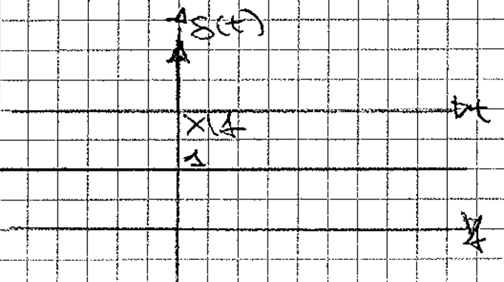
$x(t) = \delta(t)$

$$X(f) = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j2\pi f t}\, dt = e^{-j2\pi f 0} = 1 \qquad \boxed{F\{\delta(t)\} = 1}$$

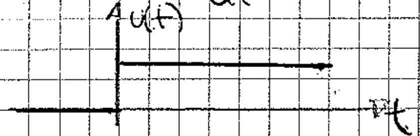


la cosa più simile è la radiazione di fondo dell'universo

- $x(t) = \delta(t - t_0) \longrightarrow X(f) = 1 \cdot e^{-j2\pi f t_0}$

$$\sum_{n=-\infty}^{+\infty} \delta(t - nt) \longrightarrow \sum_{n=-\infty}^{+\infty} e^{-j2\pi f \frac{n}{T}}$$

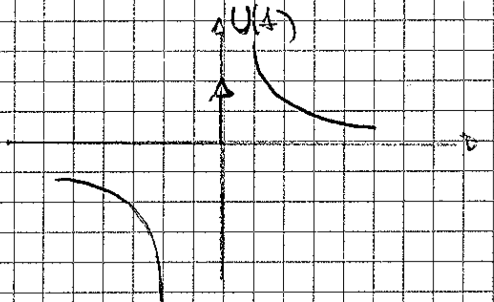
- $\delta(t) = \frac{du}{dt}$



- $\int_{-\infty}^{t} x(s)\, ds \qquad \frac{1}{2} X(0)\, \delta(f) + \frac{X(f)}{j2\pi f}$

$x(t) = \delta(t) \qquad X(f) = 1$

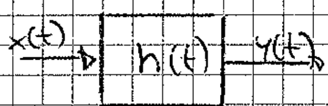
$u(t) \longrightarrow \frac{1}{2} \cdot 1\, \delta(f) + \frac{1}{j2\pi f} = U(f)$



08-03-2010

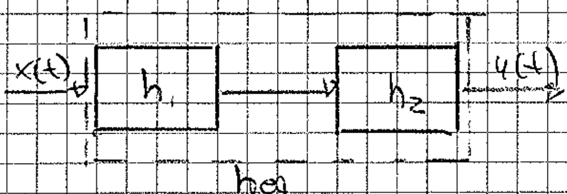
SISTEMI LINEARI

I



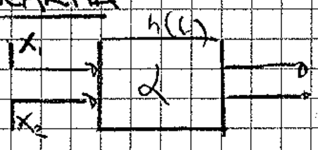
$y(t) = x(t) * h(t)$

$Y(f) = X(f) \cdot H(f)$



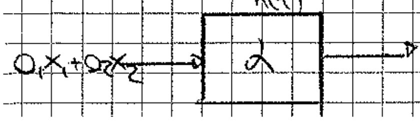
$h_{eq} = h_1 * h_2$

LINEARITÀ



$\alpha\{x_1\}a_1 +$

$\alpha\{x_2\}a_2$



$\alpha\{a_1 x_1 + a_2 x_2\}$

- $y(t) = 7x(t)$

$$a_1 7 x_1(t) + a_2 7 x_2(t) \quad \checkmark$$
$$7(a_1 x_1(t) + a_2 x_2(t)) \quad \checkmark$$

- $y(t) = |x(t)|$

$$a_1|x_1| + a_2|x_2|$$
$$|a_1 x_1 + a_2 x_2| \quad \text{NO!}$$

- $y(t) = \log(x(t))$

$$a_1 \log x_1(t) + a_2 \log x_2(t) \quad \text{NO!}$$
$$\log(a_1 x_1(t) + a_2 x_2(t))$$

TEMPO INVARIANZA

$$x(t) \rightarrow \boxed{h(t)} \xrightarrow{y(t)} \longrightarrow y(t-T)$$

$$x(t) \rightarrow x(t-T) \longrightarrow \boxed{h(t)} \longrightarrow \hat{y}(t)$$

- $x(t) \longrightarrow 7x(t) \longrightarrow 7x(t-T) \quad \checkmark$

$$x(t-T) \longrightarrow 7x(t-T) \quad \checkmark$$

- $y(t) = |x(t)|$

$$x(t) \longrightarrow \boxed{\ } \longrightarrow |x(t)| \xrightarrow{T} |x(t-T)| \quad \checkmark$$
$$x(t-T) \longrightarrow \boxed{\ } \longrightarrow |x(t-T)| \quad \checkmark$$

- $y(t) = \log(x(t))$

$$x(t) \longrightarrow \boxed{\ } \longrightarrow \log(x(t)) \xrightarrow{T} \log(x(t-T)) \quad \checkmark$$
$$x(t-T) \longrightarrow \boxed{\ } \longrightarrow \log(x(t-T)) \quad \checkmark$$

- $y(t) = t\,x(t)$

$$x(t) \longrightarrow \boxed{\ } \longrightarrow t\,x(t) \xrightarrow{T} (t-T)\,x(t-T)$$
$$x(t-T) \longrightarrow \boxed{\ } \longrightarrow t\,x(t-T)$$

NO! non è tempo invariante

- $y(t) = \int_{-\infty}^{t} x(s)\,ds$

LINEARITÀ:
$$a_1 \int_{-\infty}^{t} x_1(s)\,ds + a_2 \int_{-\infty}^{t} x_2(s)\,ds \quad \checkmark$$

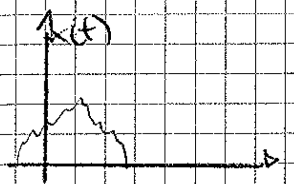
è lineare

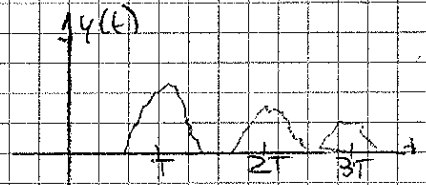
$$\int_{-\infty}^{t} a_1 x_1(s) + a_2 x_2(s)\,ds \quad \checkmark$$

TEMPO INVARIANZA:
$$\int_{-\infty}^{t} x(s)\,ds \longrightarrow \int_{-\infty}^{t-T} x(s)\,ds$$

$$\int_{-\infty}^{t} x(s-T)\,ds = \int_{-\infty}^{t-T} x(\theta)\,d\theta$$
$$s-T = \theta \quad ds = d\theta$$

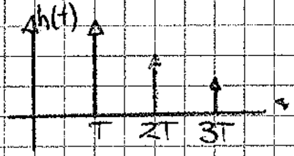
$h(t) = y(t)\Big|_{\delta(t)=x(t)}$





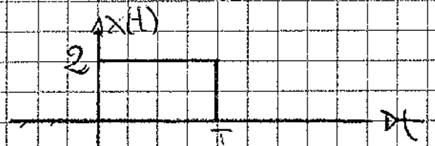
$y(t) = x(t-T) + 0.5\,x(t-2T) + 0.3\,x(t-3T)$

$h(t) = \delta(t-T) + 0.5\,\delta(t-2T) + 0.3\,\delta(t-3T)$

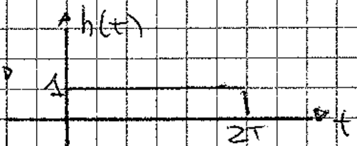


PRODOTTO DI CONVOLUZIONE:

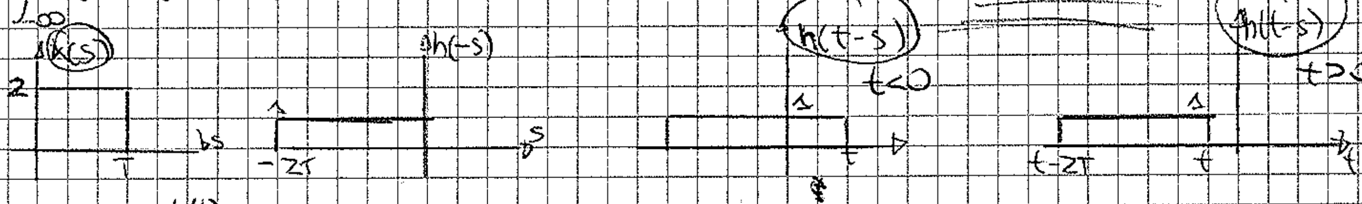
$$y(t) = x(t) * h(t)$$
$$y(t) = \int_{-\infty}^{+\infty} x(s)\, h(t-s)\, ds$$
$$y(t) = \int_{-\infty}^{+\infty} h(s)\, x(t-s)\, ds$$

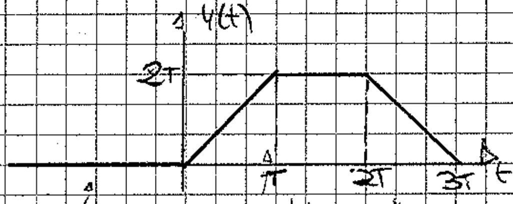


$x(t) \longrightarrow \boxed{h(t)} \longrightarrow$



$$\int_{-\infty}^{+\infty} x(s)\, h(t-s)\, ds$$





per t<0 le due porte non si accavallano

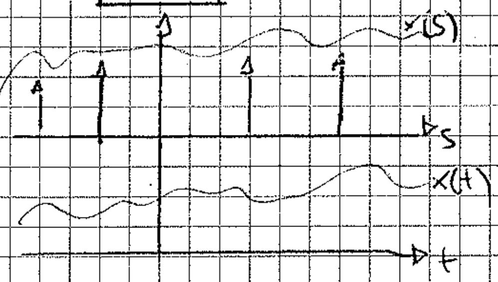
area del prodotto mutuo tra le due

$y(t) = x(t) * h(t)$

$Y(f) = X(f) \cdot H(f)$

$x(t) \longrightarrow \boxed{h(t)=\delta(t)} \longrightarrow y(t)$

$$y(t) = x(t) * \delta(t) = \int_{-\infty}^{+\infty} x(s)\, \delta(t-s)\, ds$$

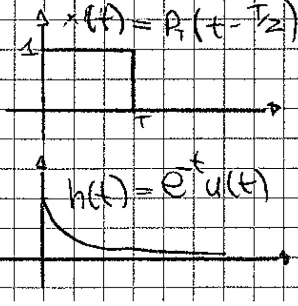


$\int x(s)\,\delta(s) = x(0) \longrightarrow \int x(s)\,\delta(s-t) = x(t)$

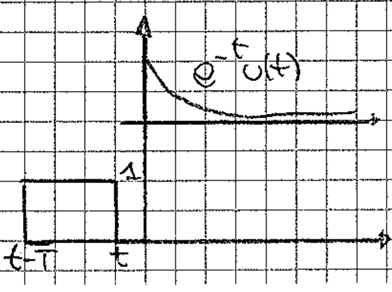
$Y(f) = X(f) \cdot \underbrace{F\{\delta(t)\}}_{=1} = X(f)$

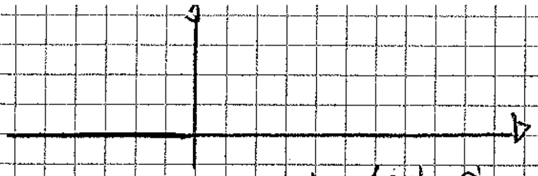
$x(t) = R_1(t - T/2)$

$h(t) = e^{-t}u(t)$



$$\int_{-\infty}^{+\infty} x(t-s)\, h(s)\, ds = $$
$$= x(t) * h(t)$$





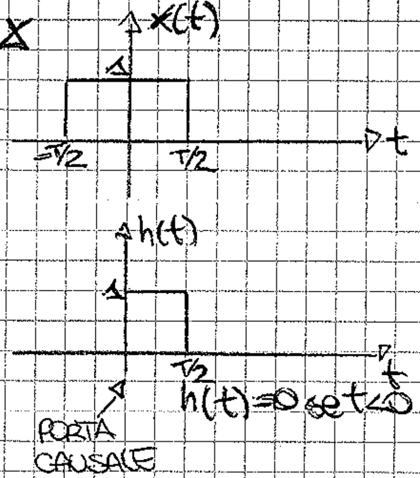
1) $\int_{-\infty}^{t} x(t-s)\,h(s)\,ds = \int_{-\infty}^{t} 1\cdot e^{-s}\,ds$

2) $\int_{t-T}^{t} e^{-s}\,ds$

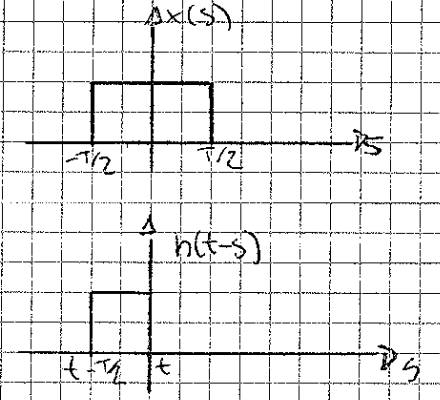
non ha finito l'esercizio

Esempi PRODOTTO DI CONVOLUZIONE

11-03-20



$h(t)=0$ se $t<0$



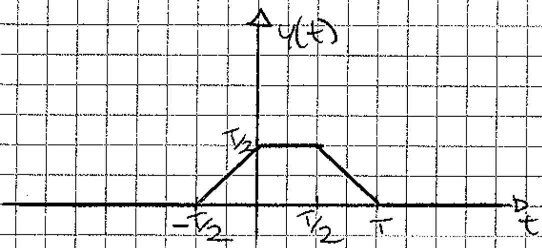
1) $\begin{cases} -\infty < t < -\frac{T}{2} \\ y(t) = 0 \end{cases}$

2) $\begin{cases} -\frac{T}{2} < t < 0 \\ y(t) = t - (-\frac{T}{2}) = t + \frac{T}{2} \end{cases}$

3) $\begin{cases} 0 < t < \frac{T}{2} \\ y(t) = \frac{T}{2} \end{cases}$

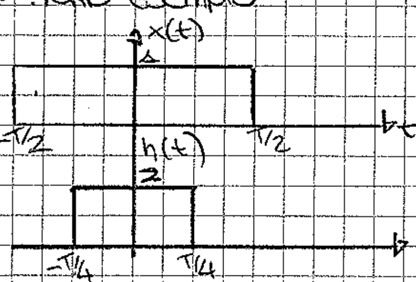
4) $\begin{cases} \frac{T}{2} < t < T \\ y(t) = \frac{T}{2} - (t - \frac{T}{2}) = T - t \end{cases}$

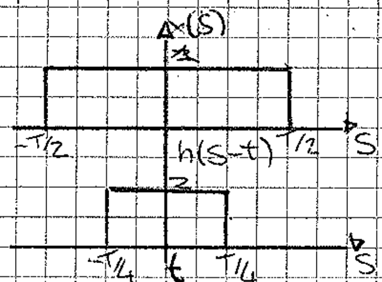
5) $\begin{cases} T < t < +\infty \\ y(t) = 0 \end{cases}$



Il prodotto di convoluzione fa salire di un grado la nostra funzione

- altro esempio





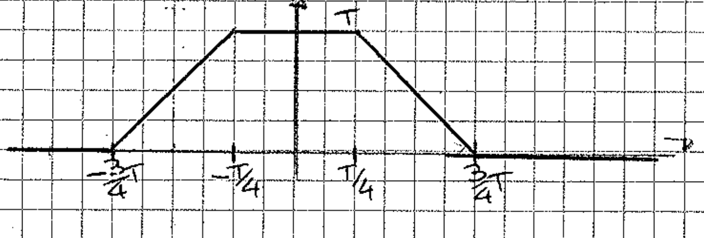
Il punto t dopo la traslazione è il punto in cui $s=0$

1) $\begin{cases} -\infty < t < -\frac{3}{4}T \\ y(t) = 0 \end{cases}$

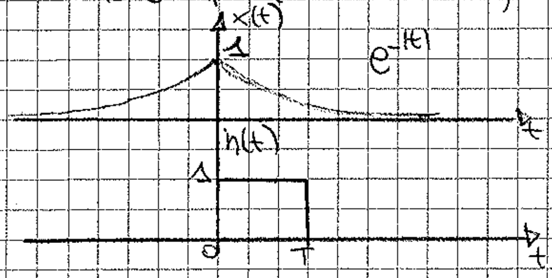
2) $\begin{cases} -\frac{3}{4}T < t < -\frac{T}{4} \\ y(t) = 2\left((t+\frac{T}{4}) - (-\frac{T}{2})\right) = 2t + \frac{3}{2}T \end{cases}$

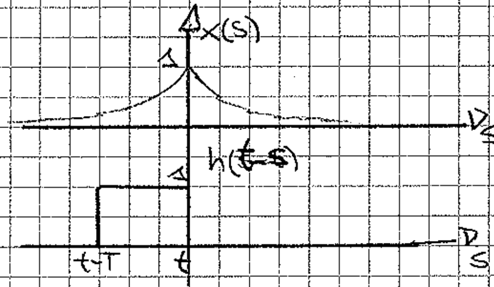
3) $\begin{cases} -\frac{T}{4} < t < \frac{T}{4} \\ y(t) = T \end{cases}$

4 $\begin{cases} \frac{T}{4} < t < \frac{3}{4}T \\ y(t) = \frac{3}{2}T - 2t \end{cases}$



• Altro esempio (TEMA D'ESAME)



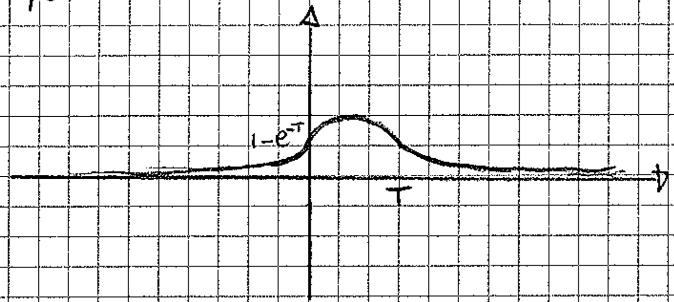


1) $\begin{cases} t<0 \\ y(t)=e^{t}(1-e^{-T}) \end{cases}$ $\quad \int_{t-T}^{t} e^{s}\,ds = \left[e^{s}\right]_{t-T}^{t} = e^{t}-e^{t-T} = e^{t}(1-e^{-T})$

3) $\begin{cases} t>T \\ y(t)=e^{-t}(e^{T}-1) \end{cases}$ $\quad \int_{t-T}^{t} e^{-s}\,ds = \left[-e^{-s}\right]_{t-T}^{t} = \left[-e^{-t}+e^{T-t}\right] = e^{-t}(e^{T}-1)$

2) $\begin{cases} 0<t<T \\ y(t)=2-e^{-t}-e^{t-T} \end{cases}$ $\quad \int_{t-T}^{0} e^{s}\,ds + \int_{0}^{t} e^{-s}\,ds = \left[e^{s}\right]_{t-T}^{0} + \left[-e^{-s}\right]_{0}^{t} = e^{0}-e^{t-T}-e^{-t}+e^{0} =$

$= 2 - e^{-t} - e^{t-T}$



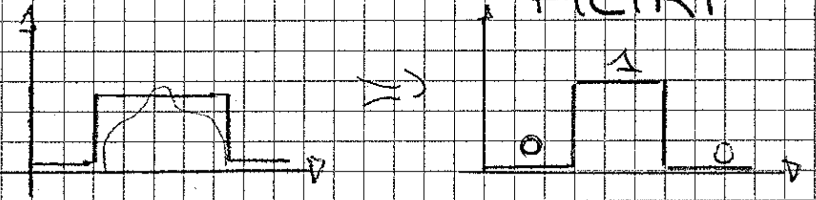
• $x(t) = e^{-\alpha t}\,u(t)$

$y(t) = x(t) * \sin\frac{\theta t}{\pi t}$

$\mathcal{E}_y = \frac{1}{2}\mathcal{E}_x$

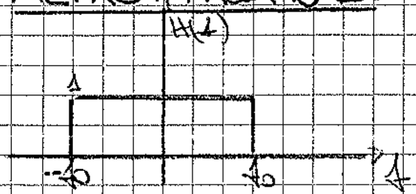
FILTRI

15-03-2010



Un filtro è un sistema tempo-invariante

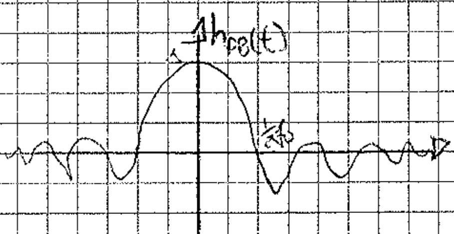
FILTRO PASSA-BASSO



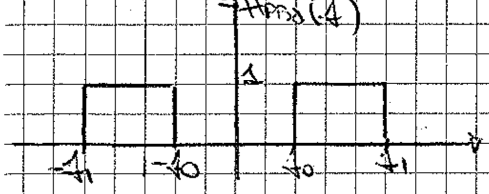
$H_{PB}(f) = p_{2f_0}(f)$

$\downarrow F^{-1}$

$h_{PB}(t) = \frac{\sin \pi 2 f_0 t}{\pi t}$



FILTRO PASSA-BANDA



$H_{PBd}(f) = p_{(f_1-f_0)}\left(f - \frac{f_1-f_0}{2}\right)$

$\downarrow F^{-1}$

$h_{PBd}(t) = \frac{\sin \pi (f_1-f_0) t}{\pi t} \cdot e^{+j2\pi \frac{f_1+f_0}{2} t}$