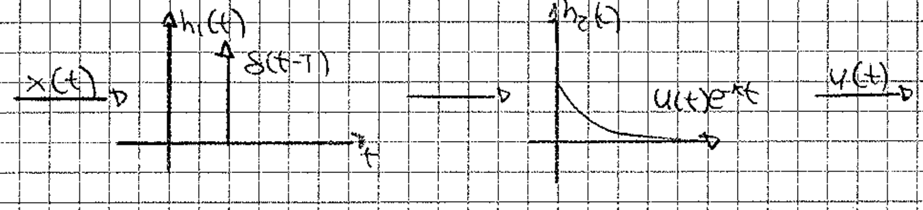
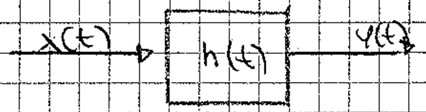
$$H(f)=\frac{Y(f)}{X(f)}=\frac{1}{1+RCj2\pi f}=\frac{1}{RC}\cdot\frac{1}{\frac{1}{RC}+j2\pi f} \rightsquigarrow h(t)=\frac{1}{RC}e^{-\frac{t}{RC}}u(t)$$

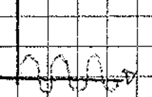
$$u(t)e^{-at}\to\frac{1}{a+j2\pi f}$$

$$Y(f)(1+RC\,j2\pi f)=X(f)$$



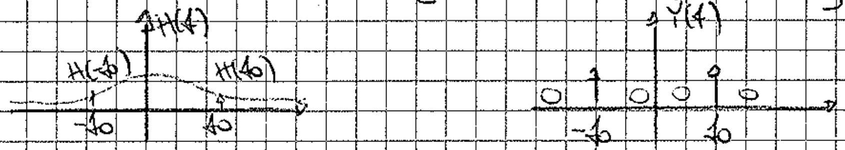
$$h_{eq}(t)=u(t-T)\,e^{-\alpha(t-T)} \qquad H_{eq}(f)=\frac{1}{\alpha+j2\pi f}e^{-j2\pi fT}$$





$x(t)=A\cos(2\pi f_0 t+\varphi)$

$$Y(f)=H(f)\cdot X(f)=H(f)\left[\frac{1}{2}\delta(f-f_0)+\frac{1}{2}\delta(f+f_0)\right]$$

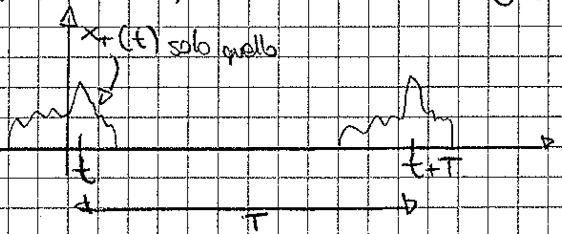


$H(f_0)\cos[2\pi f_0 t+\varphi+\varphi_+]$

SEGNALI PERIODICI

18-03-2010

$x(t+mT)=x(t)$ segnale periodico



$$\underline{x(t)=\sum_{m=-\infty}^{+\infty}x_T(t-mT)}$$ generico segnale periodico

$$x(t)=x_T(t)*\sum_{m=-\infty}^{+\infty}\delta(t-mT)$$

$$x(t)=\frac{1}{T}\sum_{m=-\infty}^{+\infty}\mu_m\,e^{j2\pi\frac{m}{T}t} \qquad X(f)=\int_{-\infty}^{+\infty}\sum\mu_m\,e^{j2\pi\frac{m}{T}t}\,e^{-j2\pi ft}\,dt=$$

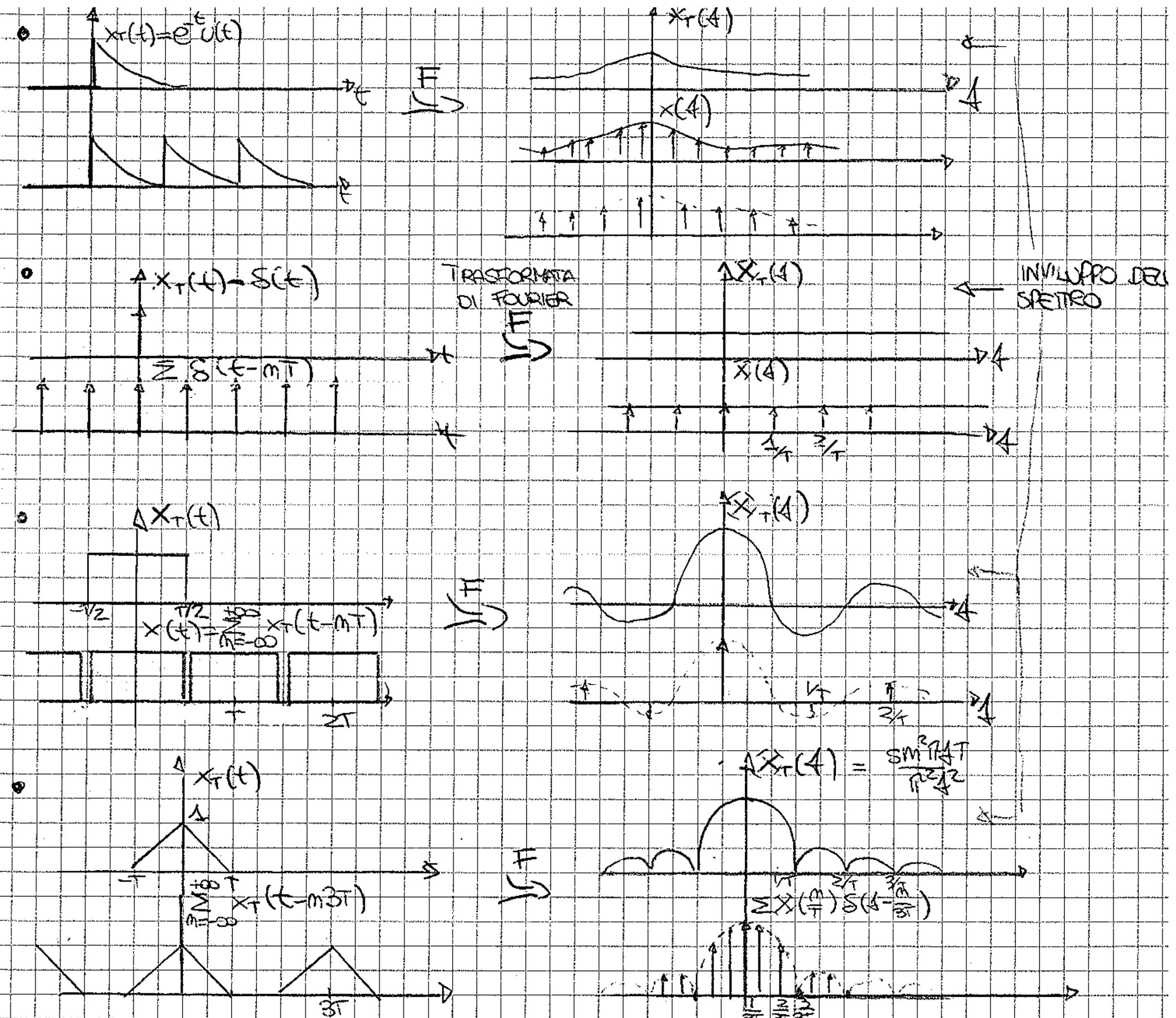
$$\mu_m=\int_{-T/2}^{T/2}x(t)\,e^{-j2\pi\frac{m}{T}t}\,dt \qquad =\sum\mu_m\int_{-\infty}^{+\infty}e^{j2\pi\frac{m}{T}t}\,e^{-j2\pi ft}=$$

$$=\sum_{-\infty}^{+\infty}\mu_m\int_{-\infty}^{+\infty}1\cdot e^{-j2\pi(f-\frac{m}{T})t}$$

$$X(f-m/T)=\delta\left(f-\frac{m}{T}\right) \rightsquigarrow X(f)=\sum_{m=-\infty}^{+\infty}\delta\left(f-\frac{m}{T}\right)$$

$$X(f)=\frac{1}{T}\sum\mu_m\,\delta\left(f-\frac{m}{T}\right)=\frac{1}{T}\sum_{m=-\infty}^{+\infty}x_T\left(\frac{m}{T}\right)\delta\left(f-\frac{m}{T}\right)$$

Dimostrazione che non capisco



Lo trasformata di un segnale periodico è uno spettro a righe

↳ UN SEGNALE PERIODICO NEL TEMPO HA UNO SPETTRO A RIGHE IN FREQ.
e viceversa (righe nel tempo → periodico in frequenza)

INVILUPPO DELLO SPETTRO = segnale da periodicizzare in frequenza

NON ESISTE UN SEGNALE DAL SUPPORTO LIMITATO SIA NEL TEMPO CHE IN FREQUENZA

SPAZIO DI HILBERT

H , $(\langle .,. \rangle)$

- N   a·b   no!

non ci devono essere salti, irregolarità

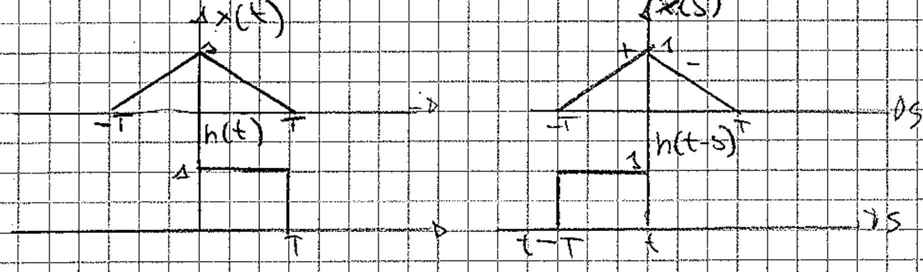
Nello spazio di Hilbert è definito il prodotto punto a punto, il prodotto

$\int f(t) g^*(t)\,dt$, il prodotto di convoluzione

$E_x = \int |x(t)|^2 dt$ ENERGIA DEL SEGNALE

Nello spazio dei segnali ci sono tutti i segnali (infiniti)

$$y(t)=\int_{-\infty}^{+\infty} x(s)\,h(t-s)\,ds=\int_{-\infty}^{+\infty} h(s)\,x(t-s)\,ds$$



$x^+(s)=\frac{s}{T}+1$

$x^-(s)=-\frac{s}{T}+1$

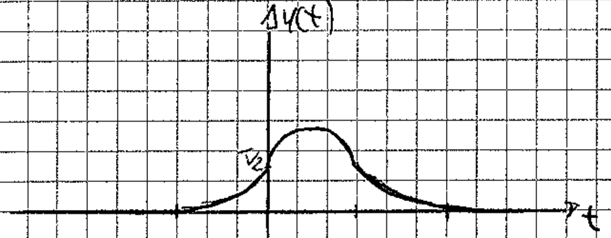
1) $\begin{cases} t<-T,\ t>2T \\ y(t)=0 \end{cases}$ 2) $\begin{cases} -T<t<0 \\ y(t)=\frac{1}{2T}t^2+t+\frac{T}{2} \end{cases}$ 3) $\begin{cases} 0<t<T \\ y(t)=-\frac{t^2}{T}+t+\frac{T}{2} \end{cases}$ 4) $\begin{cases} T<t<2T \\ y(t)=\frac{t^2}{2T}-2t+2T \end{cases}$

$$\int_{-T}^{t}\left(\frac{s}{T}+1\right)ds=\left[\frac{s^2}{2T}+s\right]_{-T}^{t}=\frac{t^2}{2T}+t-\frac{T^2}{2T}+T=\frac{1}{2T}t^2+t+\frac{T}{2}$$

$$\int_{t-T}^{0}\left(\frac{s}{T}+1\right)ds+\int_{0}^{t}\left(-\frac{s}{T}+1\right)ds=\left[\frac{s^2}{2T}+s\right]_{t-T}^{0}+\left[-\frac{s^2}{2T}+s\right]_{0}^{t}=-\frac{(t-T)^2}{2T}\cancel{-t}+T-\frac{t^2}{2T}\cancel{+t}=-\frac{t^2}{T}+t+\frac{T}{2}$$

t^2+T^2-2tT

$$\int_{t-T}^{T}\left(-\frac{s}{T}+1\right)ds=\left[-\frac{s^2}{2T}+s\right]_{t-T}^{T}=-\frac{T^2}{2T}+T+\frac{(t-T)^2}{2T}-t+T=\frac{t^2}{2T}+2T-2t$$



ho porto da e centro di simmetria in $\frac{T}{2}$

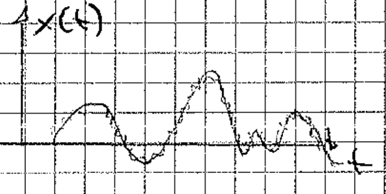
$$E_x=\int_{-\infty}^{+\infty}|x(t)|^2dt=\int_{-\infty}^{+\infty}x(t)x^*(t)\,dt$$

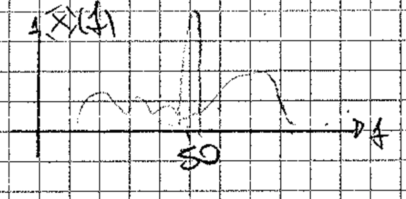
$$x(t)=\int_{-\infty}^{+\infty}X(f)e^{j2\pi ft}df$$

$$E_x=\int_{-\infty}^{+\infty}x(t)\int X^*(f)e^{-j2\pi ft}df\,dt=\iint_{-\infty}^{+\infty}x(t)e^{-j2\pi ft}X^*(f)\,dt\,df$$

$$=\int|X(f)X^*(f)|\,df$$

$$E_x=\int_{-\infty}^{+\infty}|X(f)|^2df$$





c'è un disturbo
↓
picco a 50 Hz in Fourier

$$\boxed{S_x(f)=|X(f)|^2}$$

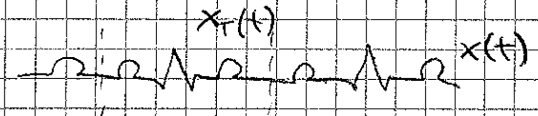
SPETTRO
DI
ENERGIA

1) $\int_{f_1}^{f_2}S_x(f)\,df=E_x\{f_1,f_2\}$ energia compresa tra f_1 e f_2

2) $S_x\geq 0$

3) Se $X(f)\cdot H(f)=Y(f)\Rightarrow S_x(f)\cdot|H(f)|^2=S_y(f)$ $\quad x(t)\rightarrow\boxed{h(t)}\rightarrow y(t)$

SEGNALI PERIODICI



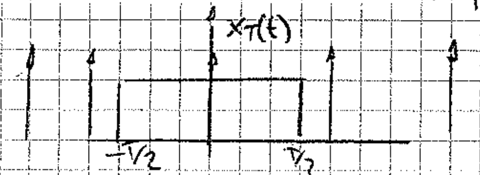
$$x(t+kT)=x(t) \qquad \forall k \in \mathbb{N}$$

$$x(t)=\sum_{m=-\infty}^{+\infty} x_T(t-mT)$$

c'è un per $*$

$$x(t)=x_T(t) * \sum_{m=-\infty}^{+\infty} \delta(t-mT)$$

$$X(f)=X_T(f)\cdot\frac{1}{T}\sum_{m=-\infty}^{+\infty} \delta\left(f-\frac{m}{T}\right)$$



$$x(t)\sum \mu_m e^{j2\pi\frac{m}{T}t}$$

$$\mu_m=\frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-j2\pi\frac{m}{T}t}dt$$

$$x(t)=\sum\frac{1}{T}\int_{-T/2}^{T/2}x(t)e^{+j2\pi\frac{m}{T}t}\cancel{dt}\cdot e^{-j2\pi\cancel{f}t}dt=$$

$$=\frac{1}{T}\sum\int\delta(t)\,e^{-j2\pi\left(f-\frac{m}{T}\right)t}dt=$$

$$=\frac{1}{T}\sum\delta\left(f-\frac{m}{T}\right)$$

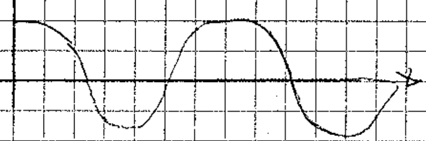
Dimostrazione che la trasformata di un treno di $\delta(t)$ è un treno di δ

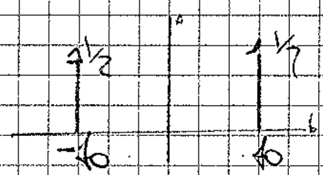
$$X(f)=\sum_{m=-\infty}^{+\infty}\mu_m\,\delta\left(f-\frac{m}{T}\right)$$

La potenza di un segnale periodico è

$$\underline{P=\sum_{-\infty}^{+\infty}|\mu_m|^2}$$

- $x(t)=\cos t$

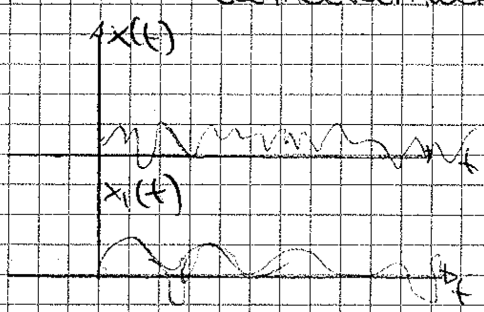


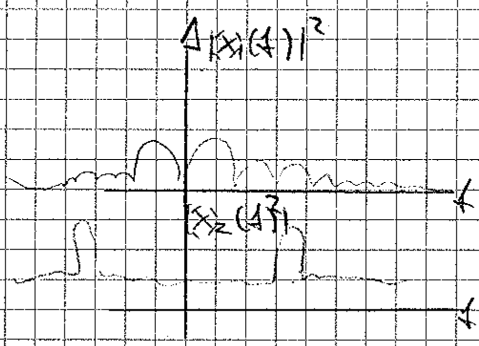


$$P=\sum_{-\infty}^{+\infty}|\mu_m|^2=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

ELETTROENCEFALOGRAMMI

25-03-2010



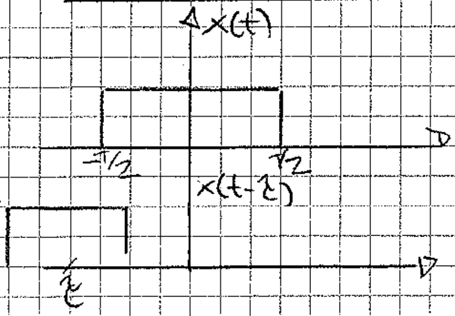


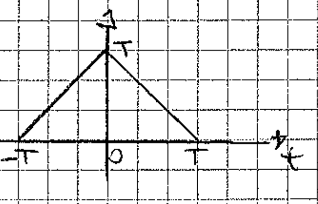
$$S_Y(f)=|H(f)|^2 S_X(f)$$

$$\int_{f_1}^{f_2} S_X(f)=E_X\Big|_{f_1,f_2}$$

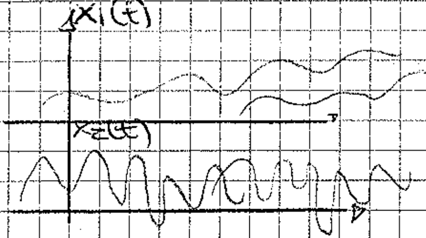
AUTOCORRELAZIONE

$$R_x(\tau)=\int_{-\infty}^{+\infty} x(t)\,x(t-\tau)\,dt$$





LA FUNZIONE DI AUTOCORRELAZIONE È LA TRASFORMATA INVERSA DI FOURIER DELLO SPETTRO DI ENERGIA



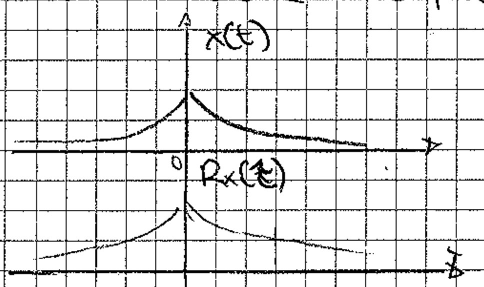
La funzione di autocorrelazione mi dice quanto un segnale è imparentato con se stesso nel tempo.

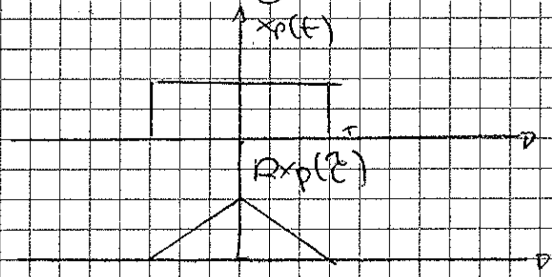
Confronto tra $x(t)$ e $x(t+\tau)$ → dopo un tempo τ

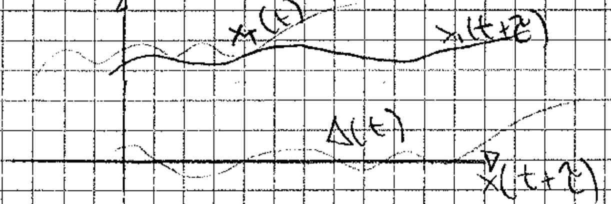
↳ quanto un segnale varia velocemente o lentamente nel tempo

Segnale che varia velocemente → autocorrelazione stretta
Segnale che varia lentamente → autocorrelazione larga

L'autocorrelazione della delta è una delta
L'autocorrelazione della porta è un triangolo







$$\Delta = x(t) - x(t+\tau)$$

$$\mathcal{E}(\Delta)=\int_{-\infty}^{+\infty} |x(t)-x(t+\tau)|^2\,dt = \int |x(t)|^2\,dt + \int |x(t+\tau)|^2\,dt - 2\int |x(t)\,x(t+\tau)|\,dt = \mathcal{E}_x+\mathcal{E}_x-2R_x(\tau)$$

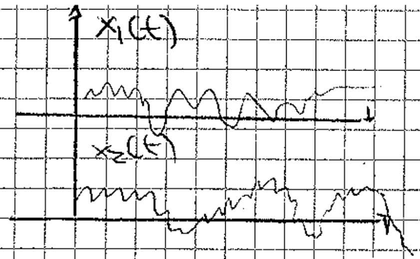
$$\mathcal{E}_\Delta = 2(\mathcal{E}_x - R_x(\tau))$$

Se $\mathcal{E}_\Delta$ piccolo → i segnali sono simili

Se $\mathcal{E}_\Delta$ grande → i segnali sono diversi

Se $R_x(\tau)=\mathcal{E}_x$ ⟺ il segnale è molto simile a se stesso traslato
NO VARIAZIONI

Se $R_x(\tau)\to 0$ ⟺ $x(t)\neq x(t+\tau)$ VARIAZIONI



1) $R_x(\tau) = F^{-1}\{S_x(f)\} \geq 0$
2) $R_x(0) = \mathcal{E}_x$
3) $R_x(\tau) \leq R_x(0)$
4) $R_x(\tau)$ è PARI

$e^{-|t|}$ può essere un autocorrelazione
la porta non può essere un autocorrelazione
il seno cardinale può essere un autocorrelazione (es. del filtro passabasso)

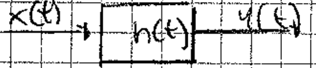
26-03-20

RIPASSO

$$R_x(\tau) = \int_{-\infty}^{+\infty} x(t)\,x(t-\tau)\,dt = \int_{-\infty}^{+\infty} x(t)\,x(t+\tau)\,dt$$

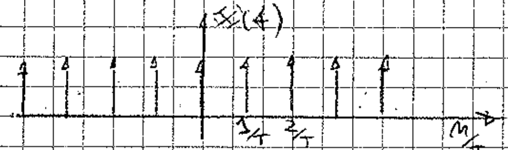
$$S_x(f) = |X(f)|^2$$

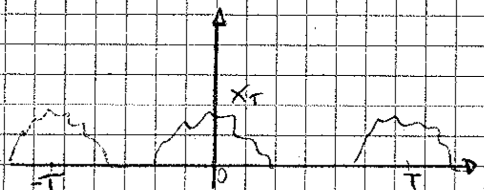
$$S_x(f) = F\{R_x(\tau)\}$$



$$S_y(f) = S_x(f)\,|H(f)|^2$$

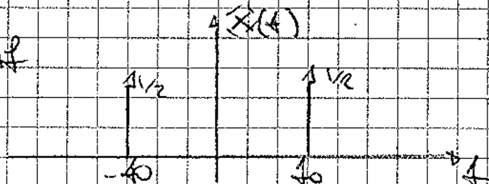
$$X(f) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X\left(\frac{n}{T}\right)\delta\left(f-\frac{n}{T}\right)$$





$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{+\infty} X_T(f)\,df$$

$$P = \sum_{n=-\infty}^{+\infty} |\mu_n|^2$$



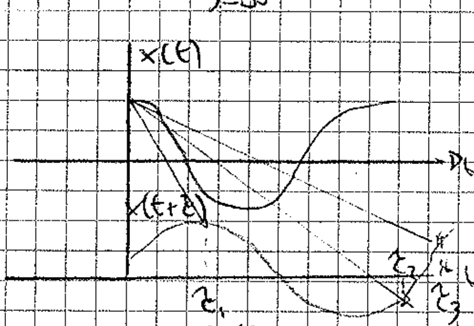
$$G_x(f) = \sum_{n=-\infty}^{+\infty} |\mu_n|^2\,\delta(f-nf_0) \qquad f_0 = \frac{1}{T}$$

spettro di potenza

$$\int_{-\infty}^{+\infty} G_x(f)\,df = P_x$$

$$G_x(f) = F^{-1}\{R_x(\tau)\}$$

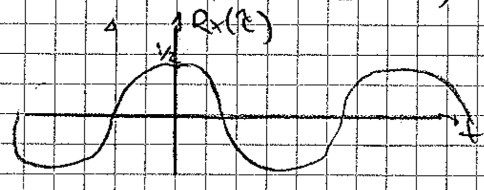
- $R_x(\tau) = \int_{-\infty}^{+\infty} \cos t \cdot \cos(t+\tau)\,dt$

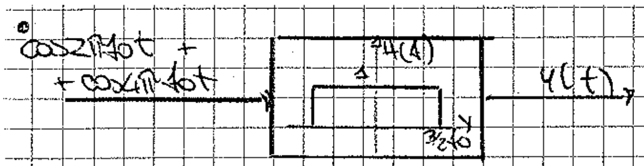


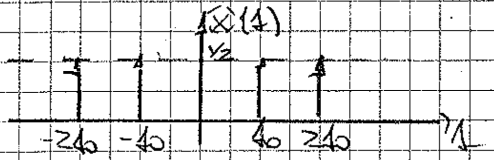
$$G_x = \frac{1}{4}\delta(f-f_0) + \frac{1}{4}\delta(f+f_0)$$

$$R_x(\tau) = F^{-1}\{G_x(f)\} = \frac{1}{2}\cos\tau$$

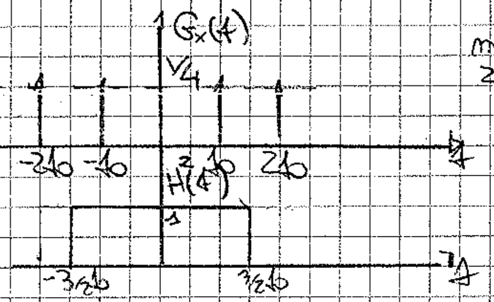
è autocorrelazione ha le 4 proprietà



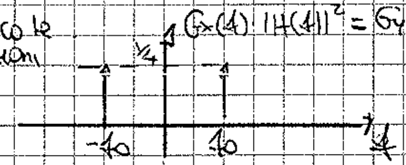




La potenza del segnale in entrata vale $\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2=1$



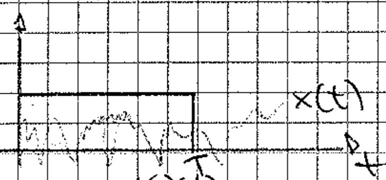
moltiplico le 2 funzioni $\Rightarrow$



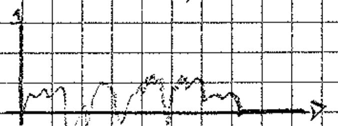
$$P=\sum \mu_n^2=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

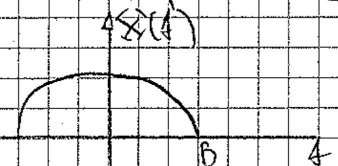
IL FENOMENO DI GIBBS

08-04-2010

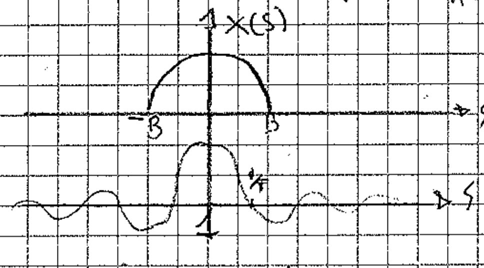


$$\hat{x}(t)=x(t)\cdot P_T\left(t-\tfrac{T}{2}\right)$$

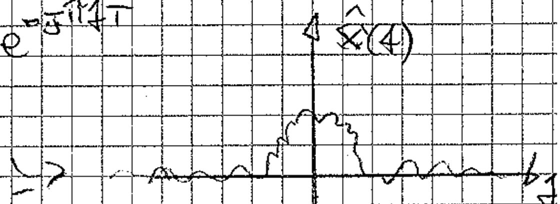




$$\hat{X}(f)=X(f) * \frac{\sin \pi f T}{\pi f}\, e^{-j\pi f T}$$



$\Rightarrow$

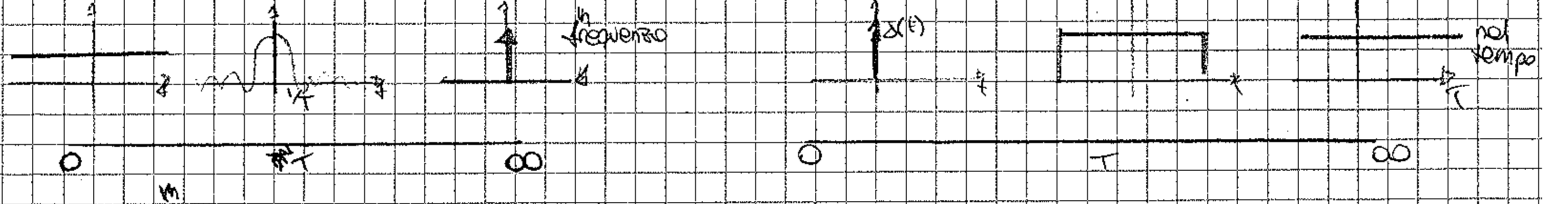


Fenomeno di Gibbs dovuto al troncamento del del segnale (→ porta)

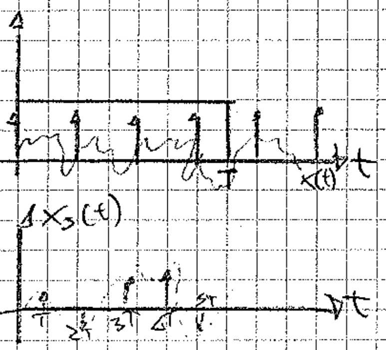
Se T diventa molto grande, il seno cardinale si stringe

Se T → ∞, il senocardinale tende ad una delta di dirac $\Rightarrow$ il segnale diventa + simile al segnale

se T → 0, la porta tende ad una delta di dirac → la sin c diventa + larga fino a diventare una costante

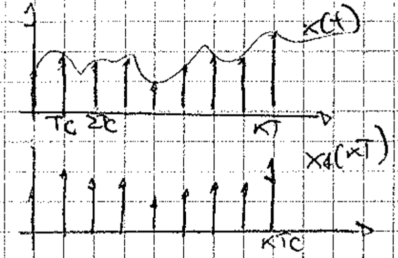


CAMPIONAMENTO



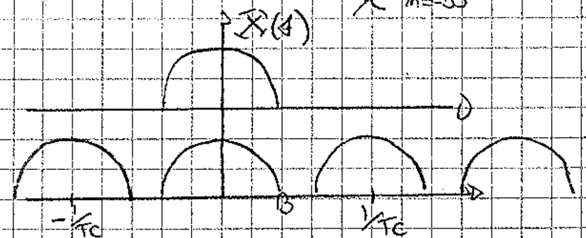
Il segnale di ingresso $x(t)$ viene campionato cioè moltiplicato per un treno di δ

$$x_s(t) = x(t)\sum\delta(t-mT)$$



$$x_s(t) = T_c\, x(t)\sum\delta(t-mT_c)$$

$$X_{x_s}(f) = T_c\, X(f) * \frac{1}{T_c}\sum_{m=-\infty}^{+\infty}\delta\left(f-\frac{m}{T_c}\right)$$



Un segnale campionato si ripete infinite volte in frequenza

TEOREMA DI NYQUIST

$$\boxed{f_c = \frac{1}{T_c} \geq 2B}$$

$\sin 2\pi f_0 T$ $\qquad T_c \leq \frac{1}{2f_0} \qquad T_c \leq \frac{T}{2} = \frac{1}{2f_0}$

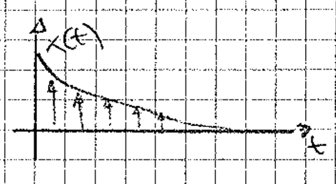
Servono almeno 2 campioni per periodo per determinare la funzione

$\sin 2\pi f_0 T \cdot \cos 2\pi f_0 T \Rightarrow f_c = 4f_0$

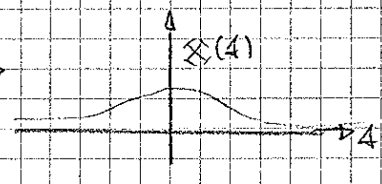
$f_c = 2f_0 \quad + \quad f_c = 2f_0$

SEGNALI ANALOGICI - SEGNALI NUMERICI

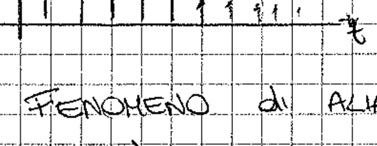
12-04-2010

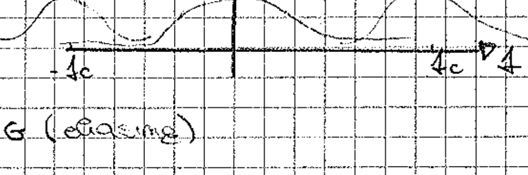


$\xrightarrow{F}$



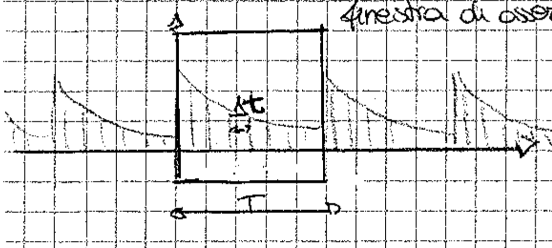
Campioniamo il segnale → treno di δ

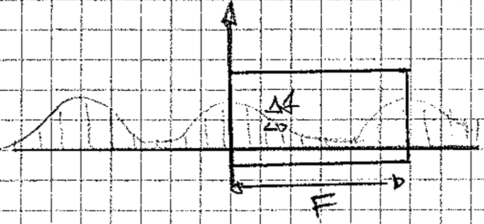




FENOMENO DI ALIASING (aliasing)

il segnale nel tempo è fatto a righe (perché il computer può leggere solo le righe) → perciò in frequenza il segnale è periodico.





1) T il più grande possibile

2) Δt il più piccolo possibile
↳ passo nel tempo

3) $N = \frac{T}{\Delta t}$ il più piccolo possibile (devo diminuire il lavoro del computer)

⇓

TRADE OFF = avere esigenze che vanno in collisione

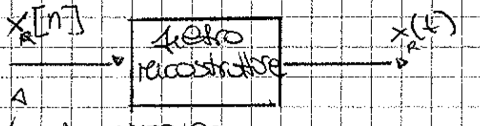
4) $F = F_c = \frac{1}{T_c} = \frac{1}{\Delta t}$

5) Δf ↳ passo in frequenza $\quad \Delta f = \frac{F}{N} = \frac{1}{\Delta t \cdot N}$

- $T = 1'40'' = 100\ s$
  $N = 1000$ campioni
  $\Delta t = \frac{T}{N} = 0.1\ s$
  $F_c = \frac{1}{\Delta t} = 10\ Hz$
  $\Delta f = \frac{10}{1000} = 0.01\ Hz$

- $T = 2.5\ s$
  $B \leq 50\ Hz$
  $10 B$ → voglio campionare a 10B → $N = ?$ $\Delta t = ?$ ⇒ $f_c = 1000\ Hz$
  $\Delta t = T_c = \frac{1}{f_c} = 0.001\ s$
  $N = \frac{T}{\Delta t} = 2500$ campioni

- $T = 10\ s$
  $N = 100$
  $T/N = \Delta t = 0.1\ s$
  $f_c = 10\ Hz$
  $\Delta f = 0.1\ Hz = \frac{1}{T}$



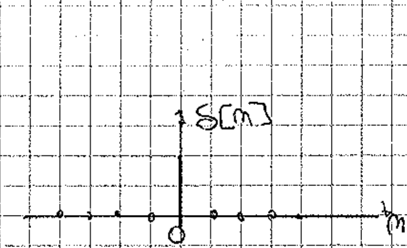
segnale NUMERICO (definito per punti)

$x[n]$ ⇒
$n=1 \rightarrow 0.7$
$n=2 \rightarrow 0.3$
$n=3 \rightarrow 0.2$
$n=4 \rightarrow 0.1$

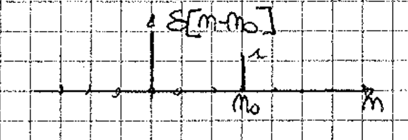
$F = \frac{1}{T_c} = \frac{1}{\Delta t} = 1$

DELTA DI KRONECKER

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{altrove} \end{cases}$$

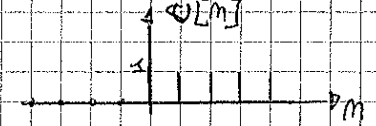


$$\delta[n - n_0] = \begin{cases} 1 & n = n_0 \\ 0 & \text{altrove} \end{cases}$$



GRADINO

$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{altrove} \end{cases}$



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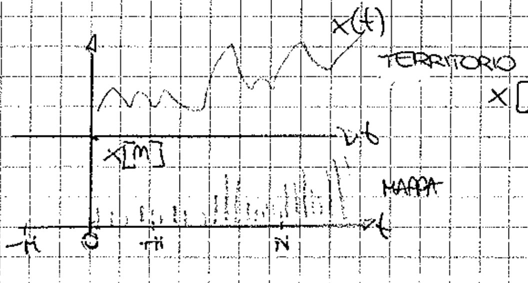


$x[n]$ è la MAPPA di $x(t)$ — Elaborazione numerica dei segnali (MARINA MONDIN)

Segnali analogici → tempo → Fourier, Laplace

Segnali digitali (numerici) → n → Fourier, Z

ENERGIA e POTENZA

- energia $E = \sum_{n=-\infty}^{+\infty} |x[n]|^2$ $\qquad E = \int_{-\infty}^{+\infty} |x(t)|^2 dt$

potenza $P = \lim_{M \to +\infty} \frac{1}{2M+1} \sum_{n=-M}^{+M} |x[n]|^2$ $\qquad P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x_T(t)|^2 dt$

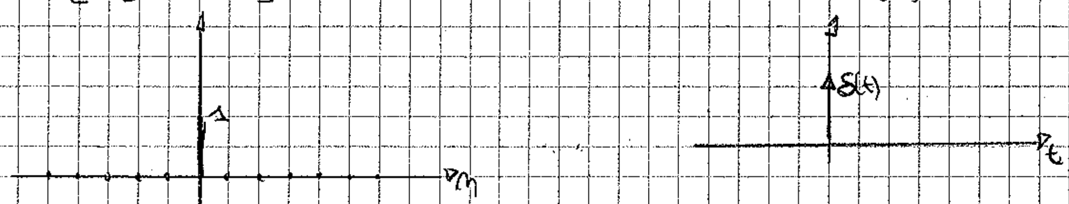
PROPRIETÀ

- PARI $x[n] = x[-n]$

DISPARI $x[n] = -x[-n]$

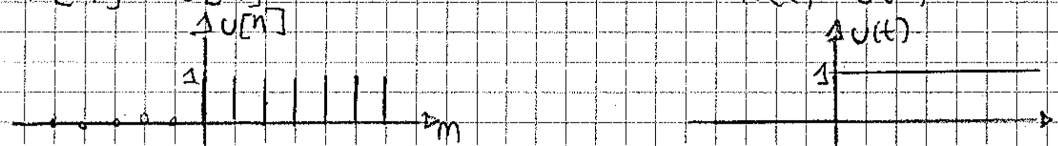
- PERIODICITÀ $x[n+N] = x[n]$

1) $x[n] = \delta[n]$ $\qquad x(t) = \delta(t)$



segnale ad energia (unitaria) $\qquad$ segnale a potenza

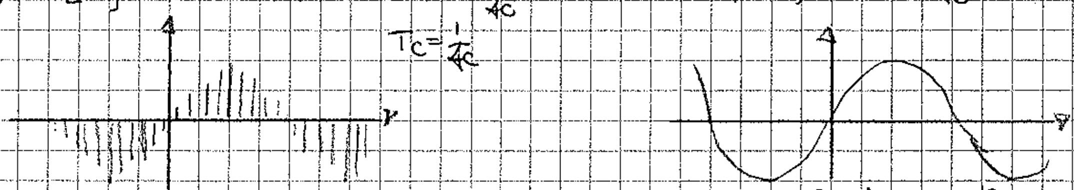
2) $x[n] = u[n]$ $\qquad x(t) = u(t)$



segnale a potenza $(=\frac{1}{2})$ $\qquad$ segnale a potenza $(=\frac{1}{2})$

3) $x[n] = \sin 2\pi f_0 n T_c = \sin 2\pi \frac{f_0}{f_c} n$ $\qquad x(t) = \sin 2\pi f_0 t$

$T_c = \frac{1}{f_c}$



segnale a potenza

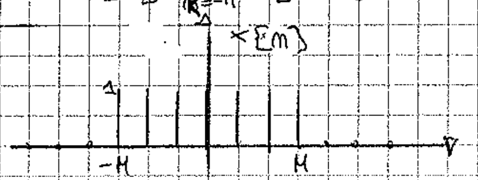
~~sono~~ $x[n] = \sin 2\pi f_N n$

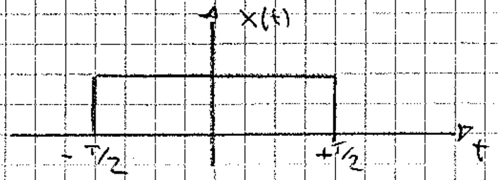
Tutte le frequenze del numerico sono normalizzate con la frequenza di campionamento.
→ $0 < f_N < 1$

• $\sin 2\pi 50 t$ (f_0) lo campiono a 100Hz e a 1000Hz ($f_c=$)

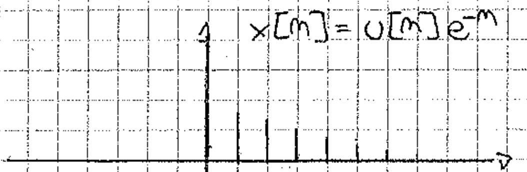
$x_1 = \sin 0.5 \cdot 2\pi n$ $\qquad$ $x_2 = \sin 0.05 \cdot 2\pi n$

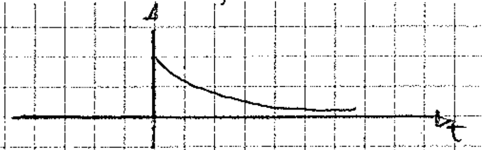
4) $x[n] = \sum_{k=-M}^{M} \delta[n-k]$ $\qquad\qquad$ $x(t) = p_T(t)$





5) $x[n] = u[n]e^{-n}$ $\qquad\qquad$ $x(t) = e^{-t}u(t)$





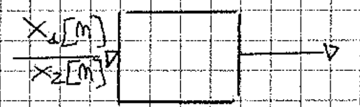
il numerico segue male i segnali brevi e segue bene i segnali lunghi

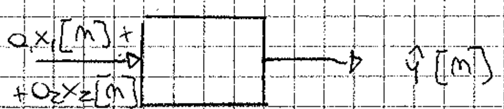
6) $x[n] = \frac{\sin \pi \frac{n}{N}}{\pi n}$ $\qquad\qquad$ $x(t) = \frac{\sin \pi t F}{\pi t}$

$x[n] = \frac{\sin \pi n \hat{F}}{\pi n}$ $\qquad$ $f = \frac{f_0}{f_c}$

se $N = 1 \Rightarrow$ il seno cardinale è una delta

LINEARITÀ e TEMPO-INVARIANZA





• $y(t) = 4x(t)$ è lineare $\qquad$ $y(t) = 4x(t) + 3$ non è lineare

$y[n] = 4x[n]$ è lineare $\qquad$ $y(n) = 4x[n] + 3$ non è lineare

• $y(t) = \cos 2\pi f_0 t \, x(t)$ è lineare

$y[n] = \cos 2\pi f_0^N n \cdot x[n]$ è lineare

$\cos 2\pi f_0^N n \cdot x[n] a_1 + \cos 2\pi f_0^N n \cdot x_2[n] a_2 = \cos 2\pi f_0^N n (a_1 x_1[n] + a_2 x_2[n])$

Se un sistema è lineare nel tempo analogico è lineare anche nel tempo numerico (perché il numerico converge al tempo analogico)

$x[n] \to \boxed{h[n]} \to y[n]$

$y[n] = x[n] * h[n]$

$h[n]$: si definisce come risposta all'impulso, cioè l'uscita quando in ingresso c'è una delta

$h[n] = y[n]\Big|_{x[n]=\delta[n]}$

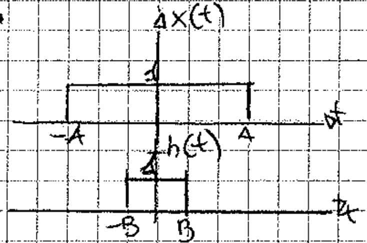
$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$ $\qquad \sum_{n=-\infty}^{+\infty} \delta(n) = 1$

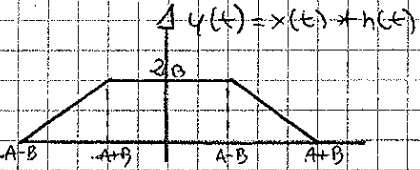
$\int \delta(t)x(t) = x(0)\delta(t)$ $\qquad \sum_{n=-\infty}^{+\infty} x[n]\,\delta[n] = x(0)\,\delta[n]$

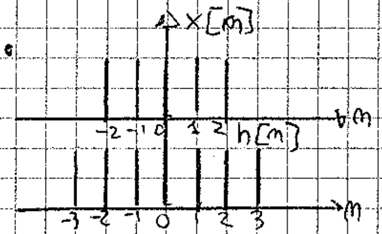
CONVOLUZIONE

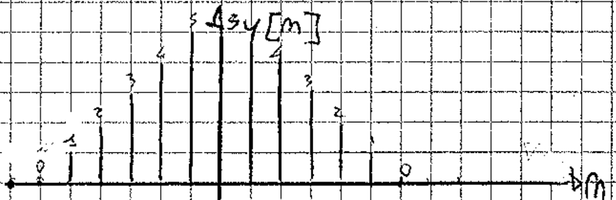
$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(s)\,h(t-s)\,ds$

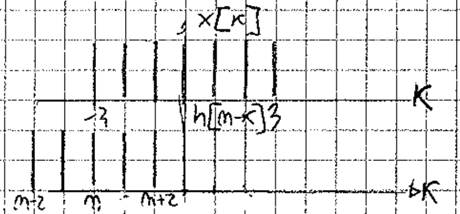
$$\boxed{y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]} \qquad x[n] * h[n]$$







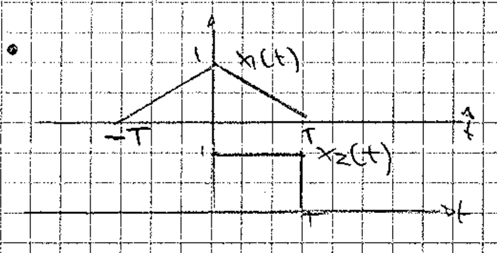


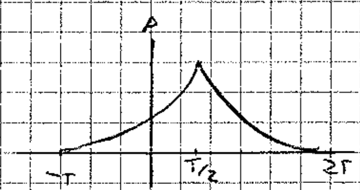


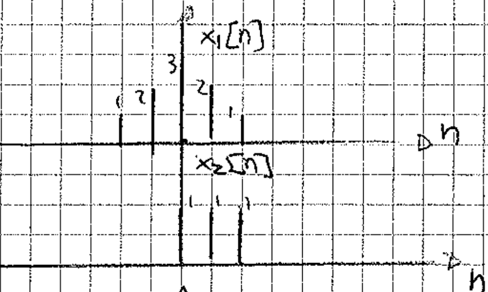
Nel numerico è spostato in alto di 1 rispetto all'analogico

Nel numerico il supporto è $[A+B-1]$

22-04-2010





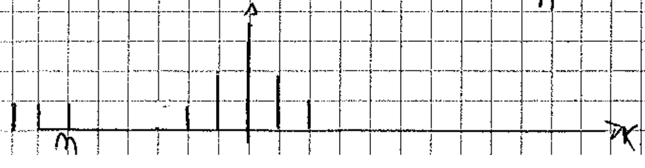


$x_1[n] = \delta[n+2] + 2\delta[n+1] + 3\delta[n] + 2\delta[n-1] + \delta[n-2]$

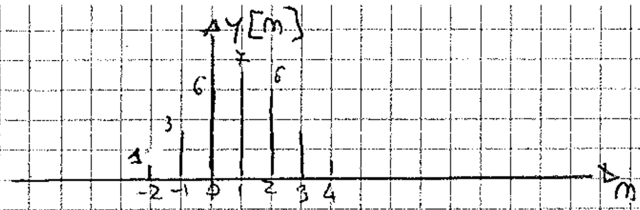
$\hookrightarrow E = \sum |x[n]|^2 \Rightarrow E_1 = 19$

$x_2[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

$\hookrightarrow E_2 = 3$



$y[n] = x_1[n] * x_2[n] = \sum_{k=-\infty}^{+\infty} x_1[k]\,x_2[n-k]$



$$y[n] = \delta[n+2] + 3\delta[n+1] + 6\delta[n] + 7\delta[n-1] + 6\delta[n-2] + 3\delta[n-3] + \delta[n-4]$$

TRASFORMATA Z

$$\delta[n] \xrightarrow{Z} 1$$

$$x[n] * y[n] \xrightarrow{Z} X(z) \cdot Y(z)$$

$$\boxed{X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}}$$

$$\sum_{0}^{+\infty} a^{-n} = \frac{1}{1-a^{-1}} \quad |a|>$$

- $X(z) = \sum_{k=-\infty}^{+\infty} x[k]\, z^{-k}$ $\qquad 1 \cdot z^0 = 1$
- $X(z) = \sum_{k=-\infty}^{+\infty} u[k]\, z^{-k} = \sum_{k=0}^{+\infty} z^{-k} \qquad |z|>1$

$$X(z) = \sum_n x[n]\, z^{-n}$$
$$Y(z) = \sum_k y[k]\, z^{-k}$$

$$z[n] = \sum x[p]\, y[n-p]$$
$$Z(z) = \sum x[p]\, y[n-p]\, z^{-n} = \underbrace{\sum x[n] z^{-n}}_{X(z)} \underbrace{\sum y[k] z^{-k}}_{Y(z)} = \sum_n \sum_k x[n] z^{-n} y[k] z^{-k} =$$
$$= \sum_{n,k} x[n]\, y[k]\, z^{-(k+n)} = \sum x[n]\, y[p-n]\, z^{-p}$$

$p = n+k$
$n = p-k$
$k = p-n$

$$x[n-N] = \sum_n x[n-N]\, z^{-n} = \sum_p x[p]\, z^{-(p+N)} = \sum_p x[p]\, z^{-p} z^{-N} = X(z)\, z^{-N}$$

$n-N=p$
$n=p+N$

$$\sum_{n=0}^{+\infty} a^n = \frac{1}{1-a} \qquad |a|<1$$
$$\sum_{n=0}^{+\infty} a^{-n} = \frac{1}{1-a^{-1}} \qquad |a|>1$$

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}$$

$$X(z) = \sum_{n=0}^{0} \delta[n]\, z^{-n} = 1$$

$$\overbrace{y[n]}^{1} = \overbrace{x[n] * h[n]}^{2}$$
$$Y(z) = X(z)\, H(z)$$

↓?

Proviamo a dimostrarlo