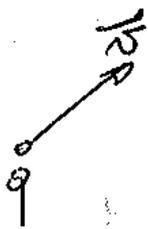


$$|F_1| = |F_2|$$

Forza esercitata da campo magnetico

Osservare una carica q che si muove con velocità \vec{v} .



$$\vec{F}_e = q\vec{E}$$

$$\vec{F}_m = q\vec{v} \wedge \vec{B}$$

forza magnetica

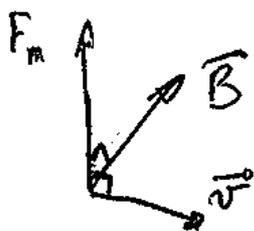
campo magnetico

Se particella è ferma nel campo magnetico non esiste del campo.

Forza magnetica non è direttamente proporzionale al campo magnetico.

Velocità e forza sono \perp

velocità e campo magnetico definiscono un piano \perp alle forze di Lorentz.



$$\vec{F}_m = q (\nu_x \vec{i} + \nu_y \vec{j} + \nu_z \vec{k}) \wedge (B_x \vec{i} + B_y \vec{j} + B_z \vec{k})$$

$$= q (\nu_x B_y \vec{k} - \nu_x B_z \vec{j} - B_x \nu_y \vec{k} + \nu_y B_z \vec{i} + B_x \nu_z \vec{j} - \nu_z B_y \vec{i})$$

$$= q \left[(v_y B_z - v_z B_y) \vec{i} + (B_x v_z - B_z v_x) \vec{j} + (v_x B_y - v_y B_x) \vec{k} \right]$$

Se $v = v_x \vec{i}$

$$F_m = q \left[-v_x B_z \vec{j} + v_x B_y \vec{k} \right]$$

Se $v = v_x \vec{i}$ e $B = B_y \vec{j}$

Allora $F_m = v_x B_y \vec{k}$

① Forza magnetica \perp velocità
 $F_m \perp \vec{B}$

2) $|F_m|$ max se $\vec{B} \perp \vec{v}$
 $|F_m|$ min se $\vec{B} \parallel \vec{v}$

Teorema energia cinetica

$$T_f - T_i = L_{A \rightarrow B}$$

Richiamo

$$E_c(t_2) - E_c(t_1) = W_{1,2} = \int_{C_{1,2}} \vec{F} \cdot d\vec{s} = \int_{t_1}^{t_2} \vec{F} \cdot \frac{d\vec{s}}{dt} dt =$$

$$= \int_{t_1}^{t_2} \vec{F} \cdot \vec{v} dt$$

Nel campo magnetico $F \perp v \Rightarrow$

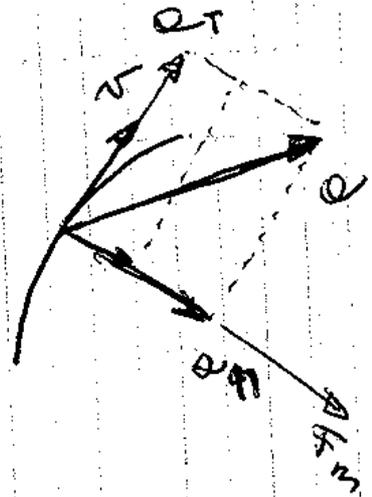
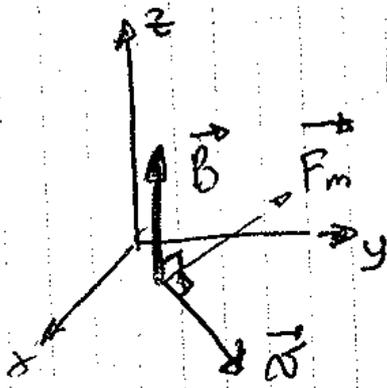
$$F \cdot v = 0 \Rightarrow W_{1,2} = 0$$

La particella passando dalla posizione 1 alla
 posizione 2 non cambia energia cinetica.
 La forza di Lorentz non può compiere
 cinetica.

$$\vec{F} = m \vec{a}$$

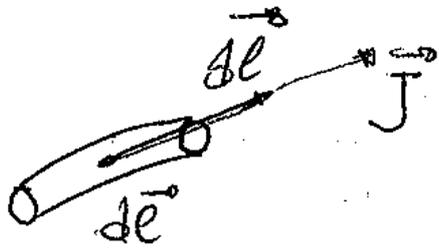
$$\vec{v} \in xy$$

$$\vec{B} \parallel z$$



$$\begin{cases} F_n = m a_n \\ F_T = m a_T \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} q v B = m \frac{v^2}{R} \\ 0 = m a_T \end{cases} \Rightarrow \begin{cases} R = \frac{m v}{q B} \\ a_T = 0 \end{cases}$$



$$\vec{J} = -Ne \vec{v}$$

(27)

N densità particelle x unità di

$$\vec{F}_1 = -e \vec{v} \wedge \vec{B}$$

$$dV = S dl$$

$$d\vec{l} = dl \frac{\vec{v}}{v}$$

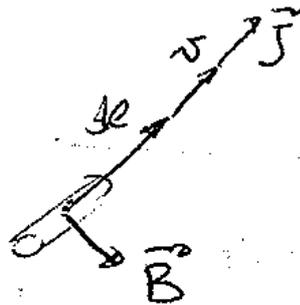
$$d\vec{F} = \vec{F}_1 N dV = -e \vec{v} \wedge \vec{B} N S dl =$$

$$= -Ne \vec{v} \wedge \vec{B} S dl = \vec{J} \wedge \vec{B} S dl =$$

$$= S \vec{J} dl \wedge \vec{B} = S \int d\vec{l} \wedge \vec{B} =$$

$$i \int d\vec{l} \wedge \vec{B}$$

$$d\vec{F} = i d\vec{l} \wedge \vec{B}$$

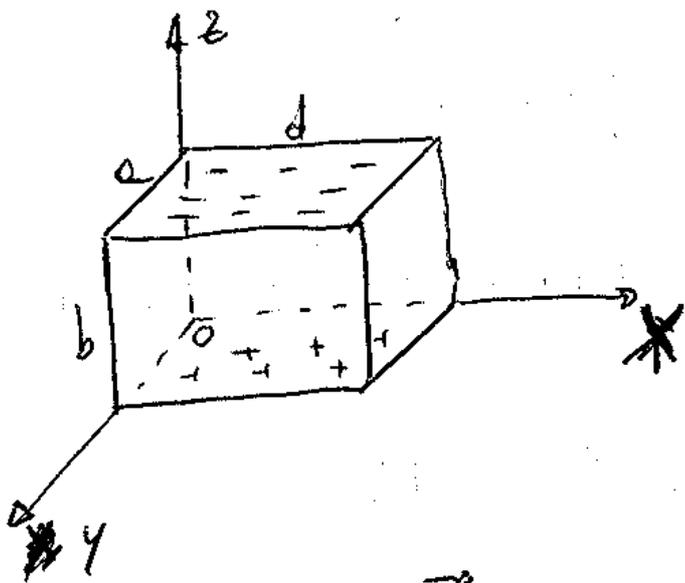


Esempio:



$$\vec{F} = \int d\vec{F} = i \int d\vec{l} \wedge \vec{B} =$$

$$i \left(\int d\vec{l} \right) \wedge \vec{B} = i \vec{l} \wedge \vec{B}$$



$$\vec{J} = \frac{i}{eb} \vec{u}_x$$

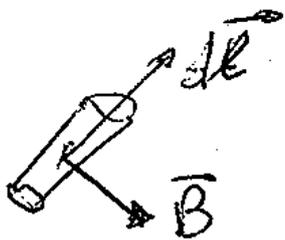
$$\vec{B} = B \vec{u}_y$$

$$F = -e \vec{v} \wedge \vec{B} = \vec{v} = \frac{i}{ebNe} \vec{u}_x$$

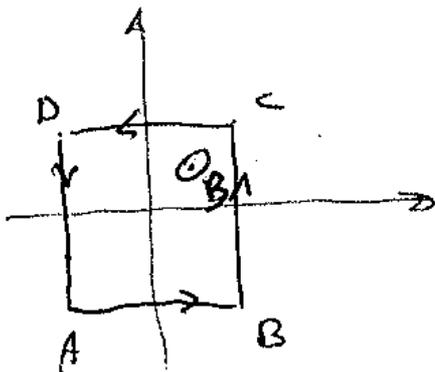
$$= -e \frac{iB}{ebNe} \vec{u}_x \wedge \vec{u}_y = \frac{iB}{ebNe} \vec{u}_z$$

$$\vec{E}_H = \frac{\vec{F}}{-e} = -\frac{iB}{ebNe} \vec{u}_z = \frac{iB}{ebN}$$

tensione $\rightarrow \int_a^b E_H dz = \frac{iB}{eNe}$ Effetto Hall



$$d\vec{F} = i dl \vec{e} \wedge \vec{B}$$



$$\vec{F} = i \vec{e} \wedge \vec{B}$$

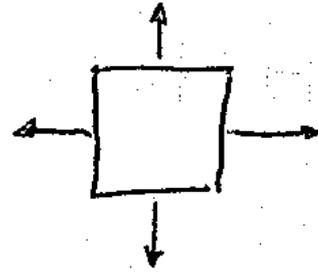
$$\vec{F}_{AB} = i e_{AB} B \vec{u}_x \wedge \vec{u}_z = -i e_{AB} B \vec{u}_y$$

$$\vec{F}_{CD} = i e_{CD} B \vec{u}_y$$

$$\vec{F}_{BC} = i l_{BC} B \vec{u}_y \wedge \vec{u}_z = i l_{BC} B \vec{u}_x$$

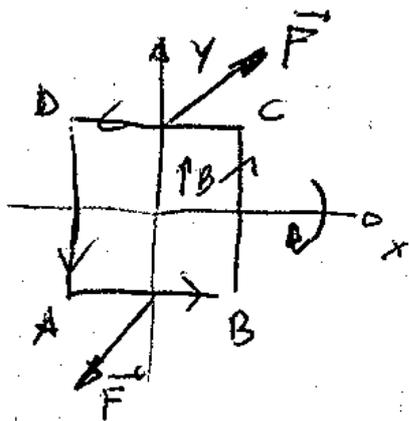
$$\vec{F}_{DA} = -i l_{BC} B \vec{u}_x$$

(98)



Se una spira percorsa da corrente i è immersa in un campo magnetico \vec{B} questi non creano forze risultanti sulla spira.

Se $\vec{B} = B \vec{u}_y$



$$\vec{F}_{AB} = i l_{AB} B \vec{u}_x \wedge \vec{u}_y = i l_{AB} B \vec{u}_z$$

$$\vec{F}_{CD} = i l_{AB} B (\vec{u}_x) \wedge \vec{u}_y = -i l_{AB} B \vec{u}_z$$

$$\vec{F}_{BC} = i l_{BC} B (\vec{u}_y) \wedge \vec{u}_y = 0$$

$$\vec{F}_{DA} = i l_{BC} B (-\vec{u}_y) \wedge \vec{u}_y = 0$$

$$\vec{M} = \vec{r}_{BC} \wedge \vec{F}_{CD} = l_{BC} \vec{u}_y \wedge (-i l_{AB} B \vec{u}_z) =$$

$$= -i l_{BC} l_{AB} B \vec{u}_y \wedge \vec{u}_z = i l_{AB} l_{BC} \vec{u}_z \wedge B \vec{u}_y$$

$$= i \vec{S} \wedge \vec{B} = i \vec{S} \wedge \vec{B}$$

$$\boxed{\vec{M} = i \vec{S} \wedge \vec{B}}$$

$$M = i S B \sin \theta$$

$$\theta = \theta(\vec{S}, \vec{B})$$

$\vec{m} = i \vec{S} = i S \vec{n}$ momento magnetico spira

$$M = \vec{m} \wedge \vec{B}$$

se $\theta = 0^\circ$ $\vec{m} \parallel \vec{B}$
 si ha equilibrio stabile

se $\theta = 180^\circ$ $\vec{m} \uparrow \vec{B}$ si ha
 equilibrio instabile

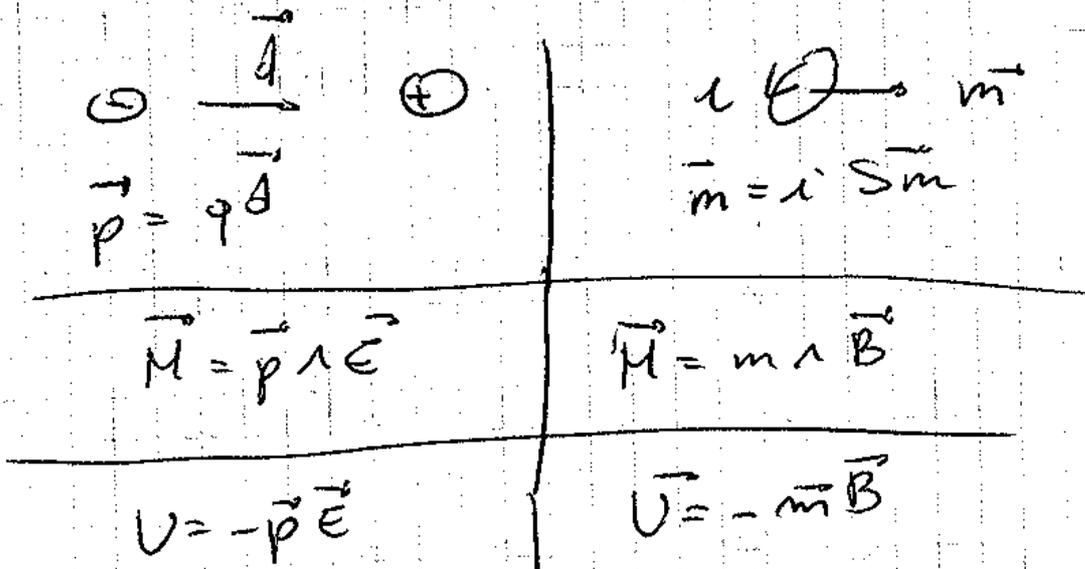
$$\vec{F} = - \frac{dU}{dx}$$

$$M = - \frac{dU}{d\theta}$$

$$M = \nu \delta B \sin \theta$$

$$U = -m B \cos \theta$$

$$U = -\vec{m} \cdot \vec{B}$$



$$\oint_S \vec{E} \cdot d\vec{\sigma} = \frac{Q}{\epsilon_0}$$

$$\oint_S \vec{E} \cdot d\vec{\sigma} = 0 \quad \text{se vi sono solo dipoli.}$$

$$\oint_S \vec{B} \cdot d\vec{\sigma} = 0$$

$$\text{div } \vec{B} = 0$$

perché non esistono monopoli magnetici

Teorema di Gauss per il campo Magnetico (II eq. di Maxwell)

$$F = q \vec{v} \wedge \vec{B}$$

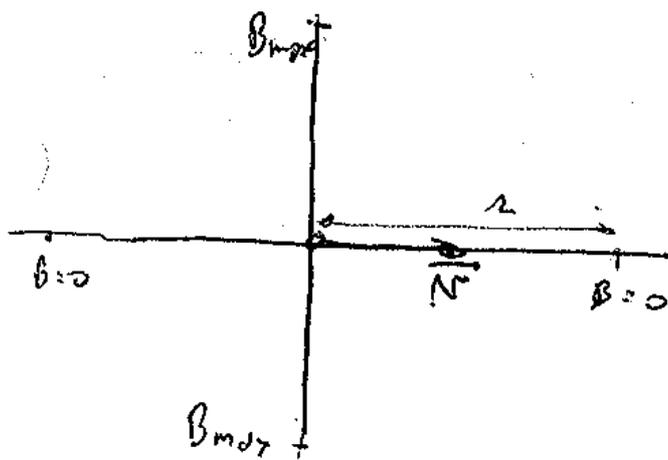
$$F = q \vec{E}$$

$$dF = i d\vec{l} \wedge \vec{B}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} q \frac{\vec{r}}{r^3}$$

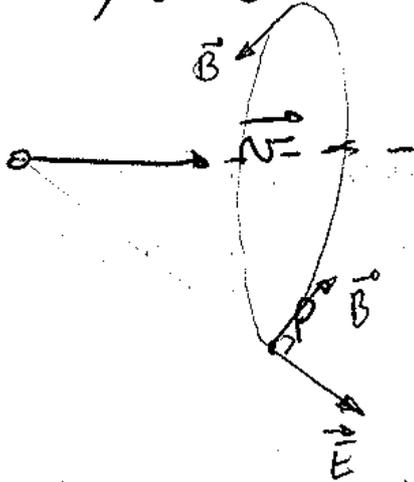
$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \wedge \vec{r}}{r^3}$$

$$B = \frac{\mu_0}{4\pi} q \frac{v \sin\theta}{r^2}$$



$$\vec{B} = \mu_0 \epsilon_0 \vec{v} \wedge \underbrace{\left(\frac{q}{4\pi\epsilon_0} \frac{\vec{r}}{r^3} \right)}_{\vec{E}}$$

$$\vec{B} = \mu_0 \epsilon_0 \vec{v} \wedge \vec{E}$$



$$\vec{B}_1 = \frac{\mu_0}{4\pi} q \frac{\vec{v} \wedge \vec{r}}{r^3}$$

$$\int d\vec{B} = N dV \vec{B}_1 = N S d\ell \frac{\mu_0}{4\pi} q \frac{\vec{v} \wedge \vec{r}}{r^3}$$

$$\vec{J} = Nq \vec{v}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} S d\ell \frac{\vec{J} \wedge \vec{r}}{r^3} = \frac{\mu_0}{4\pi} S \frac{d\ell \vec{J} \wedge \vec{r}}{r^3} =$$

$$= \frac{\mu_0}{4\pi} S \frac{\int d\vec{\ell} \wedge \vec{r}}{r^3} = \frac{\mu_0}{4\pi} S \int \frac{d\vec{\ell} \wedge \vec{r}}{r^3} =$$

$$= \frac{\mu_0}{4\pi} i \frac{d\vec{\ell} \wedge \vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} i \frac{d\vec{\ell} \wedge \vec{r}}{r^3}$$

confronto con

$$B = \frac{\mu_0}{4\pi} q \frac{\vec{v} \wedge \vec{r}}{r^3}$$

$$F = i \int_C d\vec{\ell} \wedge B$$

$$B = \frac{\mu_0}{4\pi} i \int_C \frac{d\vec{\ell} \wedge \vec{r}}{r^3}$$

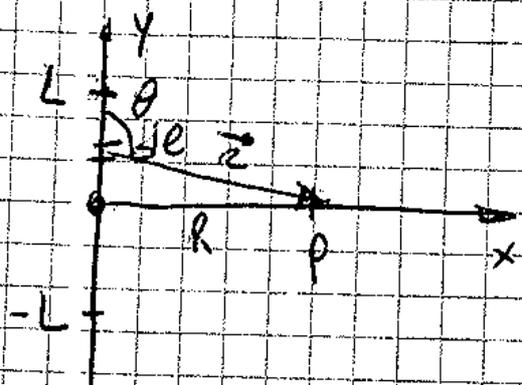
$$\vec{I} = \int_C \vec{B} \wedge d\vec{\ell}$$

$$I = \int_C B d\ell$$

$$dB = \frac{\mu_0}{4\pi} i \frac{\vec{v} \wedge \vec{r}}{r^3}$$

Legge di Ampere - Laplace

Esempio:



$$dB = \frac{\mu_0}{4\pi} i \frac{d\vec{\ell} \wedge \vec{r}}{r^3} = \frac{\mu_0}{4\pi} i \frac{d\ell \sin\theta}{r^2} \vec{u}_\varphi$$

$$B = \int_{\text{mag}} \vec{\mu}_\varphi \frac{\mu_0}{4\pi} i \frac{dl \sin \theta}{r^2} =$$

$$= \mu_\varphi \frac{\mu_0}{4\pi} i \int_0^L \frac{\sin \theta}{r^2} dl$$

$$\sin \theta = \sin(\pi - \theta) = \frac{R}{r}$$

$$\text{tg } \theta = -\text{tg}(\pi - \theta) = -\frac{R}{l}$$

$$\frac{1}{r^2} = \frac{\sin^2 \theta}{R^2}$$

$$l \text{tg}(\pi - \theta) = R$$

$$l = \frac{-R}{\text{tg}(\pi - \theta)}$$

$$dl = \frac{d}{d\theta} \left(-\frac{R}{\text{tg} \theta} \right) = \frac{R}{\sin^2 \theta} d\theta$$

$$B = \vec{\mu}_\varphi \frac{\mu_0}{4\pi} i \int_{\frac{\pi}{2}}^{\theta_1} \frac{\sin^2 \theta}{R \sin^2 \theta} d\theta =$$

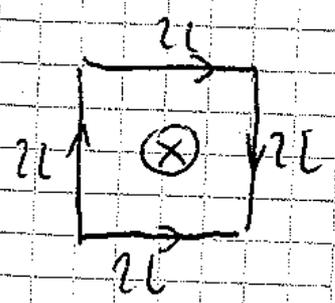
$$= \vec{\mu}_\varphi \frac{\mu_0}{4\pi} \frac{i}{R} \left[-\cos \theta \right]_{\frac{\pi}{2}}^{\theta_1} =$$

$$= -\mu_\varphi \frac{\mu_0}{4\pi} \frac{i}{R} \cos \theta_1$$

$$\cos \theta_1 = -\cos(\pi - \theta_1) = -\frac{L}{\sqrt{R^2 + L^2}}$$

$$\vec{B}_{sup} = \vec{\mu}_\varphi \frac{\mu_0}{4\pi} i \frac{L}{R \sqrt{R^2 + L^2}}$$

$$\vec{B} = 2 B_{sup} = 2 \vec{\mu}_\varphi \frac{\mu_0}{4\pi} \frac{iL}{R \sqrt{R^2 + L^2}}$$



$$B = \frac{2\mu_0 i}{\pi R} \frac{L}{\sqrt{L^2 + R^2}} =$$

$$= \frac{2\mu_0 i}{\pi R} \frac{R}{\sqrt{2}L} = \frac{\sqrt{2}\mu_0 i}{\pi L} =$$

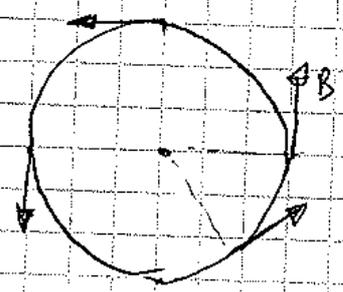
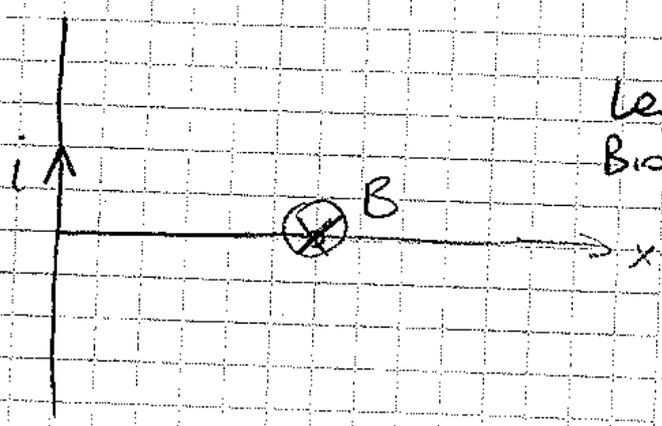
$$= \frac{\sqrt{2} 2\mu_0 i}{\sqrt{4}\pi}$$

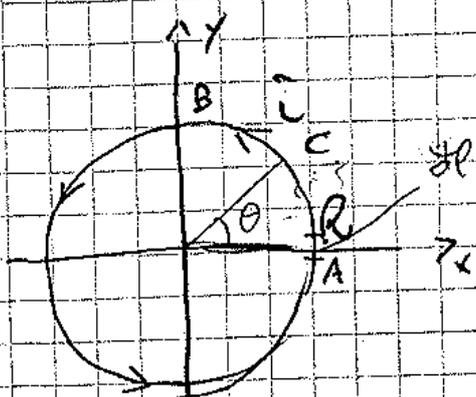
filo ha
 $L = \infty$

allora

$$B = \frac{\mu_0 i}{2\pi r} \vec{\mu}_\varphi$$

legge di
 Biot Savart





$$dB_A = \frac{\mu_0 i}{4\pi} dl \frac{\vec{i} \cdot (-R\vec{j})}{R^3} =$$

$$= \frac{\mu_0 i}{4\pi R^2} dl \vec{k}$$

$$dB_B = \frac{\mu_0 i}{4\pi R^2} dl \frac{-\vec{i} \cdot (-R\vec{j})}{R^3} = \frac{\mu_0 i}{4\pi R^2} dl \vec{k}$$

$$d\vec{B} = \frac{\mu_0 i}{4\pi R^2} dl \vec{k}$$

$$\vec{B} = \frac{\mu_0 i}{4\pi R^2} \vec{k} \int_0^{2\pi R} dl = \frac{\mu_0 i}{2\pi R^2} \cdot 2\pi R \vec{k} =$$

$$\vec{B} = \frac{\mu_0 i}{2R} \vec{k}$$

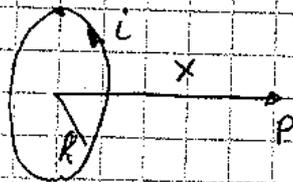
$$\vec{B} = \frac{\sqrt{\pi}}{3} \frac{\mu_0 i}{\sqrt{A}} \vec{k}$$

Spire quadrata

$$\vec{B} = \frac{2\sqrt{2}}{\pi} \frac{\mu_0 i}{\sqrt{A}} \vec{k}$$

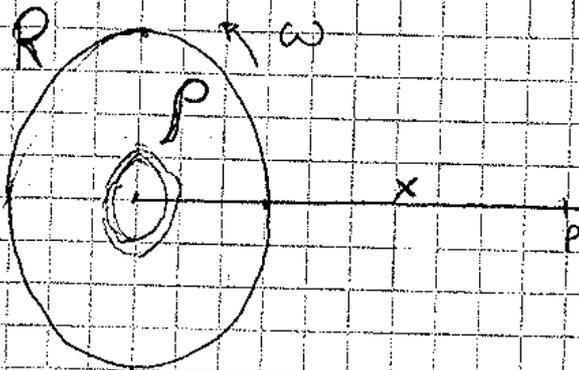
Spire circolare

$$B = \frac{\sqrt{\pi}}{2} \frac{\mu_0 i}{\sqrt{A}}$$



$$B = \frac{\mu_0 I}{2} \frac{R^2}{(x^2 + R^2)^{3/2}}$$

Esercizio:



$$Q = \pi R^2 \rho$$

$$dQ = \rho 2\pi r dr$$

$$di = \frac{dQ}{T} = \frac{\omega}{2\pi} dQ = \frac{\omega}{2\pi} 2\pi r dr =$$

$$= \omega \rho r dr$$

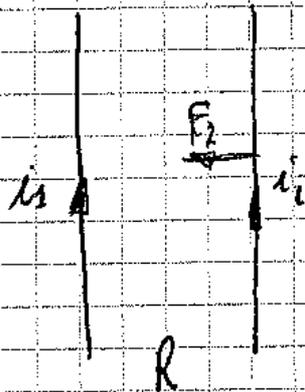
$$dB = \mu_0 \frac{M_0}{2} \omega \rho r dr \frac{r^2}{(x^2 + r^2)^{3/2}}$$

comp. moto su asse delle spire dell'anello

$$B = \mu_0 \frac{M_0}{2} \omega \rho \int_0^R \frac{r^3 dr}{(x^2 + r^2)^{3/2}}$$

Se mi sono al centro del disco $x=0$

$$\vec{B} = \vec{n} \frac{\mu_0 \omega p}{2} R$$



$$B_1 = \frac{\mu_0 i_1}{2\pi R}$$

$$\vec{F}_2 = i_2 l B_1 = \frac{\mu_0 i_1 i_2}{2\pi R} l$$

$$d\vec{F}_2 = i_2 d\vec{l}_2 \wedge \vec{B}_1$$

Teorema di Ampère

ha validità che se al di là della simmetria, si può applicare subito.

Legge di Ampère è riconducibile al campo magnetico di un filo.

Pensiamo una curva geometrica chiusa

No: dobbiamo risolvere queste vicinizzazioni

$$\oint_C \vec{B} \cdot d\vec{l}$$

$$d\vec{l} = R d\varphi \hat{u}_\varphi$$

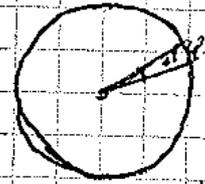
$$\oint_C \vec{B} \cdot d\vec{l} = \oint_C \frac{\mu_0 i}{2\pi R} \hat{u}_\varphi R d\varphi \hat{u}_\varphi =$$



$$dl = R d\varphi$$

relaz. diff. vale
per ogni curva
rel. finite ed
per circonferenza

$$= \frac{\mu_0 i}{2\pi} \oint_C d\varphi$$

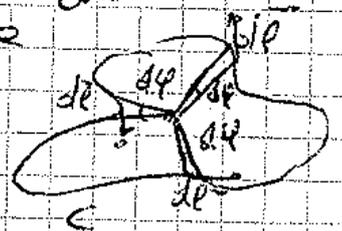


Se C è circonferenza di raggio r

$$= \frac{\mu_0 i}{2\pi} \cdot 2\pi = \mu_0 i$$

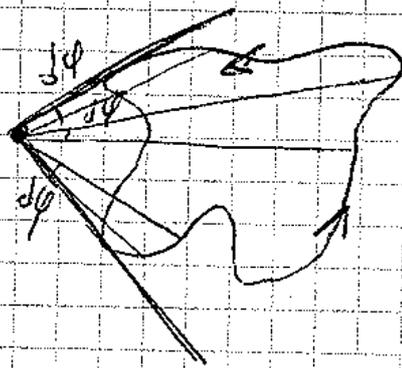
Se C non è una circonferenza che concorre

$$\oint_C d\varphi = 2\pi$$



$$\oint_C \vec{B} \cdot d\vec{\ell} = \frac{\mu_0 i}{2\pi} \cdot 2\pi = \mu_0 i$$

Se C non concorre a



$$\oint_C \vec{B} \cdot d\vec{\ell} = \frac{\mu_0 i}{2\pi} \oint_C d\varphi = \frac{\mu_0 i}{2\pi} \cdot 0 = 0$$

Quindi circuitazione del campo magnetico lungo una curva chiusa $\vec{\ell}$:

$\mu_0 i$ se curva concatenata i

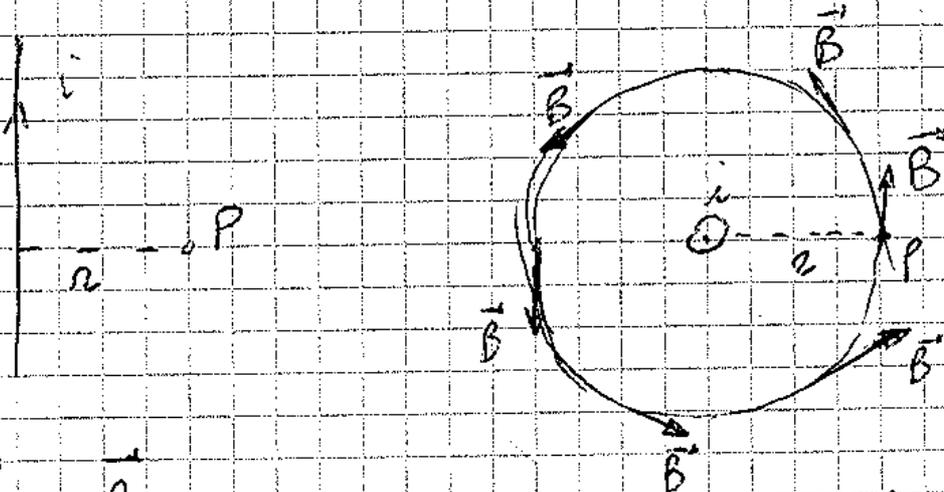
0 se curva non concatenata i

Vali anche se filo non rettilineo e + di 1 filo.

$$\oint_C \vec{B} d\vec{\ell} = \mu_0 \sum i_{int}$$

Teorema di
Ampère (in forma
integrata)

Sono parti della legge di Biot Savart e
bisogna verificare se rimane indietro o
più minore la legge di Biot-Savart.



$d\vec{\ell}$ e \vec{B} sono sempre concordi

$$\oint_C \vec{B} d\vec{\ell} = \mu_0 i$$

$$\oint_C B d\ell = \mu_0 i \Rightarrow B \oint_C d\ell = \mu_0 i \Rightarrow$$

$$\Rightarrow B \cdot 2\pi r = \mu_0 i \Rightarrow \left\{ B = \frac{\mu_0 i}{2\pi r} \right. \begin{array}{l} \text{legge} \\ \text{Biot} \\ \text{Savart} \end{array}$$

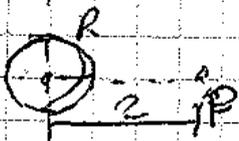
Orientazione determinata in precedenza
 quando abbiamo preso come curva C
 circonferenza.

Esempio di applicazione del teorema di Ampere

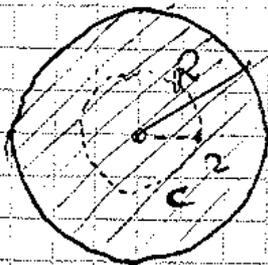
Conduttore cilindrico di raggio R percorso
 da corrente i



Determinare il campo magnetico
 in un punto a distanza r dall'
 asse.



Se $r > R$ $B = \frac{\mu_0 i}{2\pi r}$
 Per teorema di Ampere (vedi dimostrazione)



Se $r \leq R$

$$\oint_C \vec{B} d\vec{e} = \mu_0 i_c$$

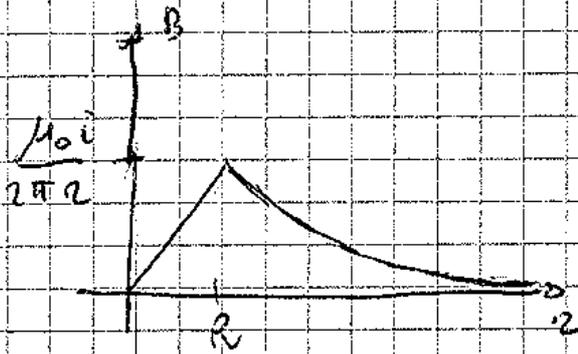
Supponiamo che corrente sia distribuita uniformemente

$$J = \frac{i}{\pi R^2}$$

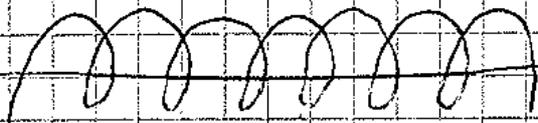
$$i_c = J \cdot \pi r^2 = \frac{i r^2}{R^2}$$

$$\oint_C \vec{B} d\vec{e} = \mu_0 i \frac{r^2}{R^2} = \oint_C B d\vec{e} = B \cdot 2\pi r$$

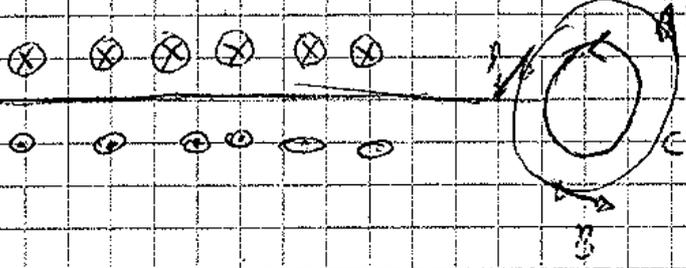
$$B \cdot 2\pi r = \mu_0 i \frac{r^2}{R^2} \Rightarrow B = \frac{\mu_0}{2\pi} i \frac{r}{R^2}$$



Esercizio:



Solenoido ideale
infinito



$$B \cdot 2\pi r = \mu_0 \cdot I$$

\downarrow

$$B = 0$$

corrente non concorre
le curve c perché
sta nel piano delle
stesse curve

È impossibile che all'interno del solenoide
il campo magnetico percorra una circonferenza

il campo magnetico può essere rappresentato
da linee rette infinite all'interno del solenoide

Scelta della curva C



$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_c \Rightarrow \int_{A_1 A_2} B dl = \mu_0 I_c \Rightarrow$$

$$B \oint_{\text{AdeL}} dl = \mu_0 i_c \Rightarrow$$

$$B L = \mu_0 i_c \Rightarrow B = \frac{\mu_0 i_c}{L}$$

Supponiamo di conoscere il numero di spire per unità di lunghezza

$$n = \frac{\text{numero spire (N)}}{\text{lunghezza (L)}}$$

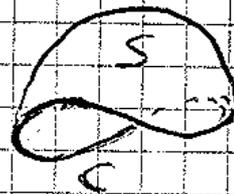
$$i_c = N i = i n L$$



$$B = \mu_0 i n$$

$$\oint_C \vec{B} \cdot d\vec{l} = \int_S \nabla \wedge \vec{B} \cdot d\vec{\sigma} \quad \times \text{ Stokes}$$

$$\mu = \int_S \vec{J} \cdot d\vec{\sigma} \quad \text{definizione}$$



Il teorema di amper effermo di

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 i$$

$$\int_S \nabla \wedge \vec{B} \cdot d\vec{\sigma} = \mu_0 \int_S \vec{J} \cdot d\vec{\sigma} \Rightarrow$$