giulio_coluccia@polito.it 26-02-201

NUMERI COMPLESSI

- $z \in \mathbb{C}$

$z = a + jb \qquad z_1, z_2 \in \mathbb{C} \qquad a, b \in \mathbb{R}$

$z_1 + z_2 = (a_1 + a_2) + j(b_1 + b_2)$

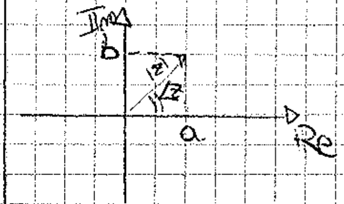
$|z| = \sqrt{a^2+b^2} \qquad |z|^2 = a^2 + b^2 = z \cdot z^* \qquad$ con $z^* = a - jb$

$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

$|z_1 + z_2| \leq |z_1| + |z_2|$

$z = |z| \, e^{j\angle z}$

$z = |z| \cos(\angle z) + j|z| \sin(\angle z)$



$\angle z = \arctan\left(\frac{b}{a}\right)$

$\angle z_1 z_2 = \angle z_1 + \angle z_2$

① $y(t) = \frac{1}{1+j2\pi t} \cdot \frac{1-j2\pi t}{1-j2\pi t} = \frac{1-j2\pi t}{1+4\pi^2 t^2} = \frac{1}{1+4\pi^2 t^2}(1 - j2\pi t)$

$|y(t)| = \frac{1}{1+4\pi^2 t^2} \cdot \sqrt{1+4\pi^2 t^2} = \frac{1}{\sqrt{1+4\pi^2 t^2}}$

$\angle y(t) = \angle 1 - \angle 1 + j2\pi t = 0 - \arctan\left(\frac{2\pi t}{1}\right)$

② $y(t) = t^j = e^{\ln(t^j)} = e^{j \ln t}$

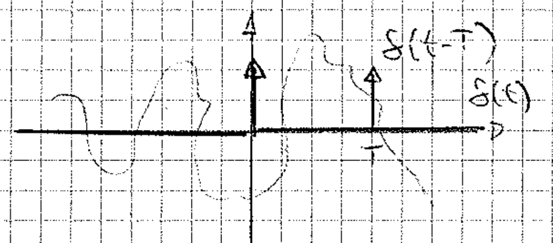
$|y(t)| = 1 \qquad \angle y(t) = \ln t$

③ Calcolare modulo e fase di $y(t) = \frac{\sin(\pi t)}{\pi t}$

DELTA DI DIRAC

- $\delta(t)$ DELTA DI DIRAC

$\int_{-\infty}^{+\infty} \delta(t)\, dt = 1$



$f(t) \cdot \delta(t) = f(0) \cdot \delta(t)$

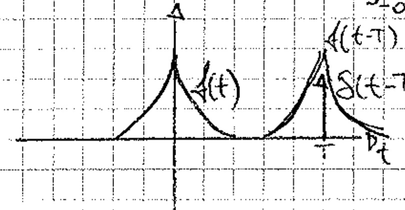
$f(t) \cdot \delta(t-T) = \underbrace{f(T)}_{\text{CAMPIONAMENTO: estrarre il valore di } f(T) \text{ in } T\text{, punto di campionamento}} \cdot \delta(t-T)$

$\int_{-\infty}^{+\infty} f(t)\, \delta(t-T)\, dt = \int_{-\infty}^{+\infty} f(T)\, \delta(t-T)\, dt = f(T) \int_{-\infty}^{+\infty} \delta(t-T)\, dt = f(T)$

PRODOTTO DI CONVOLUZIONE

- $x(t) * y(t) = y(t) * x(t) = $
$$= \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} y(\tau)\, x(t-\tau)\, d\tau$$

- $$f(t) * \delta(t-T) = \int_{-\infty}^{+\infty} \delta(\tau - T)\cdot f(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} f(t-T)\, \delta(\tau - T)\, d\tau =$$



$$= f(t-T) \int_{-\infty}^{+\infty} \delta(\tau - T)\, d\tau = f(t-T)$$

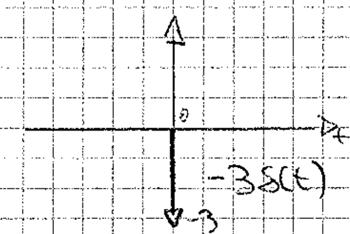
IL PRODOTTO SEMPLICE CAMPIONA

IL PRODOTTO DI CONVOLUZIONE TRASLA

(1) $y(t) = t^2 + 2t - 3$ Calcolare $y(t)\cdot \delta(t)$

$\rightarrow (t^2 + 2t - 3)\cdot \delta(t) = t^2 \cdot \delta(t) + 2t\cdot \delta(t) - 3\,\delta(t) =$

$= \cancel{t^2|_{t=0}\,\delta(t)} + \cancel{2t|_{t=0}\,\delta(t)} - 3|_{t=0}\,\delta(t) = -3\,\delta(t)$

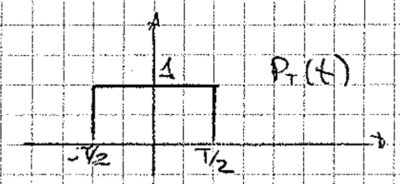


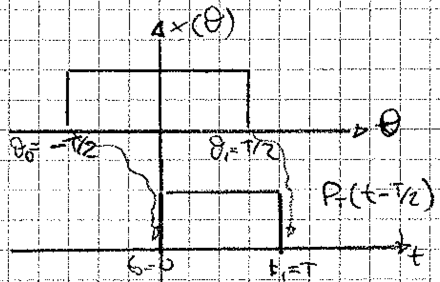
$$\int_{-\infty}^{+\infty} y(t)\,\delta(t)\, dt = \int_{-\infty}^{+\infty} -3\,\delta(t)\, dt = -3$$

Non cambia calcolare per esempio $\int_{-3}^{2} y(t)\delta(t)\, dt = \int_{-3}^{2} -3\delta(t) = -3$
(se il dominio di integrazione comprende θ o T dello $\delta(t-T)$)

PROPRIETÀ DEI SEGNALI

- $x(t) = t\, P_T(t - T/2)$

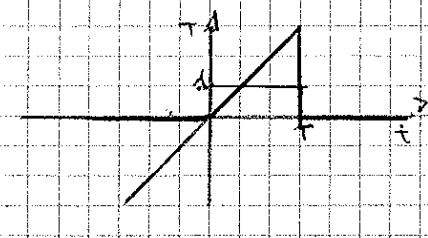




$x(t)$ $\quad 1 \quad -T/2$

$y(t) = x(\underbrace{at + b}_{\theta}) = x(\theta)$

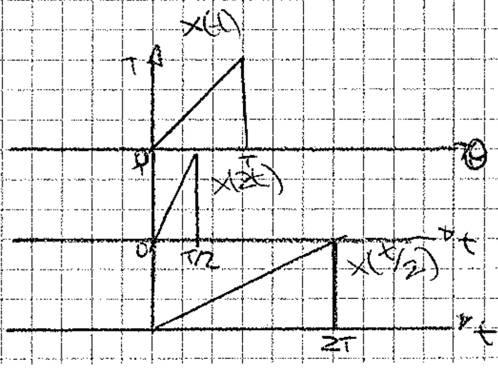
$\theta = at + b \qquad t = \dfrac{\theta - b}{a}$



$$x(t) = \begin{cases} t & 0 < t < T \\ 0 & \text{altrove} \end{cases}$$

(1) $y(t) = x(2t)$ $\qquad b = 0$, $a = 2$

$y(t) = x(t/2)$ $\qquad a = 1/2$

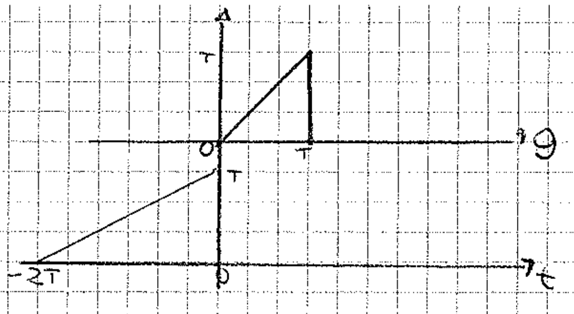


② $y(t) = x\left(\frac{t}{2}+T\right) - x(\cancel{2T})$

$\vartheta = \frac{t}{2} + T \qquad \frac{t}{2} = \vartheta - T$

$t = 2\vartheta - 2T = 2(\vartheta - T)$

$a = \frac{1}{2} \qquad b = T$



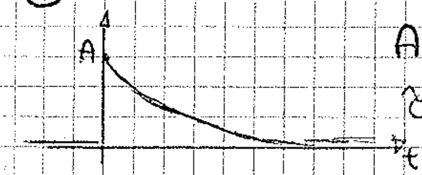
ENERGIA E POTENZA DI UN SEGNALE

$x(t) \in \mathbb{C}$

$$\mathcal{E}_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt \qquad \text{energia : è positiva}$$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \qquad \text{potenza}$$

① $x(t) = A e^{-t/\tau} \qquad t \geq 0$



$A \in \mathbb{R}$

$\tau \in \mathbb{R}^+$

$$\mathcal{E}_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} (A e^{-t/\tau})^2 dt = \int_{-\infty}^{+\infty} A^2 e^{-2t/\tau} dt = A^2 \int_{-\infty}^{+\infty} e^{-2t/\tau} dt =$$

$$= A^2\left(-\frac{\tau}{2}\right) \int_0^{+\infty} -\frac{2}{\tau} e^{-2t/\tau} dt = -\frac{A^2\tau}{2}\left[e^{-2t/\tau}\right]_0^{\infty} = -\frac{A^2\tau}{2}[0-1] = \frac{A^2\tau}{2}$$

il segnale è a ENERGIA FINITA

② $x(t) \quad | \; \mathcal{E}_x < \infty$

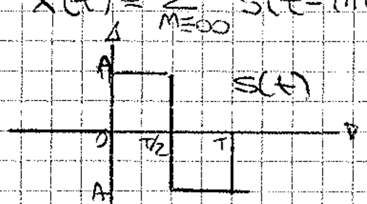
$\mathcal{E}_y = ? \qquad y(t) = x(t/2)$

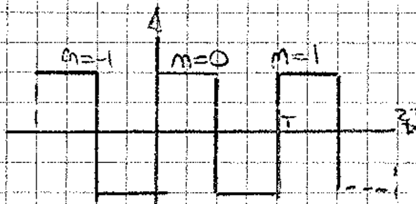
$$\mathcal{E}_y = \int_{-\infty}^{+\infty} |y(t)|^2 dt = \int_{-\infty}^{+\infty} |x(t/2)|^2 dt = \qquad \theta = t/2 \quad d\theta = \frac{1}{2} dt \quad dt = 2d\theta$$

$$= \int_{-\infty}^{+\infty} |x(\theta)|^2 2 d\theta = 2 \underbrace{\int_{-\infty}^{+\infty} |x(\theta)|^2 d\theta}_{\mathcal{E}_x} = 2\mathcal{E}_x$$

Con dilatazione di fattore a, l'energia cambia di un fattore $\frac{1}{a}$

③ $x(t) = \sum_{m=-\infty}^{+\infty} s(t - mT)$





$$\mathcal{E}_y = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} A^2 dt = \infty \qquad \text{SEGNALE A ENERGIA INFINITA}$$

$$P_y = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 dt = \lim_{T\to\infty} \frac{1}{\cancel{T}} \cdot A^2 \cancel{T} = A^2$$

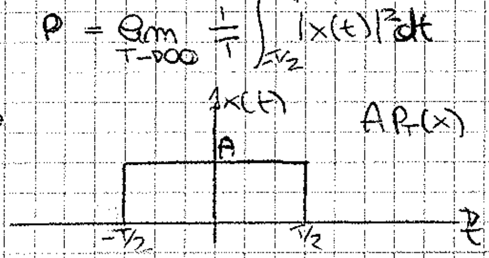
Segnale a potenza media finita

ENERGIA E POTENZA

05-03-2010

$$E=\int_{-\infty}^{+\infty}|x(t)|^2dt$$

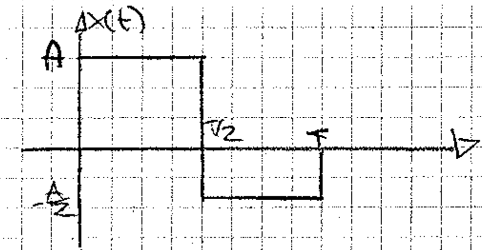
$$P=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|x(t)|^2dt$$

- 

$A\,P_T(x)$

$$\int_{-\infty}^{+\infty}|x(t)|^2dt=\int_{-T/2}^{T/2}A^2dt=A^2\big[\,t\,\big]_{-T/2}^{T/2}=A^2T$$

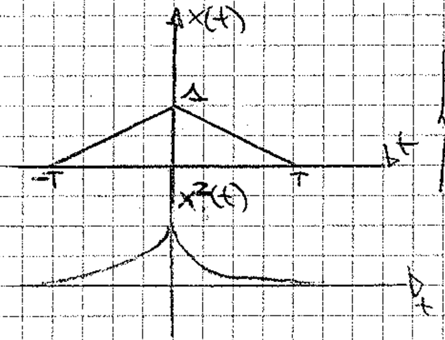
↳ il segnale porta è un segnale a energia finita

- 

$$E=\int_{-\infty}^{+\infty}|x(t)|^2dt=\int_0^{T/2}A^2dt+\int_{T/2}^{T}\frac{A^2}{4}dt=A^2\left(\frac{T}{2}\right)+\frac{A^2}{4}\left(\frac{T}{2}\right)=$$
$$=\frac{5}{8}A^2T$$

$$P=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}$$

$$x(t)=A\,p_{T/2}\left(t-\frac{T}{4}\right)+\left(-\frac{A}{2}\right)p_{T/2}\left(t-\frac{3T}{4}\right)$$

- 

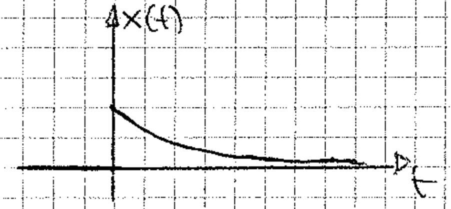
$$\begin{cases} x(t)=\frac{t}{T}+1 & -T<t<0\\ x(t)=1-\frac{t}{T} & 0<t<T\\ 0 & \text{altrove}\end{cases}$$

SEGNALE DEFINITO A TRATTI

$$\int_{-T}^{0}\left(\frac{t}{T}+1\right)^2dt+\int_0^T\left(1-\frac{t}{T}\right)^2dt=2\int_{-T}^{0}\left(\frac{t}{T}+1\right)^2dt=2\int_{-T}^{0}\left(\frac{t^2}{T^2}+1+2\frac{t}{T}\right)dt=$$

$$=2\left[t+\frac{\cancel{2}}{T}\cdot\frac{t^2}{\cancel{2}}+\frac{1}{T^2}\frac{t^3}{3}\right]_{-T}^{0}=-2\left(-\cancel{T}+\frac{\cancel{T^2}}{\cancel{T}}-\frac{T^{\cancel{3}}}{3\cancel{T^2}}\right)=\frac{2}{3}T$$

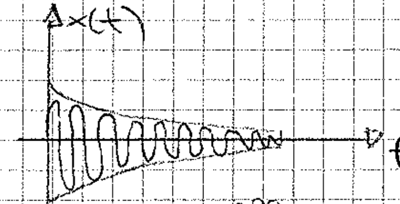
- $u(t)\,e^{-kt}=x(t)$



$$E=\int_0^{+\infty}e^{-2kt}dt=\left[\frac{e^{-2kt}}{-2k}\right]_0^{+\infty}=0-\left(-\frac{1}{2k}\right)=\frac{1}{2k}$$

segnale a energia finita

- $u(t)\,e^{-kt}\cos 2\pi f_0 t=x(t)$



$$E=\int_{-\infty}^{+\infty}\left(u(t)e^{-kt}\cos2\pi f_0t\right)^2dt=\int_0^{+\infty}e^{-2kt}\cos^2 2\pi f_0t\,dt=$$

$$=\int_0^{\infty}e^{-2kt}\left(\frac{e^{j2\pi f_0t}+e^{-j2\pi f_0t}}{2}\right)^2dt=\int_0^{\infty}e^{-2kt}\,\frac{e^{j4\pi f_0t}+e^{-j4\pi f_0t}+2}{4}\,dt=$$

$$=\frac{1}{4}\left[\int_0^{\infty}e^{-2kt}e^{j4\pi f_0t}dt+\int_0^{\infty}e^{-2kt}e^{-j4\pi f_0t}dt+\int_0^{\infty}2e^{-2kt}dt\right]=$$

$$=\frac{1}{4}\left[\int_0^{\infty}e^{-t(2k-j4\pi f_0)}dt+\int_0^{\infty}e^{-t(2k+j4\pi f_0)}dt+2\int_0^{\infty}e^{-2kt}dt\right]=$$

$$= \frac{1}{4}\left[\frac{1}{2k-j4\pi f_0}\right] + \frac{1}{4}\left[\frac{1}{2k+j4\pi f_0}\right] + \frac{1}{4}\left[\frac{2}{2k}\right] = \frac{1}{4}\,\frac{4k^2 + \cancel{j8\pi f_0 k} + 4k^2 - \cancel{j8\pi f_0 k} + 4k^2 + 16\pi^2 f_0^2}{k(4k^2 + 16\pi^2 f_0^2)} =$$

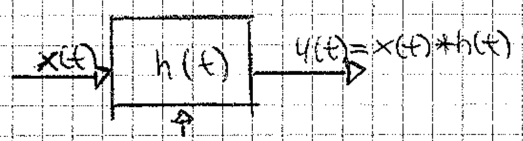
$$= \frac{1}{4}\,\frac{\cancel{4}(3k^2 + 4\pi^2 f_0^2)}{\cancel{4}k(k^2 + 4\pi^2 f_0^2)} = \frac{3k^2 + 4\pi^2 f_0^2}{4k(k^2 + 4\pi^2 f_0^2)}$$

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt$$ TRASFORMATA DI FOURIER

$$E = \int_{-\infty}^{+\infty} |X(f)|^2\, df$$ l'energia di un segnale si può calcolare anche nel dominio di Fourier

- $x(t) = u(t)\, e^{-kt}$ k, θ affinché $E_y = \frac{1}{2} E_x$

$y(t) = x(t) * \frac{\sin \theta t}{\pi t}$



FUNZIONE DI TRASFERIMENTO DEL SISTEMA

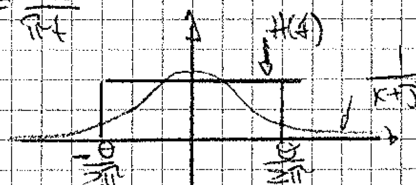
$E_x = \frac{1}{2k} \Rightarrow E_y = \frac{1}{4k}$

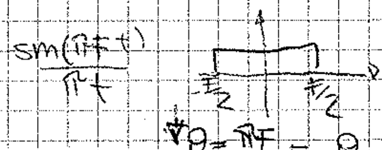
Per l'energia d'uscita conviene passare al dominio delle frequenze

$$Y(f) = X(f)\cdot H(f)$$

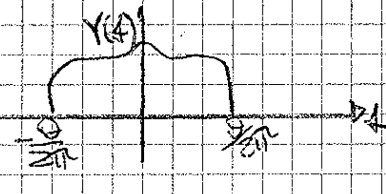
$$X(f) = \int_{-\infty}^{+\infty} u(t)\, e^{-kt} e^{-j2\pi f t}\, dt = \int_0^{\infty} e^{-t(k+j2\pi f)}\, dt = \frac{1}{k + j2\pi f}$$

$h(t) = \frac{\sin \theta t}{\pi t}$





$\theta = \pi F$ $\frac{F}{2} = \frac{\theta}{2\pi}$



$$E_y = \int_{-\theta/2\pi}^{\theta/2\pi} \left(\frac{1}{k + j2\pi f}\right)^2 df$$

$$\Downarrow$$

$$E_y = \frac{1}{2} E_x$$

$$\frac{1}{4k} = \int_{-\theta/2\pi}^{\theta/2\pi} \left(\frac{1}{k + j2\pi f}\right)^2 df = \int_{-\theta/2\pi}^{\theta/2\pi} \frac{1}{k^2 + 4\pi^2 f^2} = \frac{1}{k^2}\int_{-\theta/2\pi}^{\theta/2\pi} \frac{1}{1 + \frac{4\pi^2 f^2}{k^2}}\, df =$$

$$\frac{1}{4k} = \frac{2}{k^2}\left[\operatorname{arctg}\left(\frac{2\pi f}{k}\right)\right]_0^{\theta/2\pi} \cdot \frac{k}{2\pi}$$

$$\left[\operatorname{arctg}\frac{2\pi f}{k}\right]_0^{\theta/2\pi} = \frac{\pi}{4} \qquad \operatorname{arctg}\frac{\theta}{k} = \frac{\pi}{4} \qquad \boxed{\theta = k}$$

TRASFORMATA DI FOURIER

12-03-201

$x(t)$

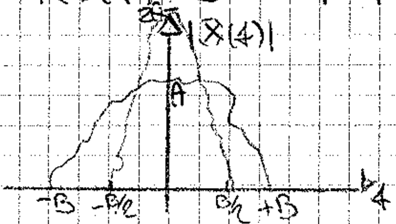
$X(f)=\int_R x(t)\,e^{-j2\pi f t}\,dt$ $\qquad F\{x(t)\}$ $\qquad$ TRASFORMATA DI FOURIER

$x(t)=\int_R X(f)\,e^{+j2\pi f t}\,df$ $\qquad F^{-1}\{X(f)\}$ $\qquad$ ANTITRASFORMATA DI FOURIER

A un comportamento nel dominio del tempo ne corrisponde uno nel dominio della frequenza opposto
↓
DUALE

① $x(t)$ $\qquad \mathcal{E}_x<\infty$

$|X(f)|=0 \qquad |f|>B$



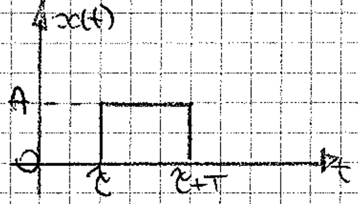
$y(t)=x(t/2) \qquad B_y=?$

$Y(f)=\int_R y(t)\,e^{-j2\pi f t}\,dt=\int_R x(t/2)\,e^{-j2\pi f t}\,dt= \qquad \theta=t/2 \quad dt=2\,d\theta$

$=\int_R x(\theta)\,e^{-j2\pi f 2\theta}\cdot 2\,d\theta = 2\int_{-\infty}^{+\infty} x(\theta)\,e^{-j2\pi \overbrace{(2f)}^{f'}\theta}\,d\theta =$

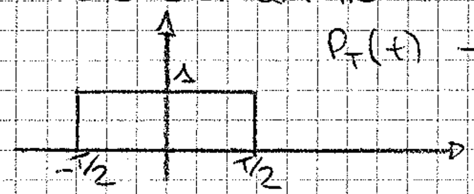
$=2X(f')=2X(2f)$

②



il centro vale $\frac{\tau+T+\tau}{2}=\frac{2\tau+T}{2}=\tau+\frac{T}{2}$

Sulle tavole abbiamo



$P_T(t) \xrightarrow{F} \frac{\sin(\pi f t)}{\pi f}$

$x(t-\theta)$
$\downarrow F$
$X(f)\,e^{-j2\pi f\theta}$

$x(t)=P_T\left(t-\left(\tau+\frac{T}{2}\right)\right)$

$X(f)=\underbrace{\frac{\sin(\pi f t)}{\pi f}}_{\text{modulo}}\ \underbrace{e^{-j2\pi f\left(\tau+\frac{T}{2}\right)}}_{\text{fase}}$

$\frac{d}{dt}x(t)$
$\downarrow F$
$j2\pi f\,X(f)$

③ $x(t)=t\,e^{-\frac{t^2}{2}}=-\frac{d}{dt}\left(e^{-\frac{t^2}{2}}\right)$

Sulle tavole abbiamo: $e^{-\alpha t^2}\xrightarrow{F}\sqrt{\frac{\pi}{\alpha}}\,e^{-\frac{\pi^2}{\alpha}f^2}$

$X(f)=-j2\pi f\sqrt{\frac{\pi}{1/2}}\,e^{-\frac{\pi^2}{1/2}f^2}=-j2\pi f\sqrt{2\pi}\,e^{-2\pi^2 f^2}$

④ $x(t) = A\left(\frac{\sin(\pi t)}{\pi t}\right)^2$

Trovare il valore di A tale che $\mathcal{E}_x = 1$

UGUAGLIANZA DI PARSEVAL $\mathcal{E}_x = \int_{\mathbb{R}} |X(f)|^2 df$

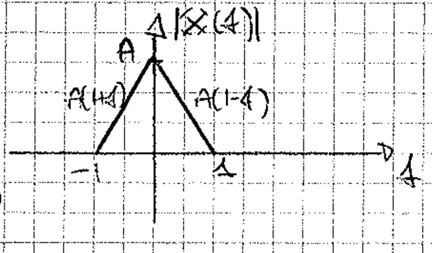
Dalle tavole abbiamo:

$$\frac{\sin^2(\pi B t)}{(\pi t)^2 B} \xrightarrow{F} t_{2B}(f) \quad \text{triangolo}$$

nel nostro caso $B = 1$

$$X(f) = A\, t_2(f) =$$

$$= \begin{cases} A(1+f) & -1<f<0 \\ A(1-f) & 0<f<1 \end{cases}$$



$$\mathcal{E}_x = \int_{-1}^{0} (A(1+f))^2 df + \int_0^1 (A(1-f))^2 df =$$

$$= A^2\left[\int_{-1}^{0}(1+2f+f^2)df + \int_0^1 (1+f^2-2f)df\right] =$$

$$= A^2\left(\left[f+f^2+\frac{f^3}{3}\right]_{-1}^{0} + \left[f+\frac{f^3}{3}-f^2\right]_0^1\right) =$$

$$= A^2\left(\cancel{1}-\cancel{1}+\frac{1}{3}+\cancel{1}+\frac{1}{3}-\cancel{1}\right) =$$

$$= A^2 \cdot \frac{2}{3}$$

$$\mathcal{E}_x = 1 = A^2\cdot\frac{2}{3} \qquad A^2 = \frac{3}{2} \qquad A = \pm\sqrt{\frac{3}{2}}$$

⑤ $x(t) = e^{-kt}u(t)$

$$y(t) = x(t) * \frac{\sin(\theta t)}{\pi t}$$

Trovare k, θ tale che $\mathcal{E}_y = \frac{1}{2}\mathcal{E}_x$

Da esercizi precedenti risulta $\mathcal{E}_x = \frac{1}{2k}$

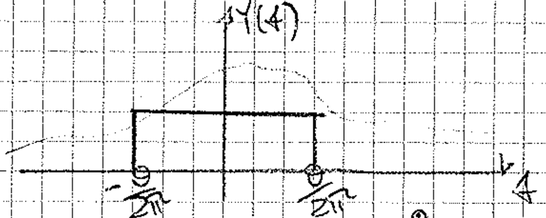
$$Y(f) = X(f)\, F\left\{\frac{\sin(\theta t)}{\pi t}\right\} \qquad X(f) = \frac{1}{k+j2\pi f} \quad \text{sulle tavole}$$

Dalle tavole: $\frac{\sin(\pi B t)}{\pi B t} \xrightarrow{F} \frac{1}{B}p_B(f)$

$$\theta = \pi B \qquad B = \frac{\theta}{\pi}$$

$$F\left\{\frac{\sin(\theta t)}{\pi t}\right\} = F\left\{\frac{\pi}{\theta}\frac{\sin(\theta t)}{\pi t}\cdot\frac{\theta}{\pi}\right\} = \frac{\theta}{\pi}\frac{1}{B}\, p_B(f) = p_{\frac{\theta}{\pi}}(f)$$

$$Y(f) = \frac{1}{k+j2\pi f}\, p_{\frac{\theta}{\pi}}(f)$$



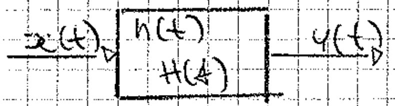
$$\mathcal{E}_y = \int_{\mathbb{R}} |Y(f)|^2 df = \int_{-\frac{\theta}{2\pi}}^{\frac{\theta}{2\pi}} \left|\frac{1}{k+j2\pi f}\right|^2 df = \int_{-\frac{\theta}{2\pi}}^{\frac{\theta}{2\pi}} \left[\frac{1}{\sqrt{k^2+4\pi^2 f^2}}\right]^2 df = \int_{-\frac{\theta}{2\pi}}^{\frac{\theta}{2\pi}} \frac{1}{k^2+4\pi^2 f^2}\, df =$$

$$= \frac{1}{4\pi^2}\int_{-\frac{\theta}{2\pi}}^{\frac{\theta}{2\pi}} \frac{1}{\frac{k^2}{4\pi^2}+f^2}\, df = \frac{1}{4\pi^2}\frac{2\pi}{k}\left[\operatorname{arctg}\left(\frac{f\,2\pi}{k}\right)\right]_{-\frac{\theta}{2\pi}}^{\frac{\theta}{2\pi}} = \ldots = \frac{1}{k\pi}\operatorname{arctg}\left(\frac{\theta}{k}\right)$$

$$\frac{1}{\pi K}\,\mathrm{arctg}\left(\frac{\theta}{K}\right)=\frac{1}{2}\cdot\frac{1}{2K} \qquad \mathrm{arctg}\left(\frac{\theta}{K}\right)=\frac{\pi}{4} \qquad \frac{\theta}{K}=\mathrm{tg}\left(\frac{\pi}{4}\right)$$

$$\frac{\theta}{K}=1 \qquad \boxed{\theta=K}$$

SISTEMI LINEARI TEMPO-INVARIANTI

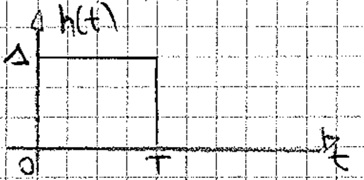


$$y(t)=x(t)*h(t)$$

$$\delta(t)*h(t)=h(t)$$

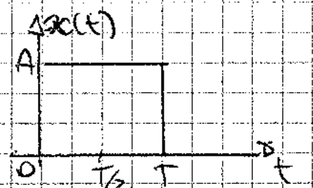
$$Y(f)=X(f)\cdot H(f)$$

① Sistema LT

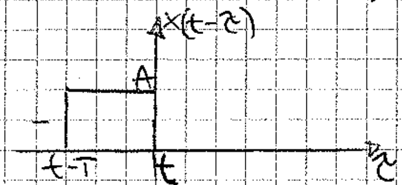


$$x(t)=A\cdot h(t)$$

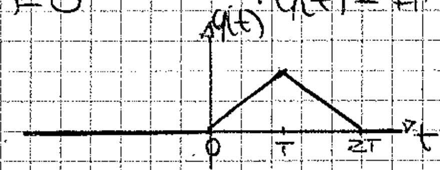
$$y(t)=?$$



$$y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}x(t-\tau)h(\tau)\,d\tau$$



① $\begin{cases} t<0,\ t>2T \\ y(t)=0 \end{cases}$ ② $\begin{cases} 0<t<T \\ y(t)=A\cdot t \end{cases}$ ③ $\begin{cases} T<t<2T \\ y(t)=A(2T-t) \end{cases}$



In frequenza:

$$y(t)=A\,h(t)*h(t) \qquad h(t)=p_T(t-T/2)$$

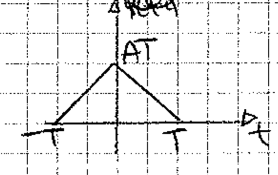
$$\downarrow F \qquad\qquad \downarrow F$$

$$Y(f)=A\,H(f)\,H(f)= \qquad H(f)=\frac{\sin(\pi f T)}{\pi f}\cdot e^{-j2\pi f T/2}$$

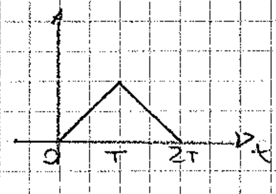
$$=A\,\frac{\sin(\pi f T)}{\pi f}\,e^{-j2\pi f T/2}\cdot\frac{\sin(\pi f T)}{\pi f}\,e^{-j2\pi f T/2}=$$

$$F^{-1}\Big\downarrow \quad =A\,\frac{\sin^2(\pi f T)}{(\pi f)^2}\,e^{-j2\pi f T}\cdot\frac{T}{T}$$

$$y(t)=AT\,t_{2T}(t-T)$$



$\xrightarrow{t-T}$



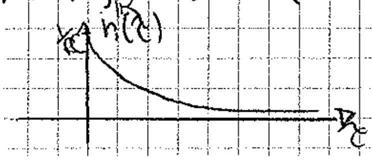
②

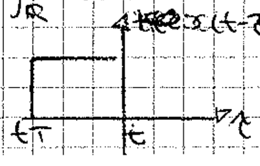
filtro passa basso

$$h(t)=\frac{1}{RC}\,e^{-t/RC}\,u(t)$$

$$x(t)=p_T(t-T/2)$$

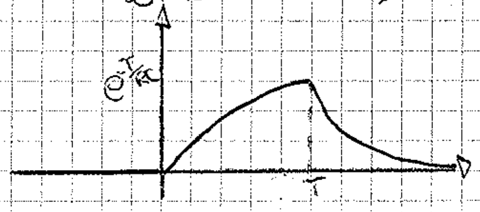
$$y(t)=\int_{\mathbb{R}}x(\tau)\,h(t-\tau)\,d\tau=\int_{\mathbb{R}}h(\tau)\,x(t-\tau)\,d\tau$$





1) $\begin{cases} t<0 \\ y(t)=0 \end{cases}$ 2) $\begin{cases} 0<t<T \\ y(t)=1-e^{-t/RC} \end{cases}$ 3) $\begin{cases} t>T \\ y(t)=e^{-t/RC}\left(e^{T/RC}-1\right) \end{cases}$

$$\int_0^t \frac{1}{RC} e^{-\tau/RC}\, d\tau = \int \ldots = 1-e^{-t/RC} \qquad \int_{t-T}^{t} e^{-\tau/RC}\cdot\frac{1}{RC}\, d\tau = \ldots = e^{-t/RC}\cdot\left(e^{T/RC}-1\right)$$

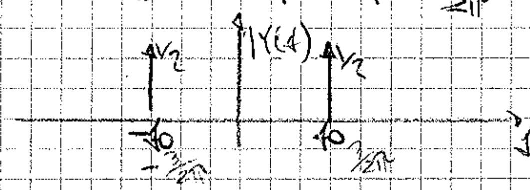


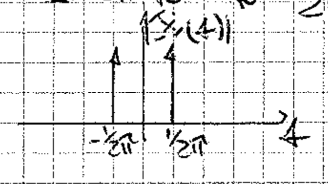
19-03-2010

(1) $y(t)=\sin(3t)$

$x(t)=\sin(t)$

$3=2\pi f_0 \quad f_0=\frac{3}{2\pi}$ $\qquad$ $1=2\pi f_0 \quad f_0=\frac{1}{2\pi}$





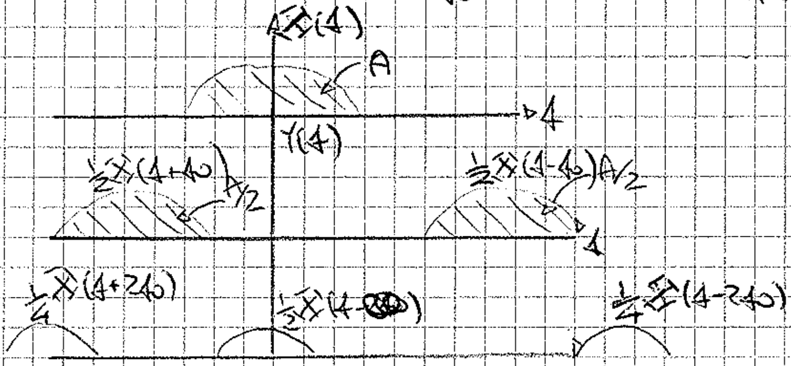
$3\, B_{\sin t} = B_{\sin 3t}$ banda

(2) $x(t) \longrightarrow \mathcal{E}_x$ $\qquad$ $X(f)$

$y(t)=x(t)\cos 2\pi f_0 t \to \mathcal{E}_y$ $\qquad$ $Y(f)=\frac{1}{2}X(f-f_0)+\frac{1}{2}X(f+f_0)$

$z(t)=x(t)\cos^2 2\pi f_0 t \to \mathcal{E}_z$ $\qquad$ $Z(f)=\frac{1}{2}Y(f-f_0)+\frac{1}{2}Y(f+f_0)=$

$=\frac{1}{4}X(f-2f_0)+\frac{1}{2}X(f)+\frac{1}{4}X(f+2f_0)$



$$\mathcal{E}_x=\int_{-\infty}^{+\infty}|x(t)|^2dt=\int_{-\infty}^{+\infty}|X(f)|^2df$$

$$\mathcal{E}_y=\int_{+f_0}\left|X(f-f_0)\tfrac{1}{2}\right|^2df+\int_{-f_0}\left|X(f+f_0)\tfrac{1}{2}\right|^2df=\frac{1}{4}\mathcal{E}_x+\frac{1}{4}\mathcal{E}_x=\frac{1}{2}\mathcal{E}_x$$

$$\mathcal{E}_z=\frac{1}{16}\mathcal{E}_x+\frac{1}{4}\mathcal{E}_x+\frac{1}{16}\mathcal{E}_x=\frac{3}{8}\mathcal{E}_x$$ ~> L'ENERGIA È UN OPERATORE NON LINEARE

(3) a) $x(t)$ supporto limitato; $X(f)$ supporto illimitato => $x(t)=\delta(t)$

b) $x(t)$ supporto illimitato; $X(f)$ supporto limitato => $x(t)=\sin(\pi F t)/\pi t$

c) $x(t)$ supporto illimitato; $X(f)$ supporto illimitato => $x(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT) \to X(f)=\sum\delta(f-\frac{n}{T})$

d) $x(t)$ supporto limitato; $X(f)$ supporto limitato => NON ESISTE NESSUN SEGNALE

④ $x(t) = \frac{t}{a + j\pi t} = t \cdot \frac{1}{a + j\pi t}$

$\frac{1}{a + j2\pi t} \xrightarrow{} e^{+af}\,u(-f)$

$e^{-at}u(t) \longrightarrow \frac{1}{a + j2\pi f}$

$x(t) \qquad X(f)$

$x(t) = t \cdot 2\,\frac{1}{2a + j2\pi t}$ dove $\frac{1}{a + j2\pi t} \xrightarrow{t = f/2} e^{+af}\,u(-f)$

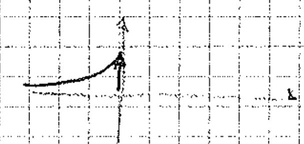
$= \underbrace{j2\pi t}_{\text{derivata}} \cdot \frac{1}{j2\pi} \cdot \frac{1}{a + j\pi t}$

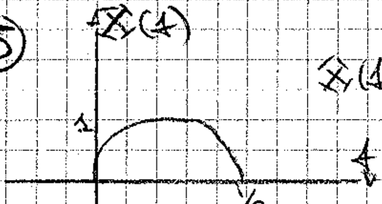
$\cancel{X(f) \to X(2f) = 2e^{2af}u(-2f)}$

$\frac{1}{a + j\pi(f/2)} \to 2e^{2af}u(-2f) = 2e^{2af}u(-f)$

$u(-2f) = u(-f)$ è un gradino

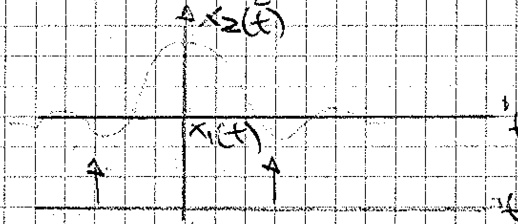
$2\,\frac{1}{j2\pi}\left(2a\,e^{2af}u(-f) - e^{2af}\delta(t)\right)$

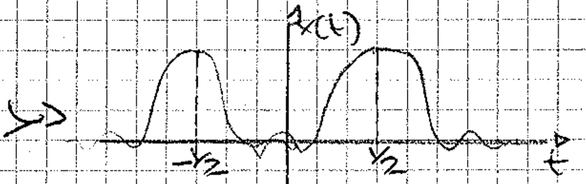


⑤ 

$X(f) = \cos 2\pi f \cdot P_{1/2}\left(f - \frac{1}{4}\right) = X_1(f) \cdot X_2(f)$

$x(t) = x_1(t) * x_2(t)$



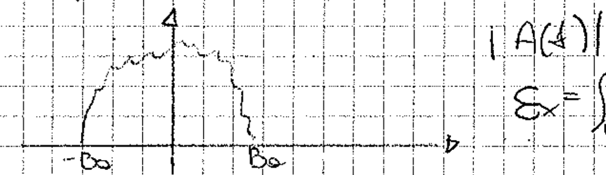


29-03-2010

① $a(t) \quad \mathcal{E}_a < \infty \quad |A(f)| = 0 \quad |f| > B_a$

$x(t) = a(t)\cos(2\pi f_0 t) + a(t)\sin(2\pi f_0 t)$

$f_0 > 2B_a \quad \mathcal{E}_x = ?$



$\mathcal{E}_x = \int_{\mathbb{R}} |X(f)|^2\,df$

$X(f) = \frac{1}{2}\left[A(f + f_0) + A(f - f_0)\right] + \frac{1}{2j}\left[-A(f + f_0) + A(f - f_0)\right]$

$|X(f)|^2 = X(f) \cdot X^*(f)$

$X^*(f) = \frac{1}{2}A^*(f + f_0) + \frac{1}{2}A^*(f - f_0) + \frac{1}{2j}A^*(f + f_0) - \frac{1}{2j}A^*(f - f_0)$

$|X(f)|^2 = \frac{1}{4}|A(f + f_0)|^2 + \cancel{\frac{1}{4j}|A(f + f_0)|^2} + \frac{1}{4}|A(f - f_0)|^2 - \cancel{\frac{1}{4j}|A(f - f_0)|^2} - \cancel{\frac{1}{4j}|A(f + f_0)|^2} +$
$+ \frac{1}{4}|A(f - f_0)|^2 + \cancel{\frac{1}{4j}|A(f - f_0)|^2} + \frac{1}{4}|A(f - f_0)|^2 = \frac{1}{2}|A(f + f_0)|^2 + \frac{1}{2}|A(f - f_0)|^2$

$$\mathcal{E}_x = \int_{\mathbb{R}} \frac{1}{2}|A(f+f_0)|^2 df + \int_{\mathbb{R}} \frac{1}{2} |A(f-f_0)|^2 df = \frac{1}{2} \underbrace{\int_{\mathbb{R}} |A(f+f_0)|^2 df}_{\mathcal{E}_a} + \frac{1}{2}\mathcal{E}_a = \mathcal{E}_a$$

$$x(t) = \sum_{n=-\infty}^{+\infty} s(t-nT) = \sum_{n=-\infty}^{+\infty} s(t) * \delta(t-nT) = s(t) * \sum_{n=-\infty}^{+\infty} \delta(t-nT)$$

$$F\{x(t)\} = S(f) \; F\left\{\sum_{n=-\infty}^{+\infty} \delta(t-nT)\right\} = S(f) \cdot \frac{1}{T}\sum_{n=-\infty}^{+\infty} \delta\left(f-\frac{n}{T}\right) =$$
$$= \frac{1}{T}\sum_{n=-\infty}^{+\infty} S\left(\frac{n}{T}\right) \delta\left(f-\frac{n}{T}\right)$$

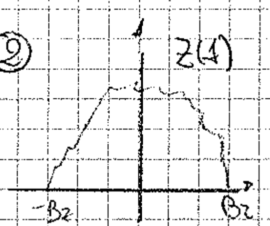
(1) $x(t) = \sum_{n=-\infty}^{+\infty} e^{-\frac{\pi}{2}(t-nT)^2}$

$s(t) = e^{-\frac{\pi}{2}t^2}$

$S(f) = \sqrt{2}\, e^{-2\pi f^2}$

$$X(f) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} \sqrt{2}\, e^{-2\pi f^2} \delta\left(f-\frac{n}{T}\right) = \frac{\sqrt{2}}{T}\sum_{n=-\infty}^{+\infty} e^{-2\pi \frac{n^2}{T^2}} \delta\left(f-\frac{n}{T}\right)$$

(2)



$y(t) = \sum_{n=0}^{N} z(t-nT)$

$Y(f) = ?$ $\qquad B_y = ?$

$Y(0) = Y(f)\big|_{f=0}$

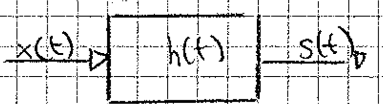
$$Y(f) = F\left\{\sum_{n=0}^{N} z(t-nT)\right\} = \sum_{n=0}^{N} F\{z(t-nT)\} = \sum_{n=0}^{N} Z(f)\, e^{-j2\pi f nT} = Z(f)\sum_{n=0}^{N} e^{-j2\pi f nT}$$

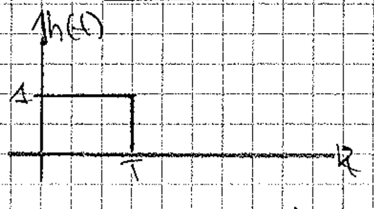
$B_y = B_z$

$Y(0) = Z(0) \sum_{n=0}^{N} 1 = (N+1)\, Z(0)$

RISPOSTA AL GRADINO

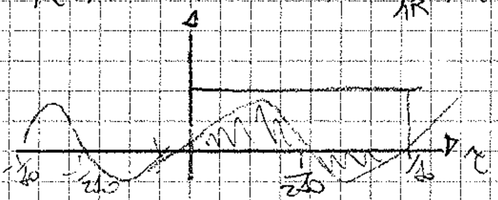
(1)





$x(t) = \sin(2\pi f_0 T)$

$$y(t) = x(t) * h(t) = \int_{\mathbb{R}} x(\tau)\, h(t-\tau)\, d\tau$$



L'uscita è zero se la porta ha ampiezza multiplo del periodo del seno

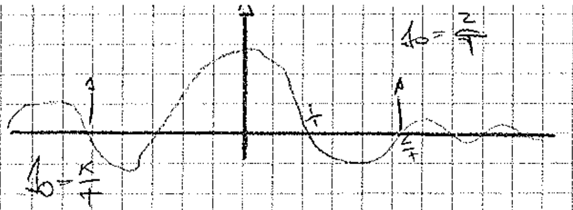
$T = k\frac{1}{f_0} \qquad f_0 = \frac{k}{T}$

- In frequenza

$Y(f) = X(f) \cdot H(f)$

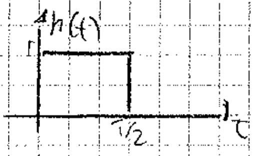
$H(f) = \frac{\sin(\pi f T)}{\pi f}\, e^{-j2\pi f \frac{T}{2}}$

$X(f) = -\frac{1}{2j}\delta(f+f_0) + \frac{1}{2j}\delta(f-f_0)$



$f_0 = \frac{2}{T}$

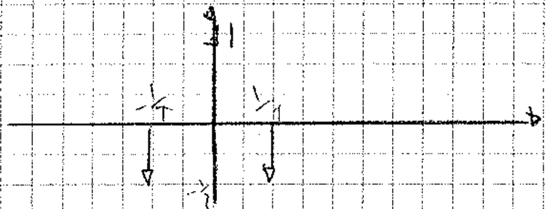
② $x(t) = 2\sin^2\left(\pi \frac{t}{T}\right)$



$H(f) = \frac{T}{2}\frac{\sin(\pi f \frac{T}{2})}{\pi f \frac{T}{2}} e^{-j2\pi f \frac{T}{4}}$

$x(t) = 2\cdot\frac{1}{2}\left(1-\cos\left(2\pi\frac{t}{T}\right)\right) = 1-\cos(2\pi t/T)$

$X(f) = \delta(f) - \frac{1}{2}\delta\left(f+\frac{1}{T}\right) - \frac{1}{2}\delta\left(f-\frac{1}{T}\right)$



$Y(f) = \frac{\sin(\pi f T/2)}{\pi f} e^{-j2\pi f T/4}\left[\delta(f) - \frac{1}{2}\delta\left(f+\frac{1}{T}\right) - \frac{1}{2}\delta\left(f-\frac{1}{T}\right)\right] =$

$= \frac{T}{2} - \frac{1}{2}\frac{\sin(\pi\frac{T}{2}\frac{1}{T})}{-\pi\frac{1}{T}}e^{j2\pi\frac{T}{4}\frac{1}{T}} - \frac{1}{2}\frac{\sin(\pi\frac{T}{2}\frac{1}{T})}{\pi\frac{1}{T}}e^{-j2\pi\frac{T}{4}\cdot\frac{1}{T}} =$

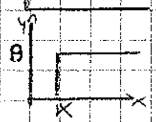
$= \frac{T}{2} + \frac{1}{2}\frac{T}{\pi}\sin(\pi/2)e^{j\pi/2} - \frac{1}{2}\frac{T}{\pi}\sin(\pi/2)e^{-j\pi/2} =$ ← è dimenticato le δ

$= \frac{T\delta(f)}{2} \frac{1}{2}\frac{T}{\pi}\left(j\delta\left(f+\frac{1}{T}\right) - j\delta\left(f-\frac{1}{T}\right)\right)\Big] =$

$= \frac{T}{2}\left[\delta(f) - \frac{j}{\pi}\delta\left(f+\frac{1}{T}\right) + \frac{j}{\pi}\delta\left(f-\frac{1}{T}\right)\right] = \frac{T}{2}\left[\delta(f) + \frac{2}{\pi}\frac{\delta(f+\frac{1}{T})-\delta(f-\frac{1}{T})}{2j}\right]$

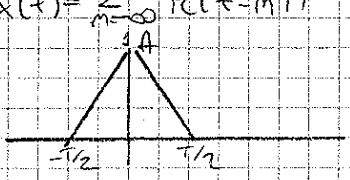
$y(t) = \frac{T}{2}\left[1+\frac{2}{\pi}\sin\left(2\pi\frac{t}{T}\right)\right]$

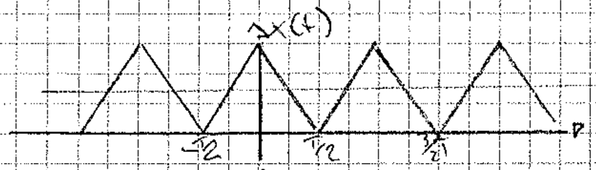
③

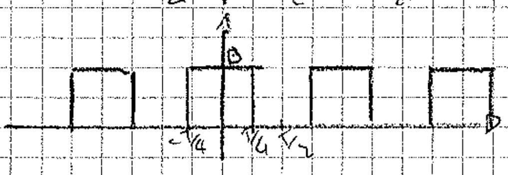


$x = A/2$

$x(t) = \sum_{n=-\infty}^{+\infty} r(t-nT)$

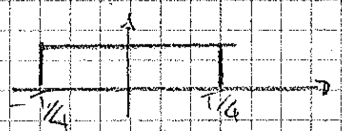






$y(t) = \sum_{n=-\infty}^{+\infty} s(t-nT)$

dove s(t)



$Y(f) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} \sin(\pi f \tfrac{T}{2})/\pi f \;\; \delta\left(f-\frac{n}{T}\right)$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

LINEARITÀ: $L\{x_1(t)\} = y_1(t)$ $\quad L\{x_2(t)\} = y_2(t)$

$$L\{a x_1 + b x_2\} = a y_1 + b y_2$$

TEMPOINVARIANZA: $\underset{y(t-T)}{L\{x(t-T)\}} = y_1(t)$

① $\cdot\; y(t) = 4 + e^{x(t)}$

$y_1(t) = 4 + e^{x_1(t)}$
$y_2(t) = 4 + e^{x_2(t)}$ $\Rightarrow y(t) = 4 + e^{a x_1(t) + b x_2(t)}$

$\neq$

$a(4 + e^{x_1(t)}) + b(4 + e^{x_2(t)})$ il sistema non è lineare

$\circ\; y(t) = 4 + e^{x(t-T)}$

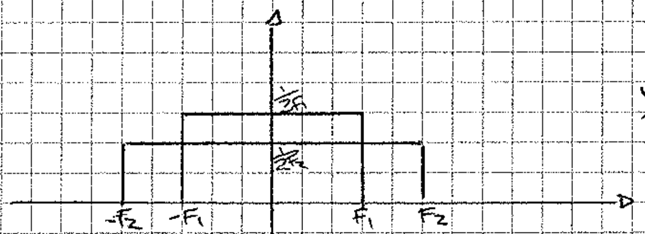
$y(t-T) = 4 + e^{x(t-T)}$ il sistema è tempo invariante

RIPASSO

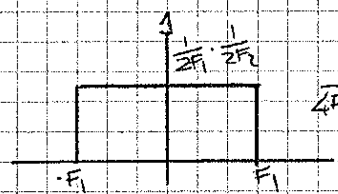
09-04-2010

① $$\frac{\sin(2\pi F_1 t)}{2\pi F_1 t} * \frac{\sin(2\pi F_2 t)}{2\pi F_2 t} = A \frac{\sin(2\pi F_A t)}{2\pi F_A t} \qquad F_1 < F_2$$

$$\frac{1}{2F_1} P_{2F_1}(f) \cdot \frac{1}{2F_2} P_{2F_2}(f) = \frac{A}{2F_A} P_{2F_A}(f)$$



$\Rightarrow$



$\frac{1}{4F_1F_2} P_{2F_1}(f)$ se $F_1 < F_2$

$$\frac{1}{4F_1F_2} P_{2\min(F_1,F_2)}(f) = \frac{A}{2F_A} P_{2F_A}(f)$$

$$\begin{cases} \cancel{2}F_A = \cancel{2}\min(F_1, F_2) \\ \frac{1}{4F_1F_2} = \frac{A}{2F_A} \end{cases} \quad \begin{cases} F_A = F_1 \\ A = \frac{F_1}{2F_1F_2} = \frac{1}{2F_2} \end{cases} \Rightarrow \begin{cases} F_A = F_{min} \\ A = \frac{1}{2F_{max}} \end{cases}$$

② $y(t) = x(t) \cdot x_s(t)$

$x(t) = e^{-t}$

$x_s(t) = \sum_{n=0}^{+\infty} \delta(t - nT)$

$$\mathcal{F}\left\{e^{-t} \cdot \sum_{n=0}^{+\infty} \delta(t-nT)\right\} = \mathcal{F}\left\{\sum_{n=0}^{+\infty} e^{-t}\delta(t-nT)\right\} =$$ CAMPIONAMENTO

$$= \mathcal{F}\left\{\sum_{n=0}^{+\infty} e^{-nT}\delta(t-nT)\right\} = \sum_{n=0}^{+\infty} \mathcal{F}\left\{e^{-nT}\delta(t-nT)\right\} =$$

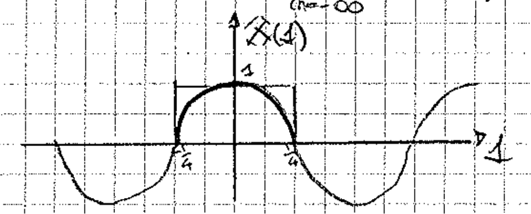
$$= \sum_{n=0}^{+\infty} e^{-nT}\,\mathcal{F}\{\delta(t-nT)\} = \sum_{n=0}^{+\infty} e^{-nT} e^{-j2\pi f nT} =$$ serie geometrica

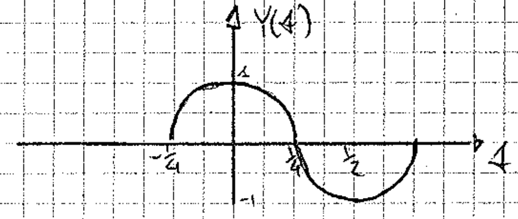
$$= \sum_{n=0}^{+\infty} e^{-nT - j2\pi f nT} = \sum_{n=0}^{+\infty} e^{n(-T - j2\pi f T)} = \sum_{n=0}^{+\infty} \Big[\overbrace{e^{-T - j2\pi f T}}^{<1}\Big]^n = \frac{1}{1 - e^{-T}e^{-j2\pi f T}}$$

③ $X(f) = \cos(2\pi f)\, P_{1/2}(f)$

$y(t) = x(t)\left[1 - e^{-j\pi t}\right]$

$z(t) = y(t) \sum_{n=-\infty}^{+\infty} \delta(t-n) = ?$





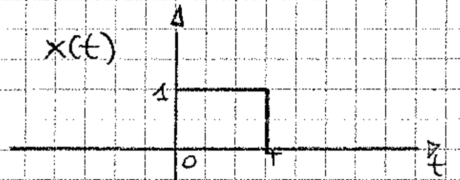
$Y(f) = X(f) - X\left(f - \frac{1}{2}\right)$

$Z(f) = Y(f) * \mathcal{F}\left\{\sum_{n=-\infty}^{+\infty} \delta(t-n)\right\} = Y(f) * \sum_{n=-\infty}^{+\infty} \delta(f-n) =$

$= \sum_{n=-\infty}^{+\infty} Y(f) * \delta(f-n) = \sum_{n=-\infty}^{+\infty} Y(f-n) = \cos(2\pi f)$ replico infinite volte $Y(f)$

$z(t) = \frac{1}{2}\delta(t+1) + \frac{1}{2}\delta(t-1)$

④ $x(t)$

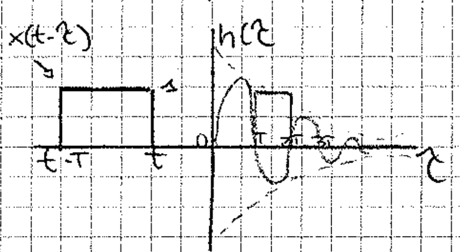


$h(t) = \sin\left(\frac{\pi t}{T}\right) e^{-at} u(t)$

$y(2T) \gtreqless 0$?

$\xrightarrow{x(t)} \boxed{h(t)} \xrightarrow{y(t)}$

$y(t) = \int_{\mathbb{R}} h(\tau)\, x(t-\tau)\, d\tau$



A noi interessa è istante a cui $t = 2T$

⇓

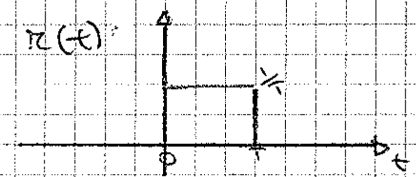
Prodotto della porta (positivo) per il seno smorzato (negativo)

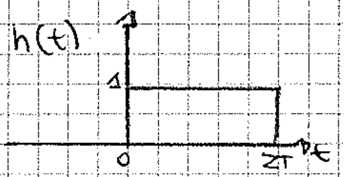
⇓

$y(2T) < 0$

⑤ $x(t) = \sum_{n=-\infty}^{+\infty} \alpha_n r(t - nT)$

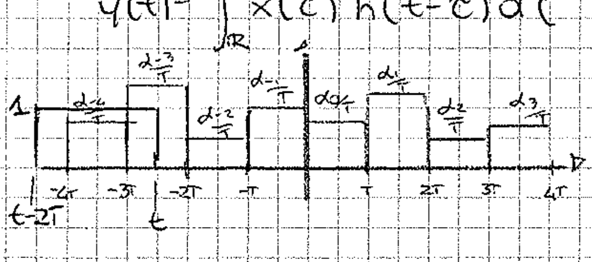
$\xrightarrow{x(t)} \boxed{h(t)} \xrightarrow{y(t)}$





$y(kT) = ?$

$y(t) = \int_{\mathbb{R}} x(\tau)\, h(t-\tau)\, d\tau$



- se ad esempio $k = 3 \rightarrow t = 3T$

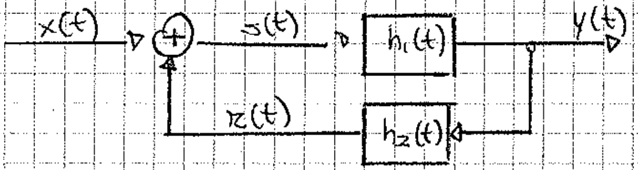
$y(3T) = T \cdot \frac{\alpha_1}{T} + T\frac{\alpha_2}{T} = \alpha_1 + \alpha_2$

- $k = 5$ $\quad y(5T) = \alpha_3 + \alpha_4$

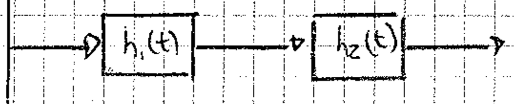
⇓

$y(kT) = \alpha_{k-2} + \alpha_{k-1}$

SISTEMI CON RETROAZIONE

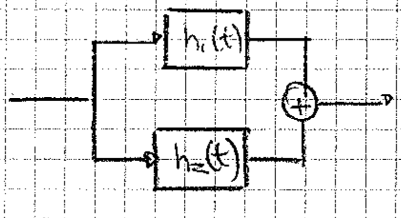


SERIE



$$h(t) = h_1(t) * h_2(t)$$
$$H(f) = H_1(f) H_2(f)$$

PARALLELO



$$h(t) = h_1(t) + h_2(t)$$
$$H(f) = H_1(f) + H_2(f)$$

$$y(t) = s(t) * h_1(t)$$
$$s(t) = x(t) + r(t)$$
$$r(t) = y(t) * h_2(t)$$
$$\Downarrow$$
$$s(t) = x(t) + y(t) * h_2(t)$$
$$y(t) = h_1(t) * [x(t) + y(t) * h_2(t)] =$$
$$= h_1(t) * x(t) + h_1(t) * y(t) * h_2(t)$$
$$Y(f) = H_1(f) \cdot X(f) + Y(f) \cdot H_1(f) \cdot H_2(f)$$
$$Y(f) - Y(f) \cdot H_1(f) \cdot H_2(f) = X(f) \cdot H_1(f)$$
$$Y(f) [1 - H_1(f) \cdot H_2(f)] = X(f) \cdot H_1(f)$$
$$\frac{Y(f)}{X(f)} = \frac{H_1(f)}{1 - H_1(f) \cdot H_2(f)}$$

① $h_1(t) = e^{-at} u(t)$ $\quad a \in \mathbb{R}_+$

$h_2(t) = \alpha\, \delta(t)$

$$\Downarrow$$

$$H_1(f) = \frac{1}{a + j2\pi f} \qquad H_2(f) = \alpha$$

risposta all'impulso

$$H(f) = \frac{\frac{1}{a+j2\pi f}}{1 - \frac{\alpha}{a + j2\pi f}} = \frac{1}{\cancel{a+j2\pi f}} \cdot \frac{\cancel{a+j2\pi f}}{a + j2\pi f - \alpha} = \frac{1}{a - \alpha + j2\pi f}$$

Verificare per quali valori di α il sistema è stabile

STABILITÀ DI UN SISTEMA LTI (lineare tempo invariante)

$$\int_{\mathbb{R}} |h(t)|\,dt < \infty$$

$$|H(f)| < +\infty \quad \forall f$$

$$a - \alpha + j2\pi f \neq 0 \qquad \alpha \neq a - j2\pi f$$ sennò $H(f)$ va a infinito e diverge

② $a(t)$: $\mathcal{E}_a < \infty$

$A(f)$ $\quad |f| > B_a$

$$x(t) = a(t)\cos(2\pi f_0 t) + a(t)\sin(2\pi f_0 t)$$

$R_x(\tau)$, $R_a(\tau)$ autocorrelazioni

$$R_x(\tau) = \mathcal{F}^{-1}\{G_x(f)\}$$

$$G_x(f) = |X(f)|^2 = \frac{1}{2}|A(f-f_0)|^2 + \frac{1}{2}|A(f+f_0)|^2$$ ← esercizio già svolto tempo prima

$$= \frac{1}{2} G_a(f-f_0) + \frac{1}{2} G_a(f+f_0)$$

$$R_x(\tau) = \frac{1}{2} R_a(\tau) e^{-j2\pi f_0 \tau} + \frac{1}{2} R_a(\tau) e^{+j2\pi f_0 \tau} = R_a(\tau)\frac{e^{j2\pi f_0 \tau} + e^{-j2\pi f_0 \tau}}{2} =$$

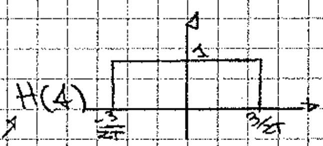
differenza di tempi

$$= R_a(\tau)\cos(2\pi f_0 \tau)$$

$$R_x(\tau)\big|_{\tau=0} = \mathcal{E}_x = \mathcal{E}_a \cdot 1$$

· FARE A CASA

③ $$x(t) = \sum_{m=-\infty}^{+\infty} e^{-\frac{|t-mT|}{T}}$$



funzione di trasferimento

$y(t) = ?$ $= 2 + \frac{4}{1+4\pi^2}\cos(2\pi t/T)$

$P_y = ?$ $= 4 + \frac{8}{(1+4\pi^2)^2}$

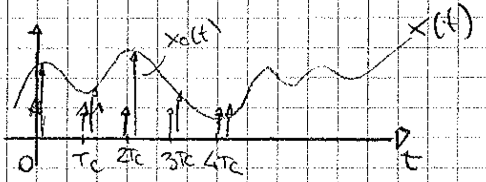
④ $$y(t) = \int_{t-1}^{t} x(\tau)\,d\tau$$ LTI? devo verificarlo : sì

trovare quale funzione trasforma $x(t)$ in $y(t)$ $\quad h(t) = ?$ $\begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{altrove} \end{cases}$

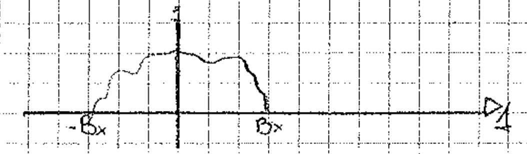
⑥ $x(t) = 1 + \cos(2\pi f_0 t)$ passa attraverso un passa-basso $RC = \frac{1}{2\pi f_0}$

$h(t) = \frac{1}{RC} e^{-t/RC} u(t)$ $\quad P_y = ?$ $\frac{5}{4}$ usare il dominio della frequenza

CAMPIONAMENTO



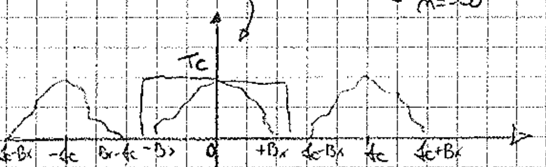
$$T_c = \frac{1}{f_c}$$



$$X(f) = 0 \qquad |f| > B_x$$

$$x_c(t) = x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_c) = \sum_{n=-\infty}^{+\infty} x(nT_c)\, \delta(t - nT_c)$$

$$X_c(f) = X(f) * \frac{1}{T_c} \sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T_c}\right) = \frac{1}{T_c} \sum_{n=-\infty}^{+\infty} X(f - nf_c)$$

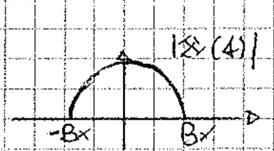


Deve essere $f_c - B_x \geq B_x \;\rightarrow\; f_c \geq 2B_x$ CRITERIO DI NYQUIST

FENOMENO DI ALIASING: se $f_c < 2B_x$, le repliche si sovrappongono e viene un segnale continuo → non può più essere ricostruito

- Un segnale a banda infinita (in frequenza) deve essere campionato a frequenza infinita
  ↳ si limita il segnale in una banda al 99%: prima di campionare si mette un filtro di aliasing (in trasmissione) → ho ridotto la frequenza di campionamento

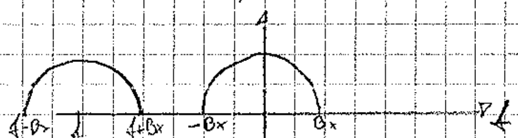
1 $x(t)$ $E_x < \infty$



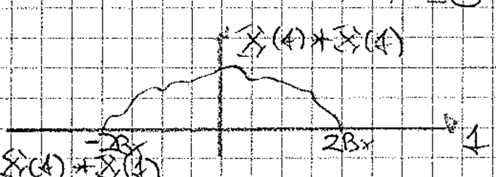
$y(t) = a\,x(t) + b\,x^3(t)$ $a, b \in \mathbb{R}$ $B_y = ?$ $f_c = ?$

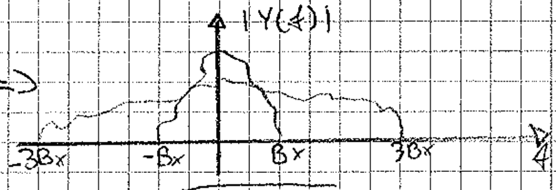
$$Y(f) = a\,X(f) + b\,X(f) * X(f) * X(f)$$

- $X(f) * X(f)$



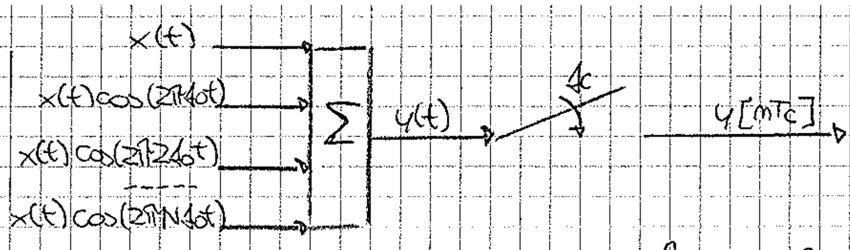
$$\begin{cases} f < -2B_x & = 0 \\ -2B_x \leq f \leq 2B_x & \neq 0 \\ f > 2B_x & = 0 \end{cases}$$



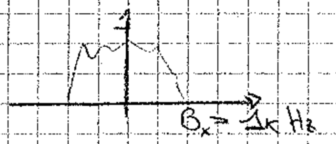


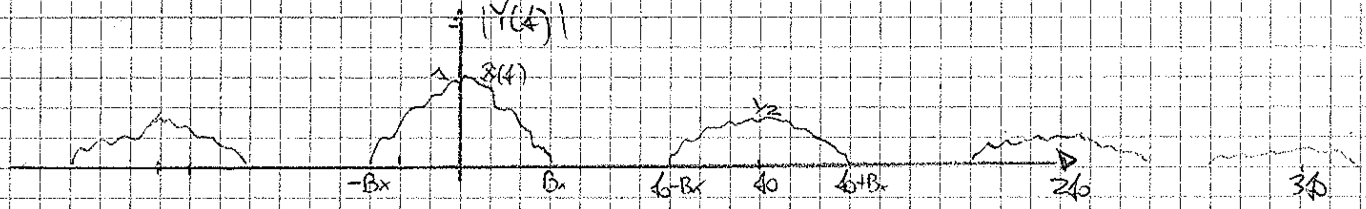
$f_c \geq 2B_y$ $\boxed{f_c \geq 6B_x}$ $\boxed{B_y = 3B_x}$

(2)



$f_c = 10$ kHz $\qquad N = ? \qquad f_0 = ?$



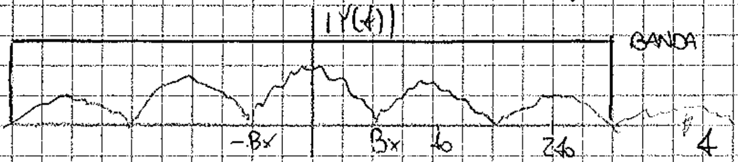


$f_0 \geq 2B_x \quad \rightarrow \quad f_0 \geq 2$ kHz

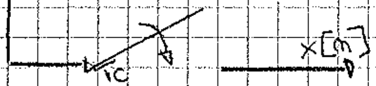
Conviene scegliere $f_0 = 2$ kHz perché rispetto il vincolo di separabilità del segnale (Nyquist)

Dato $f_c = 10$ kHz

$B_y \leq \frac{f_c}{2} = 5$ kHz $\rightsquigarrow B_y \leq 5$ kHz $\Rightarrow N = 2$



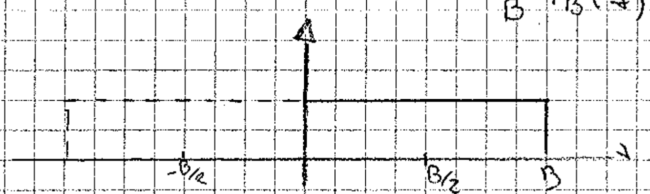
(3) $x(t) = \mathrm{sinc}(Bt)\sin(\pi Bt)$



$B \leq \frac{1}{T_c} \leq 2B$

$\frac{1}{T_c} \geq 2B_x \qquad X(f) = \mathcal{F}\left\{\frac{\sin(\pi Bt)}{\pi Bt}\right\} * \frac{1}{2j}\left[\delta\left(f - \frac{B}{2}\right) - \delta\left(f + \frac{B}{2}\right)\right] =$

$= \frac{1}{B} P_B(f) * \frac{1}{2j}\left[\delta\left(f - \frac{B}{2}\right) - \delta\left(f + \frac{B}{2}\right)\right]$

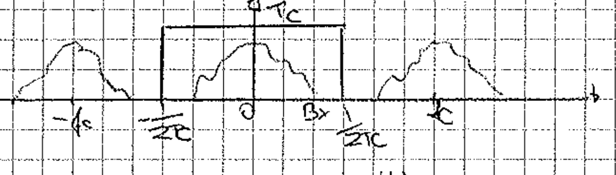


$B_x = B$

$f_c = 2B$

(4) $x(t)$

$y(t) = \sum_{n=-\infty}^{+\infty} x(nT_c)\,\delta(t - nT_c)$

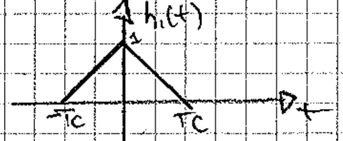


$H(f) = T_c \cdot P_{1/T_c}(f)$

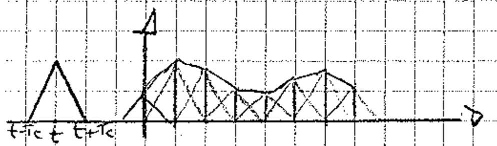
$h(t) = \frac{\sin(\pi f_c t)}{\pi f_c t}$

FILTRO NON REALIZZABILE

• $h_1(t) = \mathrm{tri}\left(\frac{t}{T_c}\right) = \begin{cases} 1 - \frac{|t|}{T_c} & |t| < T_c \\ 0 & \text{altrove} \end{cases}$

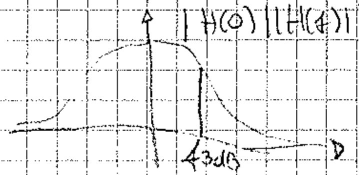


Facciamo la convoluzione del triangolo con il campione



INTERPOLAZIONE LINEARE

Ricostruzione del segnale



$$|H(f_{-3dB})|^2 = \frac{1}{2}|H(0)|^2$$

$$H(f) = \frac{\sin^2(\pi f T_c)}{T_c(\pi f)^2}$$

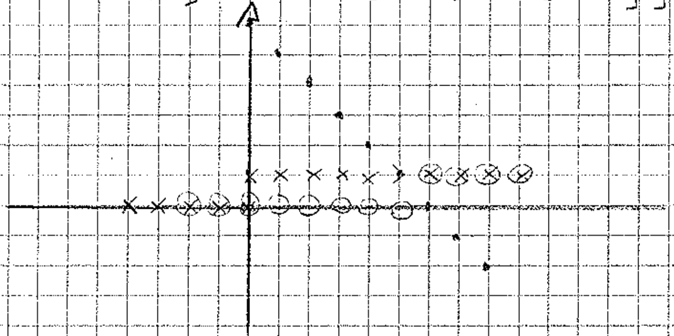
$$\left|\frac{\sin^2(\pi f_{3dB} T_c)}{T_c(\pi f_{3dB})^2}\right|^2 = \frac{1}{2}T_c^2 \qquad \frac{\sin^4(\pi f_{3dB} T_c)}{T_c^2(\pi f_{3dB})^4} = \frac{T_c^2}{2}$$

$$\sin^4(\pi f_{3dB} T_c) = \frac{(T_c \pi f_{3dB})^4}{2} \qquad \sin(\pi f_{3dB} T_c) = \frac{T_c \pi f_{3dB}}{\sqrt[4]{2}}$$

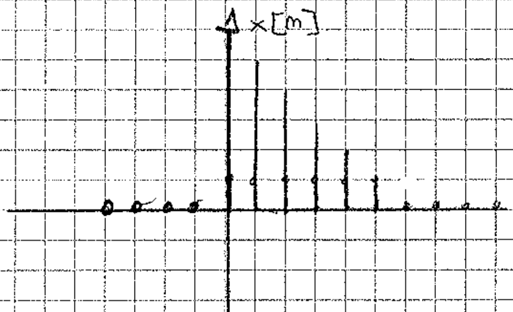
$$T_c \pi f_{3dB} \simeq 1 \qquad f_{3dB} \simeq \frac{1}{\pi T_c} \qquad \sin(x) = \frac{x}{\sqrt[4]{2}} \quad x \simeq 1$$

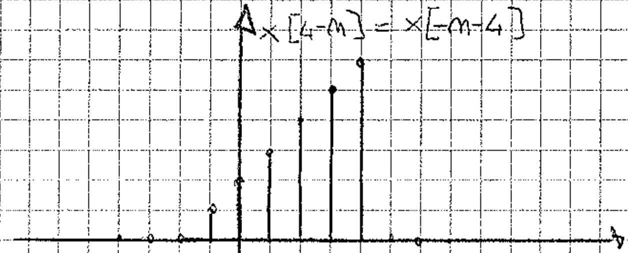
SEGNALI NUMERICI

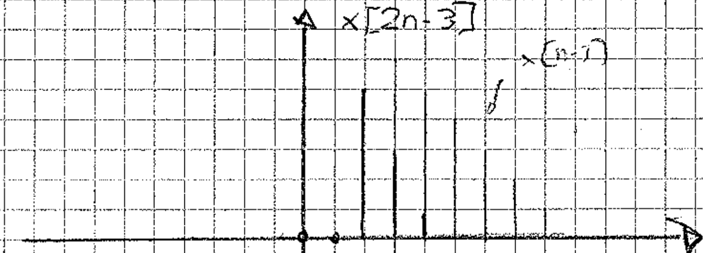
① $x[n] = (6-n)\,[u[n] - u[n-6]]$



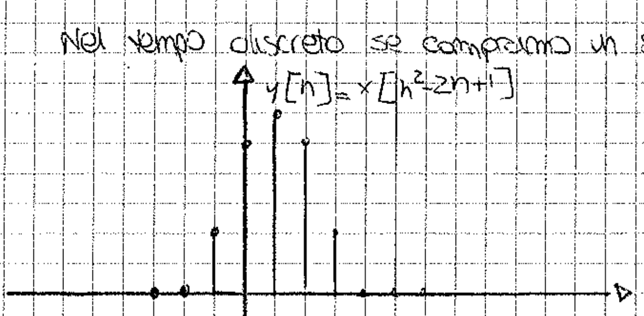
$\Rightarrow$







Nel tempo discreto se comprimiamo un segnale, perdo dei campioni ⇒ sottocampionamento



n	n^2-2n+1	$y[n]$
-3	16	0
-2	9	0
-1	4	2
0	1	5
1	0	6
2	1	5
3	4	2
4	9	0

$$E_x = \sum_{n=-\infty}^{+\infty} x^2[n] \qquad \text{ENERGIA}$$

$$x[n] = \underset{\text{pari}}{x_p[n]} + \underset{\text{dispari}}{x_d[n]}$$

ENERGIA

② $P_x = 5$ $\qquad x_p[n] = \left(\frac{1}{2}\right)^{|n|}$ $\qquad P_{x_d} = ?$

$$P_x = \sum_n x^2[n] = \sum_n (x_p[n] + x_d[n])^2 = \sum_n (x_p^2[n] + x_d^2[n] + 2x_p[n]x_d[n]) =$$

$$= \sum_n x_p^2[n] + \sum_n x_d^2[n] + 2\sum_n x_p[n]x_d[n] =$$

$$= P_{x_p} + P_{x_d} + 2\sum_n \underbrace{\cancel{x_p[n]\, x_d[n]}}_{\text{dispari}} \xrightarrow{} 0 \quad = P_{x_p} + P_{x_d}$$

è prodotto di una funzione pari per una funzione dispari è zero

$P_{x_d} = P - P_{x_p}$

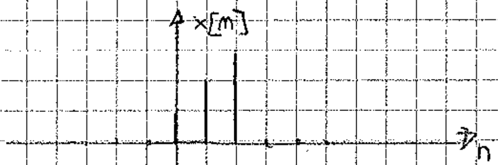
$$P_{x_p} = \sum_n \left|\left(\frac{1}{2}\right)^{|n|}\right|^2 = \sum_n \left(\frac{1}{2}\right)^{2|n|} = \sum_{n=-\infty}^{-1}\left(\frac{1}{2}\right)^{-2n} + \sum_{n=0}^{+\infty}\left(\frac{1}{2}\right)^{2n} = \sum_{n=-\infty}^{0}\left(\frac{1}{2}\right)^{-2n} - 1 + \sum_{n=0}^{+\infty}\left(\frac{1}{2}\right)^{+2n} =$$

$n = -k \quad k = -n$

$$= \sum_{k=0}^{+\infty}\left(\frac{1}{4}\right)^k - 1 + \sum_{n=0}^{+\infty}\left(\frac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}}\cdot 2 - 1 = \frac{8}{3} - 1 = \frac{5}{3}$$

$$P_{x_d} = 5 - \frac{5}{3} = \frac{10}{3}$$

③ $x[n] = \begin{cases} 1 & n=0 \\ 2 & n=1 \\ 3 & n=2 \\ 0 & \text{altrove} \end{cases}$ $\quad \rightarrow \quad x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$



$\delta[n] = u[n] - u[n-1]$

$$x[n] = u[n] - u[n-1] + 2u[n-1] - 2u[n-2] + 3u[n-2] - 3u[n-3] =$$

$$= u[n] + u[n-1] + u[n-2] - 3u[n-3]$$

$x[n] = x[n+N] \qquad N \in \mathbb{Z}$

④ $x[n] = \cos\left(\frac{\pi}{8}n\right)$

$x[n+N] = \cos\left[\frac{\pi}{8}(n+N)\right] = \cos\left[\frac{\pi}{8}n + \frac{\pi}{8}N\right]$ $\qquad \frac{\pi}{8}N = 2\pi \quad N = 16$ è periodico

④bis $x[n] = \mathrm{Re}\{e^{j\frac{\pi}{12}n}\} + \mathrm{Im}\{e^{j\frac{\pi}{18}n}\} = \cos\left(\frac{\pi}{12}n\right) + \sin\left(\frac{\pi}{18}n\right)$

$N_1 = 24 \qquad N_2 = 36$

$N = \mathrm{mcm}(24, 36) = 72$ è periodico

⑤ $x[n] = e^{j\frac{\pi}{16}n}\cos\left(\frac{\pi}{17}n\right)$ fare a casa

⑥ $x[n] = \sin\left(\frac{1}{5}n + \pi\right)$

$x[n+N] = \sin\left(\frac{1}{5}(n+N) + \pi\right) = \sin\left[\frac{1}{5}n + \pi + \frac{1}{5}N\right]$ $\qquad \frac{1}{5}N = 2\pi \quad N = 10\pi$

non è un segnale periodico perché è π è un numero irrazionale $\rightarrow$ non posso scegliere un periodo

⑦ $x[n] = \left(\frac{3}{2}\right)^n u[-n]$ $\qquad P_y = ?$

$A = \sum_n x[n]$

$y[n] = n\, x[n]$

$P = \sum_n x^2[n]$

$$A = \sum_n \left(\frac{3}{2}\right)^n u[-n] = \sum_{n=-\infty}^{0}\left(\frac{3}{2}\right)^n \overset{k=-n}{=} \sum_{k=0}^{+\infty}\left(\frac{3}{2}\right)^{-k} = \sum_{k=0}^{+\infty}\left(\frac{2}{3}\right)^k = \frac{1}{1-\frac{2}{3}} = 3$$

$$P_x = \sum_n \left[\left(\frac{3}{2}\right)^n u[-n]\right]^2 = \sum_n \left(\frac{3}{2}\right)^{2n} u[-n] = \sum_{n=-\infty}^{0} \left(\frac{3}{2}\right)^{2n} = \sum_{k=0}^{+\infty} \left(\frac{2}{3}\right)^{2k} = \sum_{k=0}^{+\infty} \left(\frac{4}{9}\right)^{k} = \frac{1}{1-4/9} = \frac{9}{5}$$

$n = -k$

$$P_y = \sum_{n=-\infty}^{+\infty} \left(n\left(\frac{3}{2}\right)^n u[-n]\right)^2 = \sum_{n=-\infty}^{0} n^2 \left(\frac{3}{2}\right)^{2n} = \sum_{k=0}^{+\infty} k^2 \left(\frac{2}{3}\right)^{2k} = \sum_{k=0}^{+\infty} k^2 \left(\frac{4}{9}\right)^{k}$$

$n = -k$

$\sum_{n=0}^{+\infty} x^n = \frac{1}{1-x}$ derivo a sinistra e destra

$\sum_{n=0}^{+\infty} n x^{n-1} = -\frac{1}{(1-x)^2}$ moltiplico e divido per x

$\sum_{n=0}^{+\infty} n x^n = -\frac{x}{(1-x)^2}$ derivo a sinistra e destra e moltiplico per x (e divido per x)

$\sum_{n=0}^{+\infty} n^2 x^n = x\frac{1+x}{(1-x)^3}$

$$P_y = \sum_{k=0}^{+\infty} k^2 \left(\frac{4}{9}\right)^k = \frac{4}{9}\,\frac{1+\frac{4}{9}}{\left(1-\frac{4}{9}\right)^3} = \frac{4}{9}\cdot\frac{13}{9}\cdot\frac{9^3}{5^3} = \frac{468}{125}$$

26-04-2010

$$x[n] = e^{j\frac{\pi}{16}n} \cos\left(\frac{n\pi}{17}\right)$$

$N_1 = 32$ $\quad N_2 = 34$

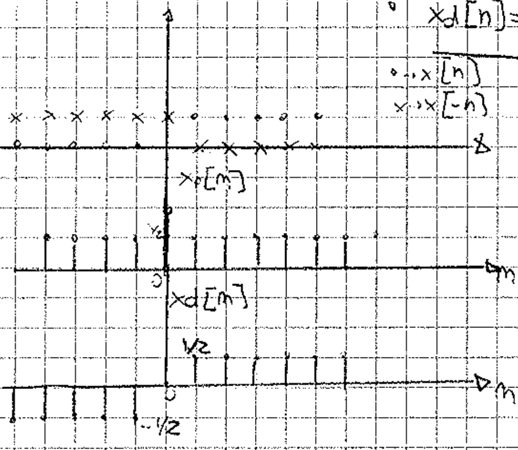
$N = \text{mcm}(32, 34) = 2^5 \cdot 17 = 544$ La funzione $x[n]$ è periodica

① $x[n] = u[n]$ $\rightarrow$ $\boxed{x_p[n] = \frac{1}{2}[x[n] + x[-n]]}$

$\boxed{x_d[n] = \frac{1}{2}[x[n] - x[-n]]}$

o → $x[n]$

x → $x[-n]$



$x_p[n] = \frac{1}{2} + \frac{1}{2}\,\delta[n]$

funzione segno

$x_d[n] = \frac{1}{2}\,\text{sgn}[n]$

② $x[n] = \alpha^n u[n]$ $\rightarrow$ $x_p[n] = \frac{1}{2}[x[n] + x[-n]] = \frac{1}{2}[\alpha^n u[n] + \alpha^{-n} u[-n]] = \begin{cases} \frac{1}{2}\alpha^n & n>0 \\ 1 & n=0 \\ \frac{1}{2}\alpha^{-n} & n<0 \end{cases}$

$= \frac{1}{2}\alpha^{|n|} + \frac{1}{2}\delta[n]$

$x_d[n] = \frac{1}{2}[\alpha^n u[n] - \alpha^{-n} u[-n]] = \begin{cases} \frac{1}{2}\alpha^n & n>0 \\ 0 & n=0 \\ -\frac{1}{2}\alpha^{-n} & n<0 \end{cases} =$

$= \frac{1}{2}\alpha^{|n|}\,\text{sgn}[n]$

③ $x[n] = 0$, $n < 0$ è possibile trovare $x[n]$ solo in funzione della sua parte dispari

$x[n] = ?$ $\quad x_p = (0.9)^{|n|}$

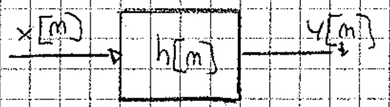
$n > 0$ $\quad x[-n] = 0$

$x_p[n] = \frac{1}{2}x[n]$ $\quad x[n] = 2x_p[n]$

$x[n] = x_p[n] + x_d[n]$ $\quad x[0] = x_p[0] + 0$

$$x[n]=\begin{cases}0 & n<0\\ 1 & n=0\\ 2(0.9)^n & n>0\end{cases} = 2(0.9)^n\,u[n-1]+\delta[n] = 2\cdot(0.9)^n\,u[n]-\delta[n]$$

CONVOLUZIONE

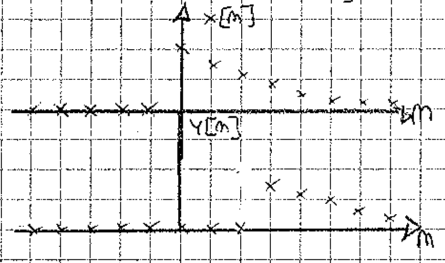


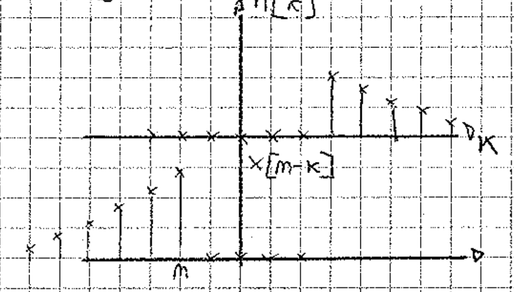
$$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{+\infty}x[n-k]\,h[k]=\sum x[k]\,h[n-k]$$

(1) $x[n]=\left(\frac{1}{6}\right)^{n-6}u[n]$

$h[n]=\left(\frac{1}{3}\right)^{n}u[n-3]$

$\rightarrow\ y[n]=\sum_{k=-\infty}^{+\infty}h[k]\,x[n-k]$





$n<3 \qquad y[n]=0$

$n\geq 3 \qquad y[n]=\sum_{k=-\infty}^{+\infty}\left(\frac{1}{3}\right)^k \cancel{u[k-3]}\left(\frac{1}{6}\right)^{n-k-6}\cancel{u[n-k]}=$

$\sum_{k=0}^{N}x^k=\frac{1-x^{N+1}}{1-x}$

$=\sum_{k=3}^{n}\left(\frac{1}{3}\right)^k\left(\frac{1}{6}\right)^{n-k-6}=\sum_{k=3}^{n}\left(\frac{1}{3}\right)^k\left(\frac{1}{6}\right)^{-6}\left(\frac{1}{6}\right)^{n}\left(\frac{1}{6}\right)^{-k}=$

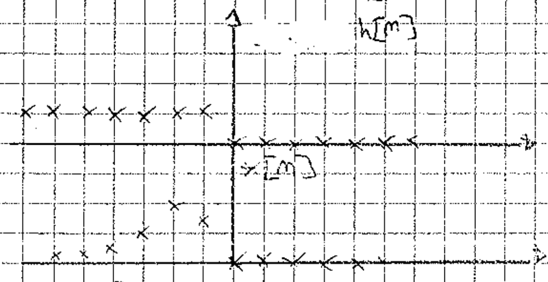
$=6^6\left(\frac{1}{6}\right)^n\sum_{k=3}^{n}\left(\frac{1}{3}\right)^k\left(\frac{1}{6}\right)^{-k}=6^6\left(\frac{1}{6}\right)^n\sum_{k=3}^{n}2^k \qquad m=k-3,\ k=m+3$

$=6^6\left(\frac{1}{6}\right)^n\sum_{m=0}^{n-3}2^{m+3}=6^6\cdot 2^3\left(\frac{1}{6}\right)^n\sum_{m=0}^{n-3}2^m=$

$=6^6 2^3\left(\frac{1}{6}\right)^n\frac{1-2^{n-2}}{-1}=6^6 2^3\left(\frac{1}{6}\right)^n\left(2^{n-2}-1\right)=2\cdot 6^6\left(\frac{1}{3}\right)^n\left[1-4\left(\frac{1}{2}\right)^n\right]$

(2) $h[n]=u[-n+1]$

$x[n]=-n3^n u[-n]$



$y[n]=\sum_{k=-\infty}^{+\infty}x[k]\,h[n-k]$

$y[n]=\sum_{k=n+1}^{-1}-k3^k\cdot 1=\sum_{k=n+1}^{0}-k3^k=\sum_{m=0}^{-n-1}m\,3^{-m}$

$\sum_{k=0}^{N-1}k\,x^k=\frac{(N-1)x^{N+1}-Nx^N+x}{(1-x)^2}$

$y[n]=\sum_{m=0}^{-n-1}m\left(\frac{1}{3}\right)^m=\frac{(-n-1)\left(\frac{1}{3}\right)^{-n+1}+n\left(\frac{1}{3}\right)^n+\frac{1}{3}}{\left(1-\frac{1}{3}\right)^2}=\ldots=$

$=\frac{3}{4}+\frac{3}{4}\left(\frac{1}{3}\right)^{-n}(2n-1)$

TRASFORMATE Z

1) $x[n] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n]$

$X(z) = \sum_{n=-\infty}^{+\infty} a^n z^{-n} = \frac{1}{1-az^{-1}} \quad |z|>|a|$

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n} = \sum_{n=0}^{+\infty} \left(\frac{1}{3}\right)^n z^{-n} - \sum_{n=0}^{+\infty} \left(\frac{1}{4}\right)^n z^{-n} =$$

$$= \sum_{n=0}^{+\infty} (3z)^{-n} - \sum_{n=0}^{+\infty} (4z)^{-n} = \frac{1}{1-\frac{1}{3z}} - \frac{1}{1-\frac{1}{4z}} \qquad |4z|>1 \quad |3z|>1$$

$$= \frac{3z}{3z-1} - \frac{4z}{4z-1} \qquad |z| > \frac{1}{3} \qquad \frac{12z^2-3z-12z^2+4z}{(3z-1)(4z-1)} = \frac{z}{(3z-1)(4z-1)}$$

2) $x[n] = 2^n u[-n]$

$$X(z) = \sum_{n=-\infty}^{0} 2^n z^{-n} \overset{m=-n}{=} \sum_{m=0}^{+\infty} 2^{-m} z^m = \sum_{m=0}^{+\infty} \left(\frac{z}{2}\right)^m = \frac{1}{1-z/2} = \frac{2}{2-z} \qquad |z|<2$$

3) $x[n] = 2^n u[-n] + \left(\frac{1}{3}\right)^n u[n]$

$$X(z) = \sum_{n=-\infty}^{0} 2^n z^{-n} + \sum_{n=0}^{+\infty} \left(\frac{1}{3}\right)^n z^{-n} = \sum_{m=0}^{+\infty} \left(\frac{z}{2}\right)^m + \sum \left(\frac{1}{3z}\right)^n =$$

$$= \frac{1}{1-z/2} + \frac{1}{1-\frac{1}{3z}} = \frac{2}{2-z} + \frac{3z}{3z-1} \qquad |z|<2 \quad |z|>\frac{1}{3}$$

4) $x[n] = \left(-\frac{1}{3}\right)^n u[-n]$

$$X(z) = \sum_{m=0}^{+\infty} \left(-\frac{1}{3}\right)^{-m} z^m = \sum_{m=0}^{+\infty} \left(-\frac{z}{3}\right)^m z^m$$

?

CIAO DENNY!

5) $x[n] = \left(\frac{1}{6}\right)^{n-1} u[n]$

$$X(z) = \sum_{n=0}^{+\infty} \left(\frac{1}{6}\right)^{n-1} z^{-n} = 6 \frac{1}{1-\frac{1}{6z}} \qquad |z| > \frac{1}{6}$$

6) $x[n] = \left(\frac{1}{3}\right)^n u[n-3] = 1 \cdot \frac{1}{3} \cdot \frac{1}{9} \left(\frac{1}{3}\right)^{n-3} u[n-3] = \frac{1}{27}$

$$X(z) = \frac{1}{27} \frac{z^{-3}}{1-\frac{1}{3z}} \qquad |z| > \frac{1}{3}$$

oppure $X(z) = Z\left\{\left(\frac{1}{3}\right)^n u[n]\right\} - \frac{1}{9}z^{-2} - \frac{1}{3}z^{-1} - 1$

ESERCIZIO:

$y[n] = x[n] - x[n-4]$

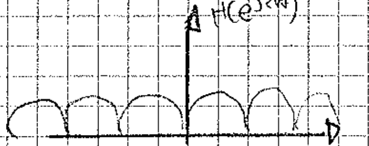
$x[n] = \sin\frac{\pi n}{2} + \sin\frac{\pi n}{4}$

Proviamo a scrivere la $h[n] = \delta[n] - \delta[n-4]$

$$H(z) = 1 - z^{-4}$$

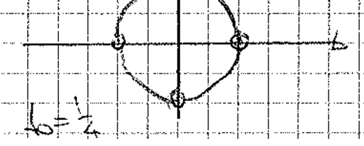
$$\sin\frac{\pi n}{2} * \delta[n-4] = \sin\frac{\pi(n-4)}{2}$$

$$H(e^{j2\pi f}) = 1 - e^{-j8\pi f} = j\, e^{-j4\pi f} \frac{e^{j4\pi f} - e^{-j4\pi f}}{2j}$$

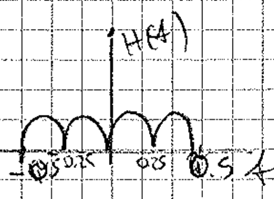


$$H(z) = (1-z^{-1})(1+z^{-1})(1+z^{-2}) = (1-z^{-2})(1+z^{-2})$$

$z_0 = 1$
$z_0 = -1$
$z_{3/4} = \pm j$



è un FILTRO NOTCH



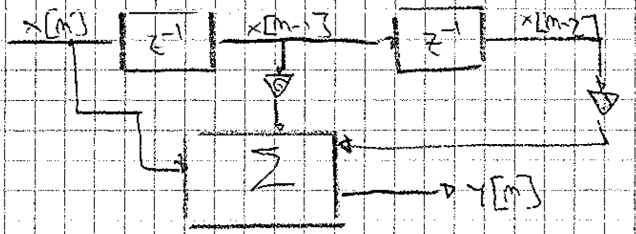
$\sin\frac{\pi}{2}n = \sin 2\pi f_0 n \qquad f_0 = \frac{1}{4}$

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n} \qquad \mathcal{Z}\{x[n-N]\} = X(z)\, z^{-N}$$

① $h[n] = \delta[n] + 6\delta[n-1] + 3\delta[n-2]$

$H(z) = 1 + 6 \cdot 1 z^{-1} + 3 z^{-2} = \dfrac{1+6z^{-1}+3z^{-2}}{1}$ filtro FIR

$y[n] = x[n] + 6x[n-1] + 3x[n-2]$



② $h[n] = \left(\frac{1}{3}\right)^{n-2} u[n-2]$

$h'[n] = h[n+2] = \left(\frac{1}{3}\right)^{n+2+2} u[n+2-2] = \left(\frac{1}{3}\right)^{n+4} u[n] = \left(\frac{1}{3}\right)^4 \left(\frac{1}{3}\right)^n u[n]$

$H'(z) = \left(\frac{1}{3}\right)^4 \dfrac{1}{1-\frac{1}{3}z^{-1}} = \left(\frac{1}{3}\right)^4 \dfrac{3z}{3z-1}$

$h[n] = h'[n-2]$

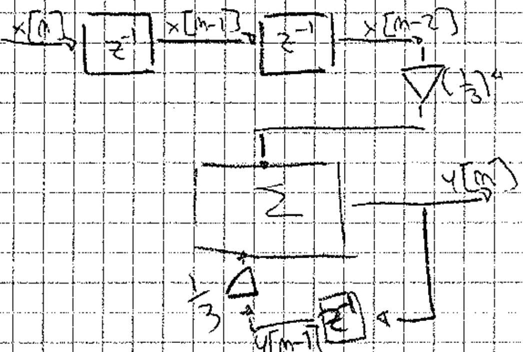
$H(z) = H'(z)\, z^{-2} = \left(\frac{1}{3}\right)^4 \dfrac{z^{-2}}{1-\frac{1}{3}z^{-1}}$

$H(z) = \left(\frac{1}{3}\right)^4 \dfrac{z^{-2}}{1-\frac{1}{3}z^{-1}} = \dfrac{Y(z)}{X(z)} \qquad Y(z)\left[1-\frac{1}{3}z^{-1}\right] = \left(\frac{1}{3}\right)^4 X(z) z^{-2}$

$Y(z) - \frac{1}{3}Y(z)z^{-1} = \left(\frac{1}{3}\right)^4 X(z) z^{-2}$

$y[n] - \frac{1}{3}y[n-1] = \left(\frac{1}{3}\right)^4 x[n-2]$

$y[n] = \left(\frac{1}{3}\right)^4 x[n-2] + \frac{1}{3} y[n-1]$

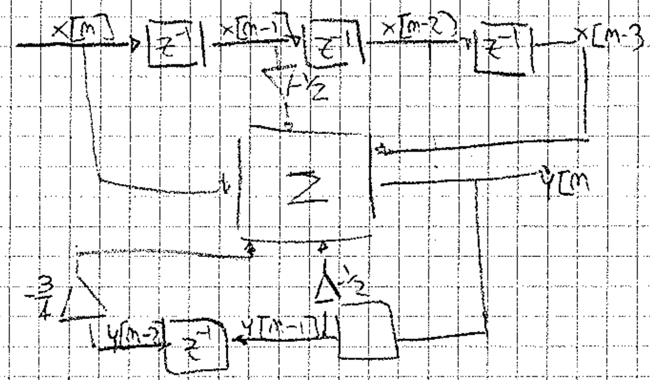


③ $H(z) = \dfrac{1-\frac{1}{2}z^{-1}+z^{-3}}{1+\frac{1}{2}z^{-1}+\frac{3}{4}z^{-2}} = \dfrac{Y(z)}{X(z)} \qquad Y(z)\left[1+\frac{1}{2}z^{-1}+\frac{3}{4}z^{-2}\right] = X(z)\left[1-\frac{1}{2}z^{-1}+z^{-3}\right]$

$Y(z) + \frac{1}{2}Y(z)z^{-1} + \frac{3}{4}Y(z)z^{-2} = X(z) - \frac{1}{2}X(z)z^{-1} + X(z)z^{-3}$

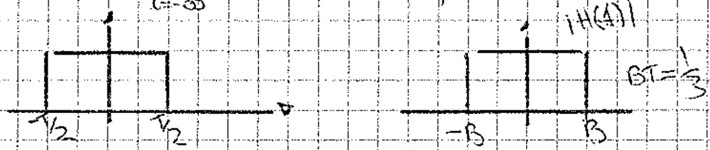
$y[n] + \frac{1}{2}y[n-1] + \frac{3}{4}y[n-2] = x[n] - \frac{1}{2}x[n-1] + x[n-3]$

$y[n] = x[n] - \frac{1}{2}x[n-1] + x[n-3] - \frac{1}{2}y[n-1] - \frac{3}{4}y[n-2]$

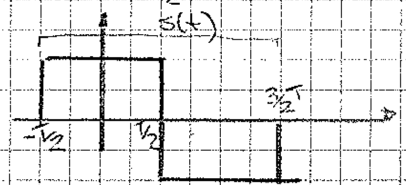


• ESERCIZIO SULL'ANALOGICO

$$x(t)=\sum_{i=-\infty}^{+\infty}(-1)^i\, p_T(t-iT)$$



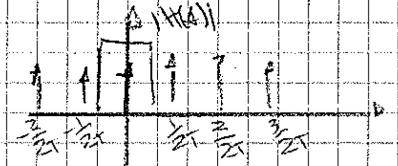
$BT=\frac{1}{3}$



periodo $T'=2T$

$\frac{1}{T'}=f_0 \qquad B=\frac{1}{3T}$

$$X(f)=f_0\sum S(f)\,\delta\!\left(f-\frac{k}{2T}\right)$$



$$S(f)=\mathrm{sinc}(f)-\mathrm{sinc}(f)\,e^{-j2\pi fT}$$

$$S(f)\Big|_{f=0}=\mathrm{sinc}(f=0)-\mathrm{sinc}(f=0)=0$$

14-05-2010

• $y[n]=x[n]-\frac{1}{4}x[n-1]+y[n-1]-\frac{1}{4}y[n-2]$

a) $h[n]=?$ b) $y[n]$, $x[n]=\left(\frac{1}{4}\right)^n u[n]$

$\frac{1}{(1-\alpha z^{-1})^2}$ $|z|>|\alpha|$ $(n+1)\alpha^n u[n]$

$y[n]-y[n-1]+\frac{1}{4}y[n-2]=x[n]-\frac{1}{4}x[n-1]$

$Y(z)-Y(z)z^{-1}+\frac{1}{4}Y(z)z^{-2}=X(z)-\frac{1}{4}X(z)z^{-1}$

$Y(z)\left[1-z^{-1}+\frac{1}{4}z^{-2}\right]=X(z)\left[1-\frac{1}{4}z^{-1}\right]$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1-\frac{1}{4}z^{-1}}{1-z^{-1}+\frac{1}{4}z^{-2}}=\frac{1-\frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})^2}=\frac{1}{(1-\frac{1}{2}z^{-1})^2}-\frac{\frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})^2}$$

$$1-z^{-1}+\frac{1}{4}z^{-2}=\left(1-\frac{1}{2}z^{-1}\right)^2$$

$p_1,\ p_2=\frac{1}{2} \qquad |p_i|=\frac{1}{2} \qquad |p_1|=|p_2|<1$

$$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}\longrightarrow n\,\alpha^n u[n] \qquad |z|>|\alpha|$$

$$h[n]=(n+1)\left(\frac{1}{2}\right)^n u[n]-\frac{1}{2}n\left(\frac{1}{2}\right)^n u[n]=\left[n\left(\frac{1}{2}\right)^n+\left(\frac{1}{2}\right)^n-\frac{1}{2}n\left(\frac{1}{2}\right)^n\right]u[n]$$

$\left[\frac{1}{2}n+1\right]\left(\frac{1}{2}\right)^n u[n]$

b) $Y(z) = H(z)\, X(z)$

$X(z) = \dfrac{1}{1-\frac{1}{4}z^{-1}}$

$H(z) = \dfrac{1-\frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})^2} \Rightarrow Y(z) = \dfrac{1}{1-\frac{1}{4}z^{-1}} \cdot \dfrac{1-\frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})^2} = \dfrac{1}{(1-\frac{1}{2}z^{-1})^2}$

$y[n] = (n+1)\left(\frac{1}{2}\right)^n u[n]$

- $x[n] = \left(\frac{1}{6}\right)^{n-6} u[n]$

$h[n] = \left(\frac{1}{3}\right)^n u[n-3]$

$x[n] \rightarrow X(z) \qquad h[n] \rightarrow H(z) \qquad Y(z) = X(z)H(z) \rightarrow y[n]$

$y[n] = (n+1)\left(\frac{1}{2}\right)^n u[n]$

$h'[n] = h[n+3] = \left(\frac{1}{3}\right)^{n+3} u[n] = \left(\frac{1}{3}\right)^3 \left(\frac{1}{3}\right)^n u[n]$

$H'(z) = \left(\frac{1}{3}\right)^3 \dfrac{1}{1-\frac{1}{3}z^{-1}} \qquad h[n] = h'[n-3]$

$H(z) = \left(\frac{1}{3}\right)^3 \dfrac{z^{-3}}{1-\frac{1}{3}z^{-1}}$

$Y(z) = 6^6 \dfrac{1}{1-\frac{1}{6}z^{-1}} \cdot \left(\frac{1}{3}\right)^3 \dfrac{z^{-3}}{(1-\frac{1}{3}z^{-1})} = 6^6 \left(\frac{1}{3}\right)^3 z^{-3} \dfrac{1}{(1-\frac{1}{6}z^{-1})(1-\frac{1}{3}z^{-1})}$

$$\frac{1}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{6}z^{-1})} = \frac{A}{(1-\frac{1}{6}z^{-1})} + \frac{B}{(1-\frac{1}{3}z^{-1})} = \frac{-1}{1-\frac{1}{6}z^{-1}} + \frac{2}{1-\frac{1}{3}z^{-1}}$$

$A = \dfrac{1}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{6}z^{-1})}(1-\frac{1}{6}z^{-1})\Big|_{z=\frac{1}{6}} = \dfrac{1}{1-\frac{1}{3}\cdot 6} = -1 \qquad B = \dfrac{1}{1-\frac{1}{6}z^{-1}}\Big|_{z=\frac{1}{3}} = \dfrac{1}{1-\frac{1}{6}3} = 2$

$Y(z) = 6^6 \left(\frac{1}{3}\right)^3 z^{-3} \left[\dfrac{2}{1-\frac{1}{3}z^{-1}} - \dfrac{1}{1-\frac{1}{6}z^{-1}}\right]$

$y[n] = 6^6 \left(\frac{1}{3}\right)^3 \left[2\left(\frac{1}{3}\right)^{n-3} u[n-3] - \left(\frac{1}{6}\right)^{n-3} u[n-3]\right] =$

$= 6^6 \left(\frac{1}{3}\right)^3 \left[2 \cdot 3^3 \left(\frac{1}{3}\right)^n - 6^3 \left(\frac{1}{6}\right)^n\right] u[n-3]$

$\left(\frac{1}{2}\right)^n \left(\frac{1}{3}\right)^n$

- $x[n] = \left(\frac{9}{10}\right)^n u[n]$

$h[n] = n\, u[n]$

$y[n] = ?$

$Y(z) = X(z)H(z) = \dfrac{1}{1-\frac{9}{10}z^{-1}} \cdot \dfrac{z^{-1}}{(1-z^{-1})^2} = z^{-1}\left[\dfrac{1}{(1-z^{-1})^2(1-\frac{9}{10}z^{-1})}\right] = z^{-1}\left[\dfrac{A}{(1-\frac{9}{10}z^{-1})} + \dfrac{B}{(1-z^{-1})^2} + \dfrac{C}{(1-z^{-1})}\right]$

$A = \left[\dfrac{1}{(1-z^{-1})^2}\right]_{z=\frac{9}{10}} = \dfrac{1}{(1-\frac{10}{9})^2} = 81 \qquad B = \left[\dfrac{1}{(1-\frac{9}{10}z^{-1})\cancel{(1-z^{-1})^2}}\cancel{(1-z^{-1})^2}\right]_{z=1} = \dfrac{1}{1-9/10} = 10$

$C = \dfrac{1}{-1}\dfrac{d}{d(z^{-1})}\left[\dfrac{\cancel{(1-z^{-1})^2}}{(1-\frac{9}{10}z^{-1})\cancel{(1-z^{-1})^2}}\right]\Big|_{z=1} = -1\dfrac{d}{ds}\left[\dfrac{1}{1-\frac{9}{10}s}\right]_{s=1} = -1\dfrac{d}{ds}\left[\left(1-\frac{9}{10}s\right)^{-1}\right]_{s=1} = -(-1)\left(1-\frac{9}{10}s\right)^{-2}\left(-\frac{9}{10}\right)$

$z^{-1} = s$

$= -\frac{9}{10}\dfrac{1}{(1-9/10)^2} = -\frac{9}{10}\cdot 100 = -90$

$$X(z)=\sum_{i=1}^{m}\sum_{p=1}^{k_i}\frac{B_{i,p}}{(1-d_i z^{-1})^p}$$

$B_{i,p}=\left[X(z)(1-d_i z^{-1})\right]\Big|_{z=d_i}$

$k_i=1$ poli 1° grado

$k_i>1$ poli di grado superiore

$$B_{i,p}=\frac{\frac{d^{k_i-p}}{ds^{k_i-p}}\left[(1-d_i s)^{k_i}X(s^{-1})\right]}{(k_i-p)!\,(-d_i)^{k_i-p}}\Bigg|_{s=\frac{1}{d_i}} \quad s=z^{-1}$$

$$Y(z)=81\frac{z^{-1}}{1-\frac{9}{10}z^{-1}}+10\frac{z^{-1}}{(1-z^{-1})^2}-80\frac{z^{-1}}{1-z^{-1}}$$

$$y[n]=81\left(\frac{9}{10}\right)^{n-1}u[n-1]+10\,n\,u[n]-80\,u[n-1]$$

- $h[n]=u[-n-1]$ $\qquad -u[-n-1] \to \frac{1}{1-z^{-1}}$ $\qquad -\alpha^n u[-n-1] \to \frac{1}{1-\alpha z^{-1}} \quad |z|<|\alpha|$

$x[n]=-n3^n u[-n]$

$$H(z)=-\frac{1}{1-z^{-1}}$$

$$-n\alpha^n u[-n-1] \qquad \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2} \qquad |z|<|\alpha|$$

$$x'[n]=x[n+1]=-(n+1)3^{n+1}u[-n-1]=$$

$$=-n3^n3u[-n-1]-3^n3u[-n-1]=$$

$$=3[-n3^n u[-n-1]-3\cdot 3^n u[-n-1]]$$

$$X'(z)=3\left[\frac{3z^{-1}}{(1-3z^{-1})^2}+\frac{1}{1-3z^{-1}}\right] \qquad |z|<3$$

$$x[n]=x'[n-1]$$

$$X(z)=3z^{-1}\left[\frac{3z^{-1}}{(1-3z^{-1})^2}+\frac{1}{1-3z^{-1}}\right]$$

$$X(z)=3z^{-1}\left[\frac{3z^{-1}+1-3z^{-1}}{(1-3z^{-1})^2}\right]=3z^{-1}\frac{1}{(1-3z^{-1})^2}$$

$$Y(z)=X(z)H(z)=\frac{-1}{1-z^{-1}}\cdot 3z^{-1}\frac{1}{(1-3z^{-1})^2}=\frac{-3z^{-1}}{(1-z^{-1})(1-3z^{-1})^2}=-3z^{-1}\left[\frac{A}{(1-z^{-1})}+\frac{B}{(1-3z^{-1})^2}+\frac{C}{(1-3z^{-1})}\right]$$

$$A=\frac{1}{(1-3z^{-1})^2}\Big|_{z=1}=\frac{1}{(1-3)^2}=\frac{1}{4} \qquad B=\frac{1}{(1-z^{-1})}\Big|_{z=3}=\frac{1}{1-\frac{1}{3}}=\frac{3}{2}$$

$$C=\frac{1}{-3}\cdot\frac{d}{ds}\left[\frac{1}{1-s}\right]_{s=\frac{1}{3}}=-\frac{1}{3}(-1)(1-s)^{-2}(-1)\Big|_{s=\frac{1}{3}}=-\frac{1}{3}\frac{1}{(1-\frac{1}{3})^2}=-\frac{3}{4}$$

$|z|<|\alpha|$

$-n\alpha^n u(-n-1)$

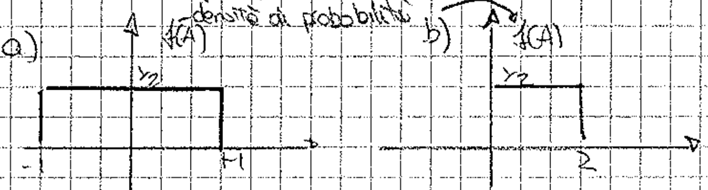
$$Y(z)=-3z^{-1}\left[\frac{1/4}{1-z^{-1}}+\frac{3/2}{(1-3z^{-1})^2}-\frac{3/4}{(1-3z^{-1})}\right]$$

$$y[n]=-\frac{3}{4}[-u[-(n-1)-1]]+\frac{9}{4}[-3^{n-1}u[-(n-1)-1]]-\frac{3}{2}[-n3^n u[-n-1]]$$

PROCESSI CASUALI

VALOR MEDIO

① $x(t) = A \cos(2\pi f_0 t)$



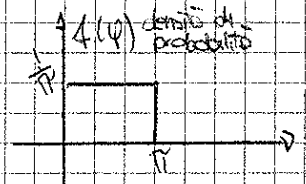
$\mathcal{E}\{x(t)\} = ?$

$$\mathcal{E}\{x(t)\} = \mathcal{E}\{A \cos(2\pi f_0 t)\} = \mathcal{E}\{A\} \cos(2\pi f_0 t)$$

a) $\mathcal{E}\{A\} = \int_{-\infty}^{+\infty} A f(A)\, dA = \int_{-1}^{+1} A \frac{1}{2}\, dA = \frac{1}{2}\left[\frac{A^2}{2}\right]_{-1}^{1} = 0$ $\qquad \mathcal{E}\{x(t)\} = 0$

b) $\mathcal{E}\{A\} = \frac{1}{2}\left[\frac{A^2}{2}\right]_0^2 = \frac{1}{2}\left[\frac{4}{2}\right] = 1$ $\qquad \mathcal{E}\{x(t)\} = \cos(2\pi f_0 t)$

② $x(t) = \cos(2\pi f_0 t + \varphi)$



$$\mathcal{E}\{x(t)\} = \mathcal{E}\{\cos(2\pi f_0 t + \varphi)\} =$$
$$= \mathcal{E}\{\cos(2\pi f_0 t)\cos(\varphi) - \sin(2\pi f_0 t)\sin(\varphi)\} =$$
$$= \mathcal{E}\{\cos(2\pi f_0 t)\cos(\varphi)\} - \mathcal{E}\{\sin(2\pi f_0 t)\sin(\varphi)\}$$
$$= \mathcal{E}\{\cos(\varphi)\} \cdot \cos(2\pi f_0 t) - \mathcal{E}\{\sin(\varphi)\} \sin(2\pi f_0 t) =$$

$$\mathcal{E}\{\cos\varphi\} = \int_{-\infty}^{+\infty} \cos(\varphi) f(\varphi)\, d\varphi = \frac{1}{\pi}\int_0^{\pi} \cos\varphi\, d\varphi = \frac{1}{\pi}[\sin\varphi]_0^{\pi} = \phi$$

$$\mathcal{E}\{\sin\varphi\} = -\frac{1}{\pi}[\cos\varphi]_0^{\pi} = +\frac{1}{\pi}[-1-1] = -\frac{2}{\pi}$$

$$\Downarrow$$

$$\mathcal{E}\{x(t)\} = -\frac{2}{\pi}\sin(2\pi f_0 t)$$

FUNZIONE DI AUTOCORRELAZIONE

$$\underline{R_x(t_1, t_2) = \mathcal{E}\{x(t_1)\, x^*(t_2)\}}$$

WSS: $m_x(t) = m_x$ $\qquad$ SEGNALI STAZIONARI IN SENSO LATO

$$R_x(t_1, t_2) = R_x(t_2 - t_1)$$
$$\Downarrow$$
$$t_2 - t_1 = \tau$$
$$\underline{R_x(\tau) = \mathcal{E}\{x(t)\, x^*(t+\tau)\}}$$
$$\underline{R_x(0) = \mathcal{E}\{x(t)\, x^*(t)\} = \mathcal{E}\{|x(t)|^2\}}$$

DENSITÀ SPETTRALE DI POTENZA $\quad S_x(f) = \mathcal{F}\{R_x(\tau)\}$

PROCESSI FILTRATI

$$x(t) \to \boxed{h(t)} \to y(t)$$

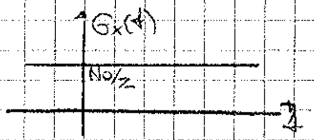
SISTEMA LTI
$x(t)$ e $y(t)$ sono processi congiunti

$R_y(\tau) = R_x(\tau) * R_h(\tau)$ $\qquad R_h(\tau) = h(\tau) * h^*(-\tau)$

$G_y(f) = G_x(f) \cdot H(f) \cdot H^*(f) = G_x(f) \cdot |H(f)|^2$

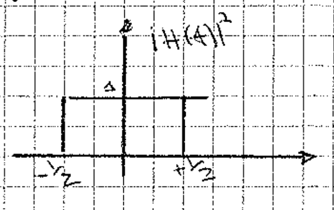
1

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$ $\qquad \mathcal{E}\{x(t)\}=0$



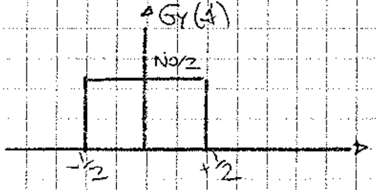
$h(t) = \sin(\pi t)/\pi t$

$P_Y = ?$



$G_Y(f) = G_X(f)\,|H(f)|^2$

$\Downarrow$



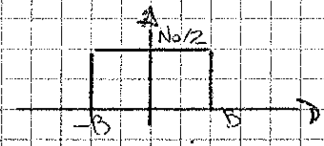
$P_Y = \int_{-1/2}^{1/2} \frac{N_0}{2}\, df = \frac{N_0}{2}$

oppure $\quad R_Y(\tau) = F^{-1}\{G_Y(f)\} = \frac{N_0}{2}\frac{\sin(\pi\tau)}{\pi\tau} \qquad$ per $\tau=0 \quad R_Y(0) = \frac{N_0}{2}$

2

$m(t)$

$G_m(f) = \frac{N_0}{2} \qquad |f|<B$



$B = \frac{3}{4T}$

$m(t) \rightarrow \boxed{h(t)} \xrightarrow{x(t)} \boxed{(\cdot)^2} \xrightarrow{z(t)}$

$h(t) = \delta(t) + \frac{1}{2}\delta(t-T)$

$\mathcal{E}\{z(t)\} = ?$

$x(t) = m(t) * h(t) = m(t) * [\delta(t) + \frac{1}{2}\delta(t-T)] = m(t) + \frac{1}{2}m(t-T)$

$z(t) = x^2(t) = m^2(t) + \frac{1}{4}m^2(t-T) + m(t)\,m(t-T)$

$\mathcal{E}\{z(t)\} = \mathcal{E}\{m^2(t) + \frac{1}{4}m^2(t-T) + m(t)m(t-T)\} =$

$= \mathcal{E}\{m^2(t)\} + \mathcal{E}\{\frac{1}{4}m^2(t-T)\} + \mathcal{E}\{m(t)m(t-T)\} =$

$= \mathcal{E}\{m(t)m(t)\} + \frac{1}{4}\mathcal{E}\{m(t-T)m(t-T)\} + \mathcal{E}\{m(t)m(t-T)\} =$

$= R_m(0) + \frac{1}{4}R_m(0) + R_m(T) =$

$= \frac{5}{4}R_m(0) + R_m(T)$

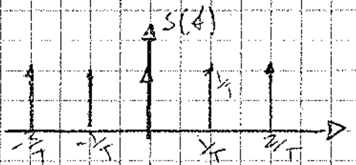
$R_m(\tau) = F^{-1}\{G_m(f)\} = \frac{N_0}{2}\frac{\sin(\pi 2B\tau)}{\pi\tau}$

$\begin{cases} R_m(0) = N_0 \cdot B \\ R_m(T) = \frac{N_0}{2}\sin(\pi 2BT)/\pi T = -\frac{N_0}{2}\frac{4B}{3\pi} = -\frac{2}{3}\frac{N_0 B}{\pi} \end{cases}$

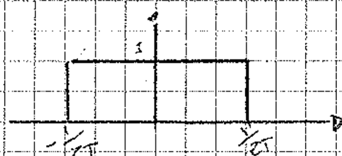
$\mathcal{E}\{z(t)\} = N_0 \cdot B\left(\frac{5}{4} - \frac{2}{3\pi}\right)$

4

$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT)$



$h(t) = \frac{\sin \pi t/T}{\pi t}$



$X(f) = \frac{1}{T}\delta(f) \qquad x(t) = \frac{1}{T}$

5

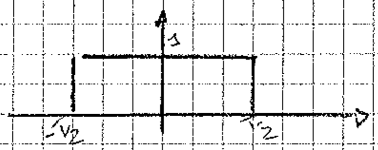
$y[n] - \frac{1}{a}y[n-1] = x[n] - \frac{1}{2}x[n-1] \qquad Y(z) - \frac{1}{a}Y(z)z^{-1} = X(z) - \frac{1}{2}X(z)z^{-1}$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{1}{2}z^{-1}}{1-a^{-1}z^{-1}} \qquad |a^{-1}|<1 \quad |a|>1 \qquad \frac{1}{1-a^{-1}z^{-1}} \rightarrow a^{-n}u[n] - \frac{1}{2}a^{-(n-1)}u[n-1]$

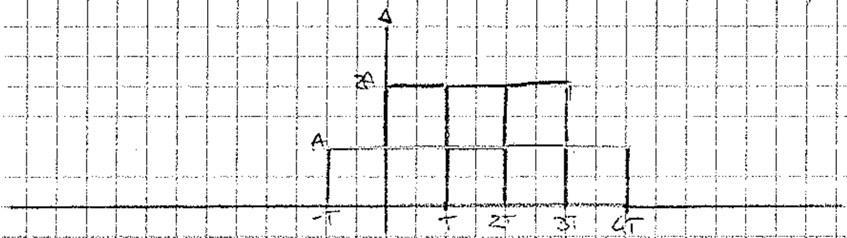
PROVA D'ESAME: GIUGNO 2008

(1) $x(t) = \sum_{m=0}^{3} x_0(t - mT)$

$x_0 = P_T(t/2)$



~~x(t) = ...~~



$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 = 5T \cdot A^2 + 3T \cdot 3A^2 =$
$= 14 A^2 T$

$x(t) = x_0(t) + x_0(t-T) + x_0(t-2T) + x_0(t-3T)$

$\hat{X}(f) = 2A\left[\frac{\sin 2\pi f T}{2\pi f} + \frac{\sin 2\pi f T}{2\pi f} \cdot e^{-j2\pi f_0 T} + \frac{\sin 2\pi f T}{2\pi f} \cdot e^{-j4\pi f_0 T} + \frac{\sin 2\pi f T}{2\pi f} e^{-j6\pi f_0 T}\right] =$

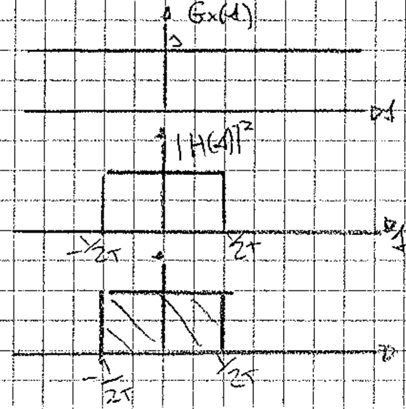
$= A \frac{\sin 2\pi f_0 T}{\pi f_0}\left[1 + e^{-j2\pi f_0 T} + e^{-j4\pi f_0 T} + e^{-j6\pi f_0 T}\right] = 2A \cdot T \operatorname{sinc}(2fT)\left[\sum_{m=0}^{3} e^{-j2\pi f_0 T \cdot m}\right]$

(2) $x(t) = 2\cos(t) + 5$

$X(f) = 5\delta(f) + \delta\left(f - \frac{1}{2\pi}\right) + \delta\left(f + \frac{1}{2\pi}\right)$

$B = \frac{1}{2\pi}$ banda del segnale $\Rightarrow f_c = \frac{1}{\pi}$

(3) la densità spettrale di potenza è costante



$P = \int_{-\infty}^{+\infty} S_x(f)\, df$

$P = \frac{1}{T}$

PROVA D'ESAME - 29 GENNAIO 2007

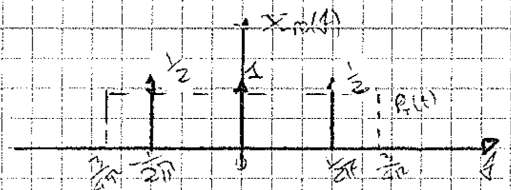
$$x(t) = \sum \pi(t - mT)$$
$$\bar{X}(f) = \frac{1}{T}\sum R\left(\frac{m}{T}\right)\delta\left(f-\ldots\right)$$

① $S(f) = \pi \dfrac{2}{1+4\pi^2 f^2}$

$$\bar{X}(f) = \frac{1}{2\pi}\sum \frac{2\pi}{1+\frac{4\pi^2 m^2}{4\pi^2}}\,\delta\left(f-\frac{m}{2\pi}\right) =$$

$$= \sum_{m=-\infty}^{+\infty} \frac{1}{1+m^2}\,\delta\left(f-\frac{m}{2\pi}\right)$$

$$X_{PB}(f) = \sum_{m=-1}^{+1} \frac{1}{1+m^2}\,\delta\left(f-\frac{m}{2\pi}\right)$$



$x(t) = \cos t + 1$

② $\bar{X}(f) = S_n(f)$, $H(f) = \frac{N_0}{2} e^{-f^2}$

$\cancel{x(\tau) = \frac{N_0}{2}\sqrt{\frac{1}{2}}\sqrt{2\pi}\, e^{-2\pi^2\tau^2\frac{1}{2}}} \rightarrow \cancel{x(0) = \frac{N_0}{2}\sqrt{\pi}}$

$$S_x(f) = S_n(f)\,|H(f)|^2 = \frac{N_0}{2} e^{-2f^2}$$

$2T^2 = \frac{1}{2}$

$T^2 = \frac{1}{4}$ $T = \frac{1}{2}$

$T\sqrt{2\pi}\, e^{-2\pi^2 T^2 f^2} \longrightarrow e^{-t^2/2T^2}$ $\quad T = \frac{1}{\pi}$

$\frac{N_0}{2}\frac{1}{2\cdot2}\sqrt{2\pi}\, e^{-\pi^2 t^2 \frac{1}{4}}$

$S_x(\tau) = \frac{N_0}{2}\sqrt{\frac{\pi}{2}}\, e^{-\frac{\pi^2\tau^2}{2}}$ $\rightarrow S_x(0) = \frac{N_0}{2}\sqrt{\frac{\pi}{2}}$
$R_x(\tau)$

$\frac{N_0}{\ldots}\sqrt{2\pi}\, e^{-\frac{1}{2}\pi^2 t^2}$

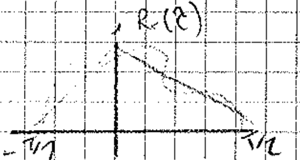
③ $f_c = 200\ \text{Hz}$

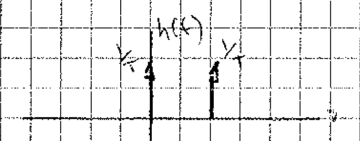
$T_c = \frac{1}{200} = 0.005\ \text{s}$ $\quad \gg 5\cdot10^{-3}\cdot N = 10$

$T = 10\ \text{s}$

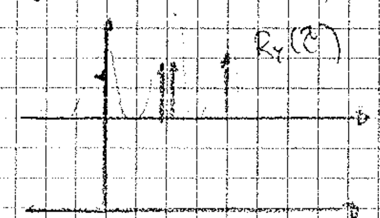
$N = 2000$

④

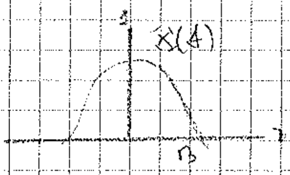


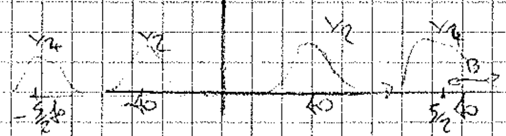


$$R_y(\tau) = R_x(\tau) * R_h(\tau)$$



⑤



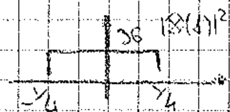


Banda totale $\frac{5}{2}f_0 + B$

$\Downarrow$

$f_c = 5f_0 + 2B$

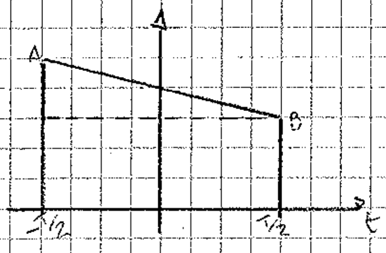
⑥

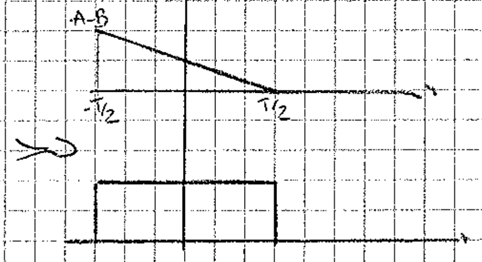


$E(x) = 18$

ESERCIZI A CASO

•





Trovo l'equazione della retta $\frac{y-A}{B-A} = \frac{x+T/2}{T}$ $\quad y = \frac{B-A}{T}\left(x+\frac{T}{2}\right)+A$

$y = \frac{B-A}{T}\cdot x + \frac{B+A}{2}$

$$x(t) = \left[\frac{B-A}{T}\cdot t + \frac{B-A}{2} + A\right] p_T(t)$$

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt = \int_{-T/2}^{T/2} \left(\frac{B-A}{T}\right) t\, e^{-j2\pi f t}\, dt + \int_{-T/2}^{T/2} \left(\frac{B+A}{2}\right) e^{-j2\pi f t}\, dt \quad \ldots\ldots$$

↑ per parti

× L'ESAME

$x(t) = p_T(t)$

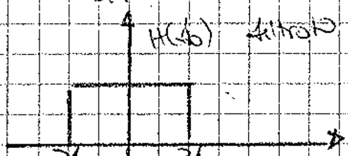
$y(t) = x(t/2)$

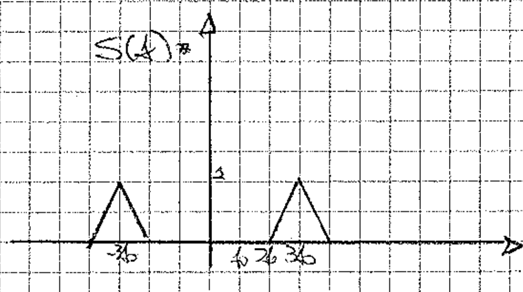
$E_x \overset{?}{=} E_y$

• $s(t) = 2f_0\, \text{sinc}^2(f_0 t)\cos(6\pi f_0 t)$

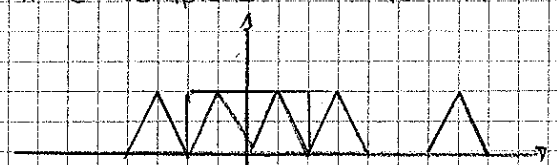
$f_C = \frac{1}{4f_0}$ campionato

$H(f)$ filtrato





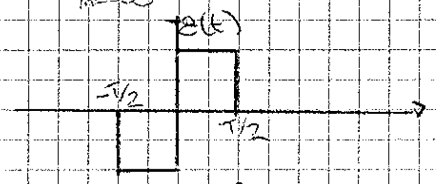
Viene campionato a $f_C = 4f_0$

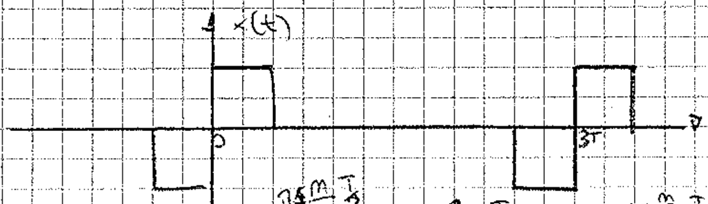


Dal filtro esce il segnale

$y(t) = 2f_0\, \text{sinc}(f_0 t)\cos(2\pi f_0 t)$

• $x(t) = \sum_{n=-\infty}^{+\infty} g(t-3nT)$





$$X(f) = \frac{1}{3T}\sum_{n=-\infty}^{+\infty} G\left(\frac{n}{3T}\right)\delta\left(f-\frac{n}{3T}\right) = \frac{1}{3T}\sum \frac{\sin \pi \frac{n}{3T}\frac{T}{2}}{\pi \frac{n}{3T}}\left(e^{-j2\pi\frac{n}{3T}\frac{T}{4}} - e^{+j2\pi\frac{n}{3T}\frac{T}{4}}\right)\delta\left(f-\frac{n}{3T}\right)$$

$$G(f) = \frac{\sin \pi f T/2}{\pi f}\left(e^{-j2\pi f T/4} - e^{+j2\pi f T/4}\right)$$

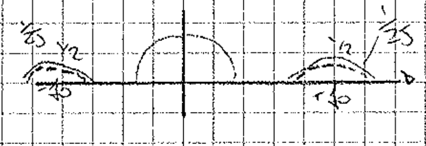
• $x(t)$ banda B

$E\{x(t)\} = E_x$

$y(t) = x(t)\sin 2\pi f_0 t + x(t)\cos 2\pi f_0 t$ $\quad E_y\{E_x\}$?

$x(t) \to f_c = 2B$

$y(t) \to f_c = 2(f_0+B)$

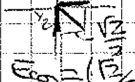


⇓

$E_x = E_y$

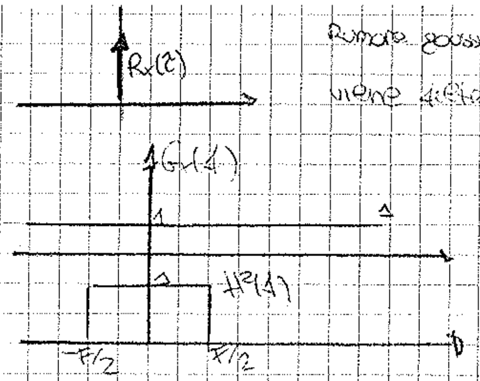
$E_{\sin} = \frac{1}{4}$

$E_{\cos} = \frac{1}{4}$



$E_{\sin} + E_{\cos} = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

Rumore gaussiano bianco con delta come autocorrelazione viene filtrato: $h(t) = \frac{\sin \pi F t}{\pi t}$ $P_y = ?$



$P_y = F$