^2}$

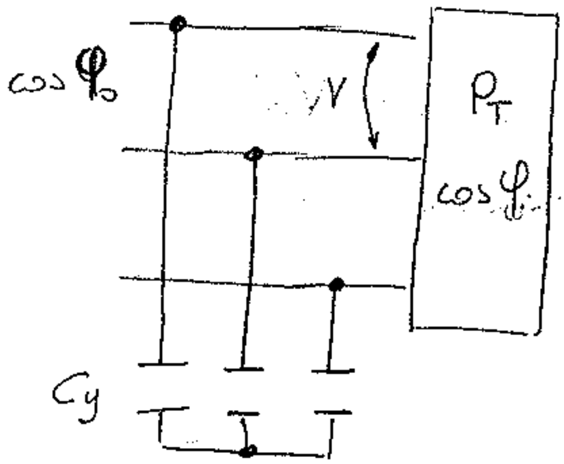
$I_b = \dfrac{S_H}{E_G} \qquad P_b = R_b I_b^2 \qquad Q_b = X_b I_b^2$

$P_i = P_H + P_b \qquad Q_i = Q_H + Q_b$

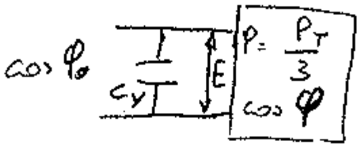
$S_i = \sqrt{P_i^2 + Q_i^2} \qquad E_i = \dfrac{S_i}{I_b}$

$V_i = \sqrt{3}\, E_i$

Rifasamento sistema trifase simmetrico ed equilibrato



Metodo rid. a monofase

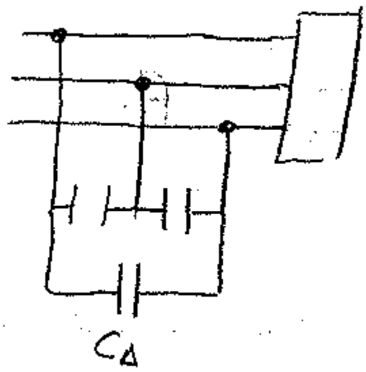


$$C_Y = \frac{P(tg\,\varphi - tg\,\varphi_0)}{2\pi f E^2} = $$

$$= \frac{P_T/\cancel{3}\,(tg\,\varphi - tg\,\varphi_0)}{2\pi f\,\frac{V^2}{\cancel{3}}} = $$

$$= \frac{P_T\,(tg\,\varphi - tg\,\varphi_0)}{2\pi f\,V^2}$$

Potrei collegare i tre condensatori a triangolo



$$C_\Delta = \frac{1}{3} C_Y$$

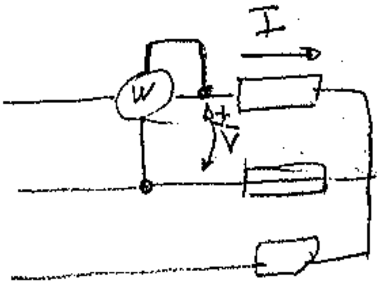
$$\overline{Z}_\Delta = 3\,\overline{Z}_Y \qquad \overline{Z}_\Delta = \frac{1}{2\pi f C_\Delta}$$

$$\frac{1}{\cancel{2\pi f} C_\Delta} = \frac{3}{\cancel{2\pi f} C_Y} \Rightarrow C_\Delta = \frac{C_Y}{3}$$

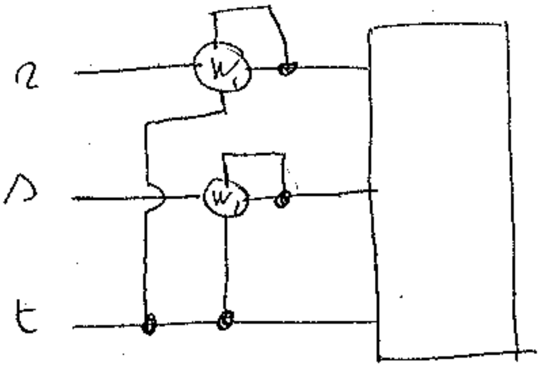
In bassa tensione conviene rifasamento a Δ

In alta tensione conviene rifasamento a Y

Misure di potenze nei sistemi trifase



$$W = VI\cos\widehat{VI}$$



$$W_1 = \overline{V}_{rt}\cdot\overline{I}_r$$

$$W_2 = \overline{V}_{st}\cdot\overline{I}_s$$

$$W_1 + W_2 = \overline{V}_{rt}\cdot\overline{I}_r + \overline{V}_{st}\cdot\overline{I}_s =$$

$$= (\overline{E}_r - \overline{E}_t)\cdot I_r + (E_s - E_t)\cdot I_s$$

$$\overline{E}_r\cdot\overline{I}_r + \overline{E}_s\cdot\overline{I}_s - E_t(I_r + I_s)$$

$$\overline{I}_r + \overline{I}_s + \overline{I}_t = 0 \qquad I_t = -\overline{I}_r - \overline{I}_s$$

$$W_1 + W_2 = \overline{E}_r\cdot\overline{I}_r + \overline{E}_s\overline{I}_s + \overline{E}_t\cdot\overline{I}_t = P_T$$

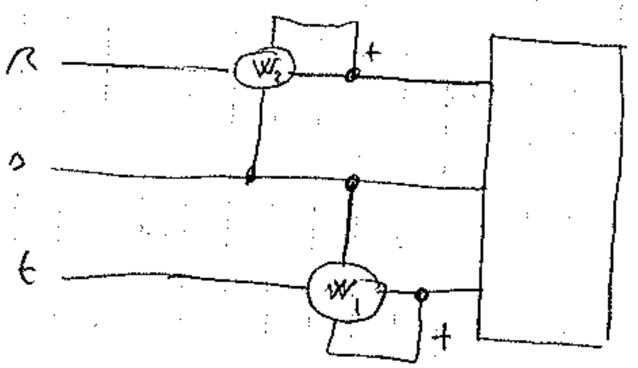
Inserzione ARON

① Unica ipotesi è che il sistema sia a 3 fili

② Non sono obbligato a disporre i wattmetri in questo modo

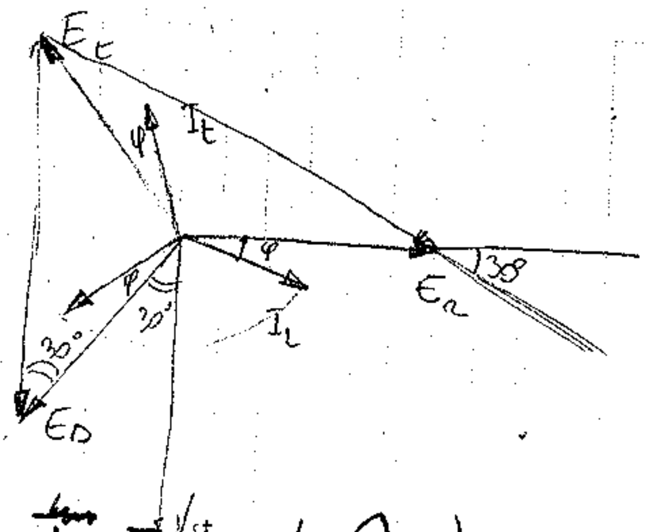
W_1		W_2	
A	V	A	V
r	r-t	s	s-t
s	s-r	t	t-r
t	t-s	r	r-s

Inserzioni ARON



Anche questa è una inserzione Aron.

$W_1 - W_2$ se sistema simmetrico ed equilibrato



$$W_1 - W_2 = V_{rt} \cdot I_r \cos(\widehat{V_{rt} I_r}) - V_{st} \cdot I_s \cos(\widehat{V_{st} I_s})$$

$$= V_{rt} I_r \cos(\varphi - 30°) - VI \cos(\varphi + 30°)$$

$$= VI\left(\cos(\varphi - 30°) - \cos(\varphi + 30°)\right) =$$

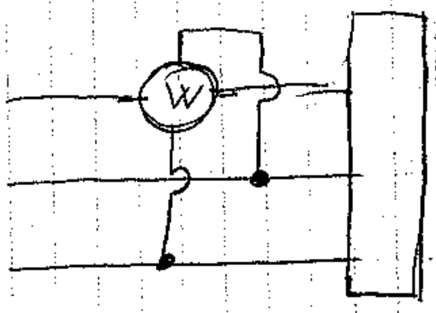
$$= -2VI \sin\varphi \left(-\frac{1}{2}\right) = VI \sin\varphi = \frac{Q_T}{\sqrt{3}}$$

Sistema a 3 fili

$W_1 + W_2 = P_T$ sempre

$W_1 - W_2 = Q_T / \sqrt{3}$ sistema simmetrico ed equilibrato con terna diretta

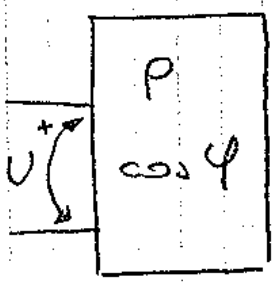
Quadratura

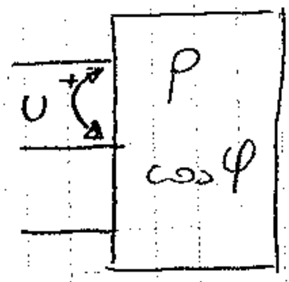


$W = VI \sin \varphi = \frac{Q}{\sqrt{3}}$

x sistemi simmetrici ed equilibrati

— . — . — . — . —





$\overleftrightarrow{\quad e \quad}$ ρ P_J

$\overleftrightarrow{\quad e \quad}$ ρ P_J

$I_m = \frac{P}{U \cos \varphi}$

$I_t = \frac{P}{\sqrt{3} U \cos \varphi}$

R_m

R_t

$P_J = 2 R_m I_m^2 =$
$= 2 R_m \frac{P^2}{U^2 \cos^2 \varphi}$

$P_J = 3 R_t I_t^2 = \cancel{3} R_t \frac{P^2}{\cancel{3} U^2 \cos^2 \varphi}$

$$2 R_m \frac{\cancel{P^2}}{\cancel{U^2 \cos^2\varphi}} = R_T \frac{\cancel{P^2}}{\cancel{U^2 \cos^2\varphi}}$$

$$2 R_m = R_T$$

$$2 \cancel{\rho} \frac{\cancel{\ell}}{S_m} = \cancel{\rho} \frac{\cancel{\ell}}{S_t} \qquad S_t = \frac{S_m}{2}$$

$$V_m = \ell S_m \cdot 2$$

$$V_t = \ell S_t \cdot 3 = \ell S_m \cdot \frac{3}{2}$$

$$\frac{V_t}{V_m} = \frac{3 \cancel{S_m \ell}}{2 \cdot 2 \cancel{S_m \ell}} = \frac{3}{4}$$

$$V_t = \frac{3}{4} V_m$$

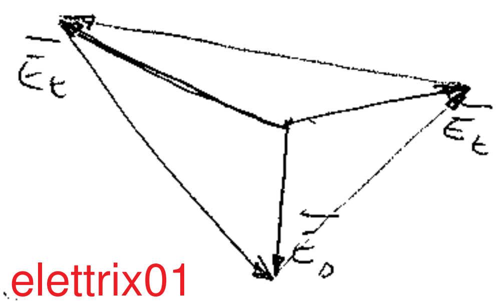
Sistemi dissimmetrici e squilibrati

Si trattano con teorema di Millman

$$\varphi_1 \neq \varphi_s \neq \varphi_t$$

$$\cos \Phi = \frac{P_T}{\sqrt{P_T^2 + Q_T^2}}$$

consideriamo



$$\bar{V}_{2s} = \bar{E}_1 - \bar{E}_s$$

$$\bar{V}_{st} = \bar{E}_s - \bar{E}_t$$

$$\bar{V}_{t1} = \bar{E}_t - \bar{E}_1$$

Se abbiamo solo tensioni di linea (ma non di fase a stella) ci serve una relazione aggiuntiva

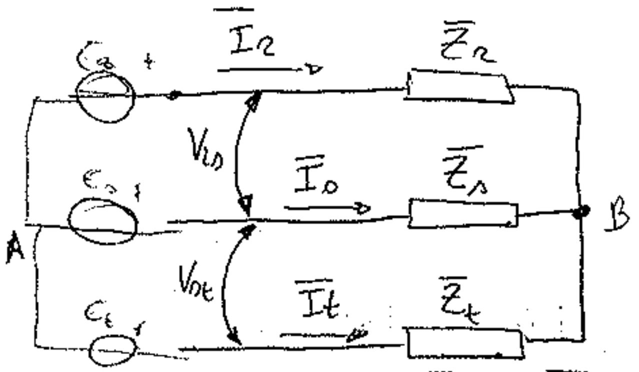
$$\overline{E}_r + \overline{E}_s + \overline{E}_t = \overline{E}_0$$ — valore arbitrario

Nei sistemi simmetrici $\overline{E}_0 = 0$

$$\overline{E}_r = \frac{1}{3}\left(2\overline{V}_{rs} + \overline{V}_{st} + \overline{E}_0\right)$$

$$\overline{E}_s = \frac{1}{3}\left(-\overline{V}_{rs} + \overline{V}_{st} + \overline{E}_0\right)$$

$$E_t = \frac{1}{3}\left(-V_{rs} - 2V_{st} + E_0\right)$$



I_r, I_s, I_t dipendono da E_0?

$$\overline{V}_{AB} = \frac{\frac{\overline{E}_r}{\overline{Z}_r} + \frac{\overline{E}_s}{\overline{Z}_s} + \frac{\overline{E}_t}{\overline{Z}_t}}{\frac{1}{\overline{Z}_r} + \frac{1}{\overline{Z}_s} + \frac{1}{\overline{Z}_t}} \qquad \overline{I}_r = \frac{\overline{E}_r - \overline{V}_{AB}}{Z_r}$$

A questo punto E_0 non influenza il comportamento del circuito.