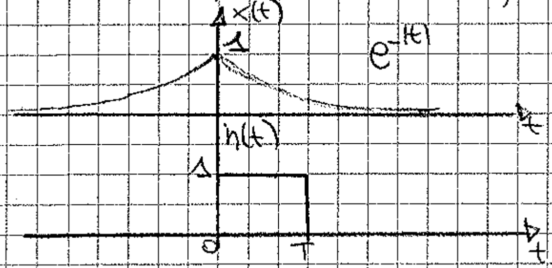
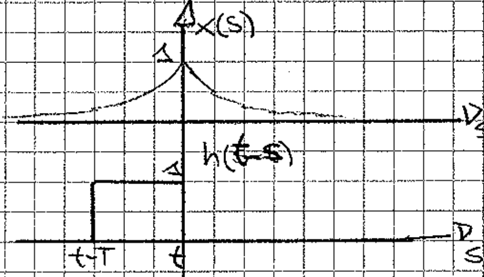
• Altro esempio (TEMA D'ESAME)



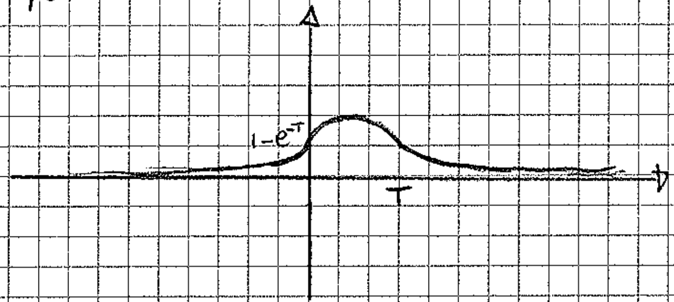


1) $\begin{cases} t<0 \\ y(t)=e^{t}(1-e^{-T}) \end{cases}$ $\quad \int_{t-T}^{t} e^{s}ds = \left[e^{s}\right]_{t-T}^{t} = e^{t}-e^{t-T} = e^{t}(1-e^{-T})$

3) $\begin{cases} t>T \\ y(t)=e^{-t}(e^{T}-1) \end{cases}$ $\quad \int_{t-T}^{t} e^{-s}ds = \left[-e^{-s}\right]_{t-T}^{t} = \left[-e^{-t}+e^{T-t}\right] = e^{-t}(e^{T}-1)$

2) $\begin{cases} 0<t<T \\ y(t)=2-e^{-t}-e^{t-T} \end{cases}$ $\quad \int_{t-T}^{0} e^{s}ds + \int_{0}^{t} e^{-s}ds = \left[e^{s}\right]_{t-T}^{0} + \left[-e^{-s}\right]_{0}^{t} = e^{0}-e^{t-T}-e^{-t}+e^{0} =$

$= 2 - e^{-t} - e^{t-T}$



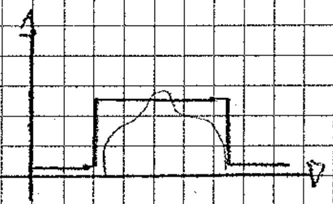
• $x(t) = e^{-\alpha t}u(t)$

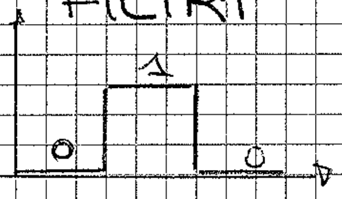
$y(t) = x(t) * \sin\frac{\theta t}{\pi t}$

$\mathcal{E}_y = \frac{1}{2}\mathcal{E}_x$

FILTRI

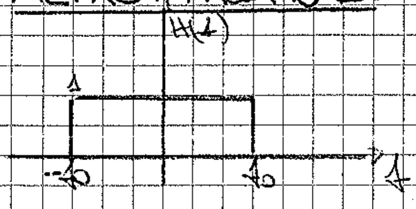
15-03-2010





Un filtro è un sistema tempo-invariante

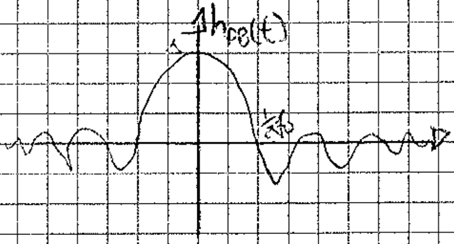
FILTRO PASSA-BASSO



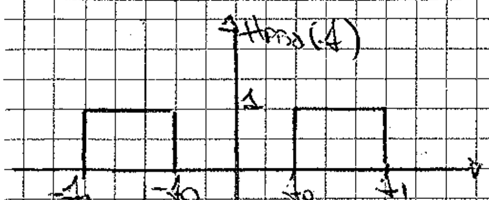
$H_{PB}(f) = p_{2f_0}(f)$

$\downarrow F^{-1}$

$h_{PB}(t) = \frac{\sin \pi 2 f_0 t}{\pi t}$



FILTRO PASSA-BANDA

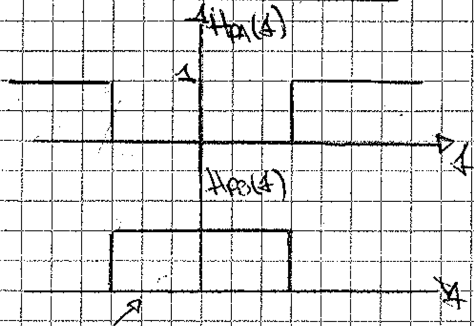


$H_{PBa}(f) = p_{(f_1-f_0)}\left(f - \frac{f_1-f_0}{2}\right)$

$\downarrow F^{-1}$

$h_{PBa}(t) = \frac{\sin \pi (f_1-f_0)t}{\pi t} \cdot e^{+j2\pi\frac{f_1+f_0}{2}t}$

FILTRO PASSAALTO

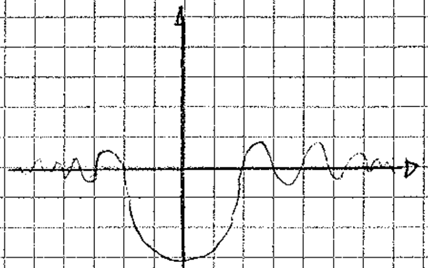


FILTRO PASSABASSO

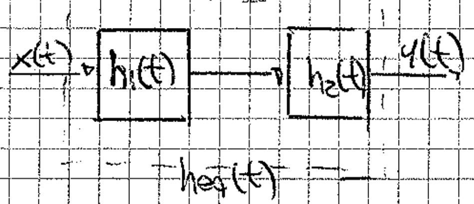
$H_{PA}(f) = 1 - P_{2f_0}(f)$

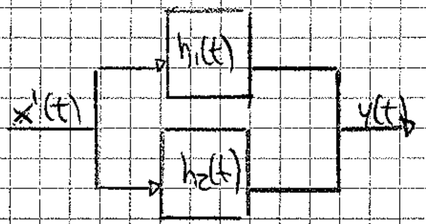
$\downarrow$

$h_{PA}(t) = \delta(t) - \frac{\sin 2\pi f_0 t}{\pi t}$



FILTRI IN SERIE E PARALLELO





$h_{eq} = h_1(t) * h_2(t)$

$H_{eq}(f) = H_1(f) \cdot H_2(f)$

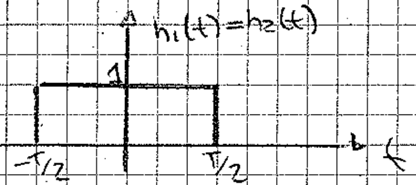
$h_{eq}(t) = h_1(t) + h_2(t)$

$H_{eq}(f) = H_1(f) + H_2(f)$

•

$h_{eq}(t)$

$h_2(t) = h_1(t) = P_T(t)$



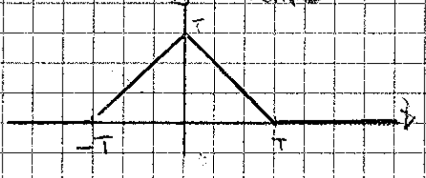
$h_{eq} = h_1(t) * h_2(t)$

① $\begin{cases} t < -T,\ t > T \\ y(t) = 0 \end{cases}$

② $\begin{cases} -T < t < 0 \\ y(t) = t + T \end{cases}$

③ $\begin{cases} 0 < t < T \\ y(t) = T - t \end{cases}$

Tempo



Frequenza

$\rightarrow \quad H_{eq} = \frac{\sin^2 \pi f T}{\pi^2 f^2}$

$TRI(t/T) \xrightarrow{F} \frac{1}{T} \frac{\sin^2 \pi f T}{\pi^2 f^2}$

• $h_1(t) = \delta(t - T)$

$h_2(t) = P_T(t)$

$\overbrace{\rightarrow h_1(t) \rightarrow h_2(t) \rightarrow}^{h_{eq}}$

tempo

$h_{eq} = \delta(t - T) * P_T(t) =$
$= P_T(t - T)$

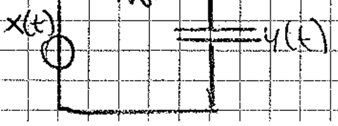
frequenza

$H_1(f) = e^{-j2\pi f T}$

$H_2(f) = \frac{\sin \pi f T}{\pi f}$

$H_{eq} = \frac{\sin \pi f T}{\pi f} e^{-j2\pi f T}$

•

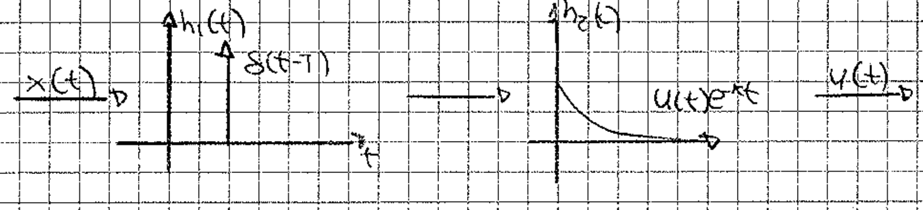


$y(t) + RC \frac{dy(t)}{dt} = x(t)$

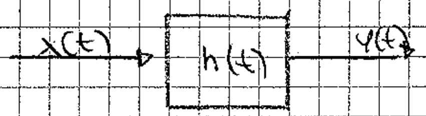
$Y(f) + RC\, j2\pi f\, Y(f) = X(f)$

$H(f) = \frac{Y(f)}{X(f)} = \frac{1}{1+RCj2\pi f} = \frac{1}{RC}\cdot\frac{1}{\frac{1}{RC}+j2\pi f} \rightsquigarrow h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$ | $u(t)e^{-at} \to \frac{1}{a+j2\pi f}$

$Y(f)(1+RCj2\pi f) = X(f)$

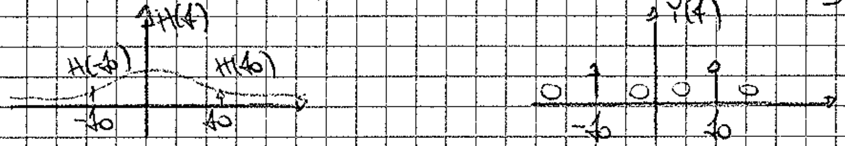


$h_{eq}(t) = u(t-T)e^{-\alpha(t-T)}$ $\quad H_{eq}(f) = \frac{1}{\alpha+j2\pi f}e^{-j2\pi f T}$



$x(t) = A\cos(2\pi f_0 t + \varphi)$

$Y(f) = H(f)\cdot X(f) = H(f)\left[\frac{1}{2}\delta(f-f_0)+\frac{1}{2}\delta(f+f_0)\right]$

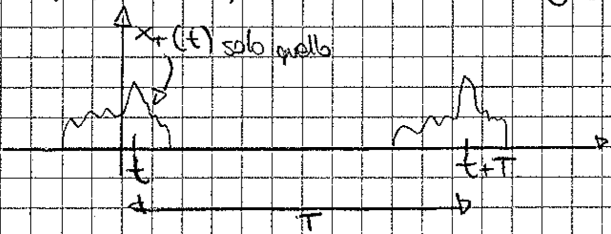


$H(f_0)\cos[2\pi f_0 t+\varphi+\varphi_+]$

SEGNALI PERIODICI

18-03-2010

$x(t+nT)=x(t)$ segnale periodico



$x(t)=\sum_{n=-\infty}^{+\infty} x_T(t-nT)$ generico segnale periodico

$x(t)=x_T(t) * \sum_{n=-\infty}^{\infty}\delta(t-nT)$

$x(t)=\frac{1}{T}\sum_{n=-\infty}^{+\infty}\mu_n e^{j2\pi\frac{n}{T}t}$ $\quad X(f)=\int_{-\infty}^{+\infty}\sum \mu_n e^{j2\pi\frac{n}{T}t}e^{-j2\pi f t}\,dt =$

$\mu_n=\int_{-T/2}^{T/2}x(t)e^{-j2\pi\frac{n}{T}t}\,dt$ $\quad =\sum\mu_n\int_{-\infty}^{+\infty}e^{j2\pi\frac{n}{T}t}e^{-j2\pi f t} =$

$=\sum_{-\infty}^{+\infty}\mu_n\int_{-\infty}^{+\infty}1\cdot e^{-j2\pi(f-\frac{n}{T})t}$

$X(f-n/T) = \delta(f-\frac{n}{T}) \rightsquigarrow X(f)=\sum_{n=-\infty}^{+\infty}\delta(f-\frac{n}{T})$

$X(f)=\frac{1}{T}\sum\mu_n\delta(f-\frac{n}{T}) = \frac{1}{T}\sum_{n=-\infty}^{+\infty}x_T(\frac{n}{T})\delta(f-\frac{n}{T})$

Dimostrazione che non capisco